Due at 1700, Fri. Feb. 24 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 7,8.

1. (15 pts) Steady state error for unity feedback (Nise 7.4)
For the system in Fig. 1, let $G_1(s) = \frac{k}{s^2}$, $G_2(s) = \frac{s+1}{(s+3)}$ and $H(s) = 1$, $D(s) = 0$. $E = R - C$.
[3pts] a. What is the system type?
[4pts] b. What is the appropriate static error constant?
[3pts] c. What is the value of the appropriate static error constant?
[5pts] d. What is the steady state error for a unit step input? For a unit ramp input? For $r(t) = t^2 u(t)$?

2. (20 pts) Steady state error (Nise 7.8)
[10pts] a) Find steady state error for $r(t)$ a unit step input, using input substitution.
[10pts] b) Find steady state error for $r(t)$ a unit ramp input, using input substitution.
Given system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \text{ , and } \; y = [1 \quad 0 \quad 0]\mathbf{x}$$

3. (15 pts) Steady state error (Nise 7)
Consider a roll-to-roll fabrication system where flat material passes through processing steps at constant velocity. Every roller must have the same tangential velocity to prevent the material tearing. Consider a plant $G_2(s) = \frac{1}{Js^2+Bs}$ with proportional controller $G_1(s) = k_p$ which tracks a reference angle $r(t) = \theta(t)$, where $r(t) = 100tu(t)$ and $H(s) = 1$.
[5pts] a. For the given $r(t)$, find the steady state error.
[10pts] b. For the given $r(t)$, find a new $G_1(s)$ which will have zero steady state error. (Hint: add something else in the controller.)

4. (30 pts) Root locus sketching (Nise 8.6)
For each part below with open loop transfer function $G(s)$ in unity gain feedback (Fig.2):
[5] i) Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.
[2] ii) Find $j\omega$ axis intercepts if any.
[1] iii) Hand sketch root locus.
[1] iv) Specify range of k for stability.
[1] v) Verify your root locus using MATLAB.

a) $G(s) = \frac{k(s+10)}{(s+5)(s+2)}$ b) $G(s) = \frac{k(s+10}{(s^2+4s+8)(s+20)}$ c) $G(s) = \frac{k(s+10)}{(s^2+4s+8)(s^2+8s+20)(s+20)}$

5. (20 pts) Generalized Root locus (Nise 8.8)
Given the unity gain feedback system in Fig. 2, where

$$G(s) = \frac{100(s+\alpha)(s+20)}{s(s+1)(s+10)}$$

[4] a) Determine the characteristic equation for the closed loop system.
[16] b) Sketch the root locus with respect to positive values of α, showing direction in which α increases on the locus.

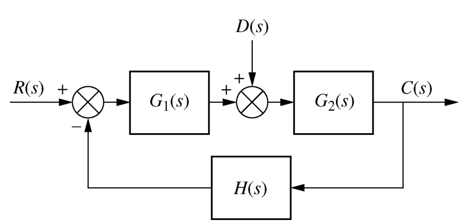

Fig. 1. Control System Block Diagram.

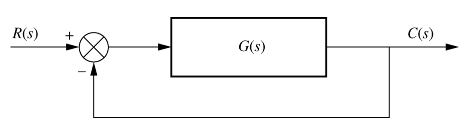

Fig. 2. Unity Gain Feedback.