1. (15 pts) Steady state error for unity feedback (Nise 7.4)
For the system in Fig. 1, let \( G_1(s) = \frac{k}{s^2} \), \( G_2(s) = \frac{s+1}{(s+3)(s+4)} \) and \( H(s) = 1 \). \( D(s) = 0 \). \( E = R - C \).

[3pts] a. What is the system type?
[4pts] b. What is the appropriate static error constant?
[3pts] c. What is the value of the appropriate static error constant?
[5pts] d. What is the steady state error for a unit step input? For a unit ramp input? For \( r(t) = t^2 u(t) \)?

2. (20 pts) Steady state error (Nise 7.8)
[a) Find steady state error for \( r(t) \) a unit step input, using input substitution.
[b) Find steady state error for \( r(t) \) a unit ramp input, using input substitution.

Given system:
\[
\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r, \text{ and } y = [1 \ 0 \ 0]x
\]

3. (15 pts) Steady state error (Nise 7)
Consider a roll-to-roll fabrication system where flat material passes through processing steps at constant velocity. Every roller must have the same tangential velocity to prevent the material tearing. Consider a plant \( G_2(s) = \frac{1}{s^2 + 2s + 1} \) with proportional controller \( G_1(s) = k_p \) which tracks a reference angle \( r(t) = \theta(t) \), where \( r(t) = 100t^2 u(t) \) and \( H(s) = 1 \).

[5pts] a. For the given \( r(t) \), find the steady state error.
[10pts] b. For the given \( r(t) \), find a new \( G_1(s) \) which will have zero steady state error. (Hint: add something else in the controller.)

4. (30 pts) Root locus sketching (Nise 8.6)
For each part below with open loop transfer function \( G(s) \) in unity gain feedback (Fig.2):
[i] i) Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.
[ii] ii) Find \( j\omega \) axis intercepts if any.
[iv] iv) Specify range of \( k \) for stability.
[v] v) Verify your root locus using MATLAB.

a) \( G(s) = \frac{k(s+10)}{(s+5)(s+2)} \)

b) \( G(s) = \frac{k(s+10)}{(s^2+4s+8)(s+20)} \)

c) \( G(s) = \frac{k(s+10)}{(s^2+4s+8)(s^2+8s+20)(s+20)} \)

5. (20 pts) Generalized Root locus (Nise 8.8)
Given the unity gain feedback system in Fig. 2, where
\[
G(s) = \frac{100(s + \alpha)(s + 20)}{s(s + 1)(s + 10)}
\]

[4] a) Determine the characteristic equation for the closed loop system.
[16] b) Sketch the root locus with respect to positive values of \( \alpha \), showing direction in which \( \alpha \) increases on the locus.