EECS C128/ ME C134
Midterm (v. 1.01)
Thurs. Oct. 22, 2015
1540 - 1700 pm

Name:_____
SID:_____

- Closed book. One page formula sheet. No calculators.

- There are 5 problems worth 100 points total.

Problem	Points	Score
1	18	
2	19	
3	23	
4	23	
5	17	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

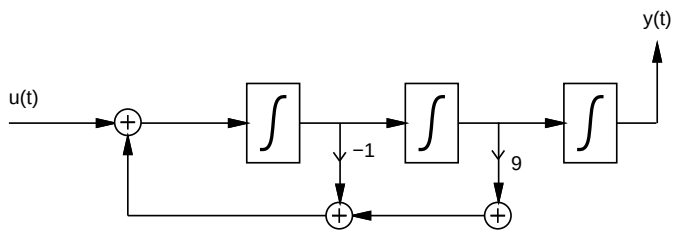
$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 (18 pts)

Each part is independent.

[3 pts] a) Consider a single-input single-output system with input u and output y shown in the block diagram below. Assuming zero initial conditions, find the **differential equation** relating $u(t)$ and $y(t)$.



$\frac{d^3y(t)}{dt^3} =$ _____

[8 pts] b) A system with input u and state x is described by the differential equation

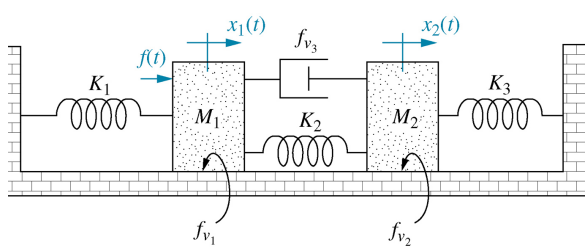
$$\dot{x} = \frac{1}{(x-4)^2} + 10(u-x)^3$$

Linearize the system about $x = 2$, $u = 2$ and express in the form $\dot{\delta}x = A\delta x + B\delta u$.

$A =$ _____

$B =$ _____

[7 pts] c) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to force and current to velocity. Let $f_{vi} = R_i$ for $i = 1, 2, 3$, $\dot{x}_i = i_i$ for $i = 1, 2$, $C_i = \frac{1}{K_i}$, for $i = 1, 2, 3$, $L_1 = M_1$, $L_2 = M_2$.



Problem 2 Steady State Error (19 pts)

[7 pts] a) For the system below, let $H(s) = 1, G_1(s) = \frac{k(s+10)}{s}$, and $G_2(s) = \frac{1}{s+4}$.

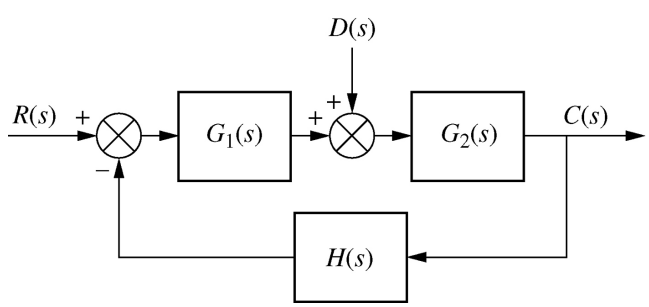
For $d(t) = 0$, and $r(t) = tu(t)$ a unit ramp, determine the static error constant, K_v. $K_v =$ _____

[7 pts] b) For the system below, let $H(s) = 1, G_1(s) = \frac{k(s+10)}{s}$, and $G_2(s) = \frac{1}{s+4}$.

For $d(t) = tu(t)$, a unit ramp, and $r(t) = 0$, find the steady state expression for $c(t)$ for large t.

$c(t) =$ _____

[5 pts] c) For the system below, let $H(s) = \frac{N_H}{D_H}$, $G_1(s) = \frac{N_{G1}}{D_{G1}}$, and $G_2(s) = \frac{N_{G2}}{D_{G2}}$.
Assume $d(t) = 0$
Let $y(t) = r(t) - c(t)$. Find $\frac{Y(s)}{R(s)}$. $\frac{Y(s)}{R(s)} =$ _____



Problem 3. Root Locus Plotting (23 pts)
Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s-1)}{(s^2+2s+2)(s+4)}$$

For the root locus $(1 + kG(s) = 0)$ with $k > 0$:

[1 pts] a) Determine the number of branches of the root locus = ___

[2 pts] b) Determine the locus of poles on the real axis _____

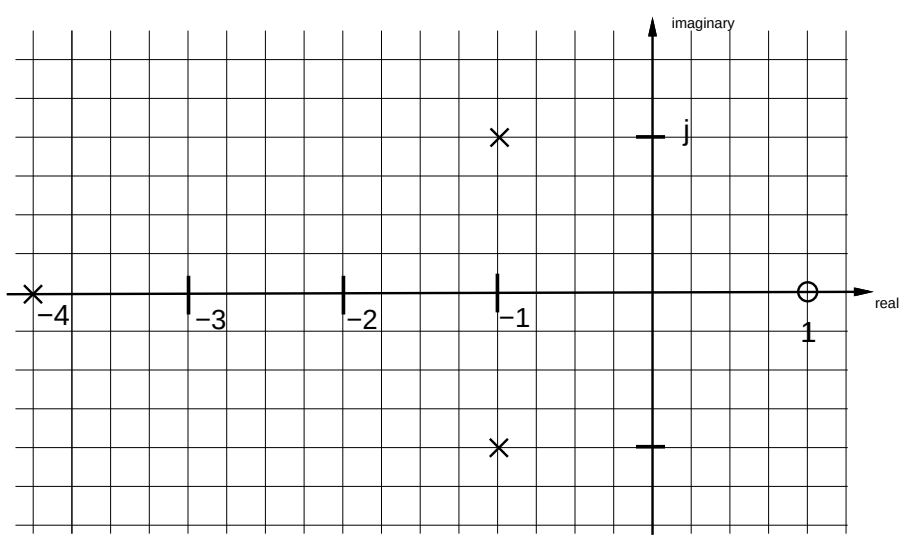
[2 pts] c) Determine the angles for each asymptote:_____

[3 pts] d) determine the real axis intercept for the asymptotes $s =$_____

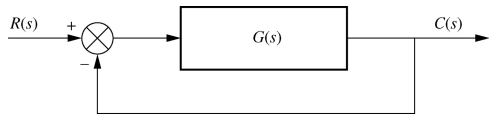
[6 pts] e) Determine the angle of departure for the root locus for the pole at $s = -1 + j =$_____

[5 pts] f) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)

[4 pts] g) Mark the point on the root locus where the system is first unstable. Estimate the minimum value of $k > 0$ for which the closed loop system would be unstable. $k =$ _____



Problem 4. Root Locus Compensation (23 pts)



Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{1}{(s+2)^3(s+2+5\sqrt{3})}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a lead compensation of the form $G_1(s) = k\frac{s+z_c}{s+p_c}$.

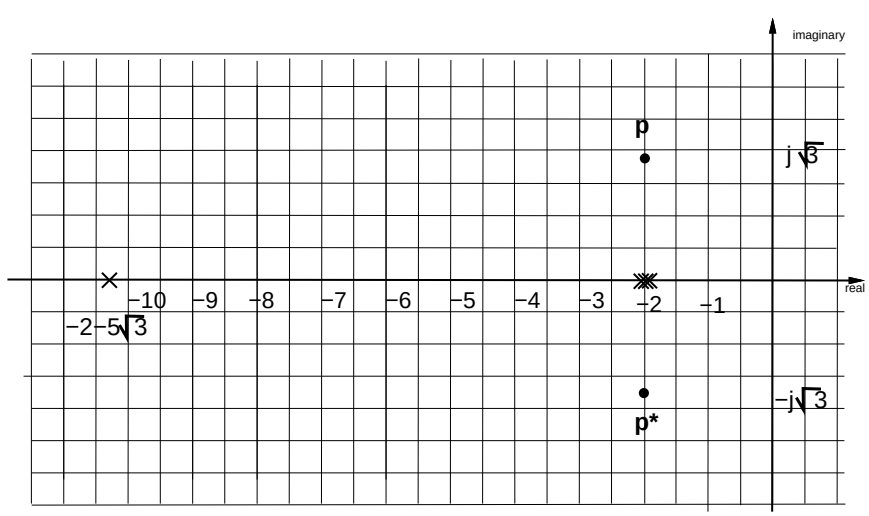
The closed loop system, using unity gain feedback and the lead controller, should have a pair of poles at $p = -2 \pm j\sqrt{3}$.

[4 pts] a. Show that for closed loop poles p, that the angle contribution contribution from $G_3(p)$ is $\approx -280°$.

[9 pts] b. Find a lead network pole p_c and zero location z_c such that p is approximately on the root locus, within $\pm$ 10 degrees.

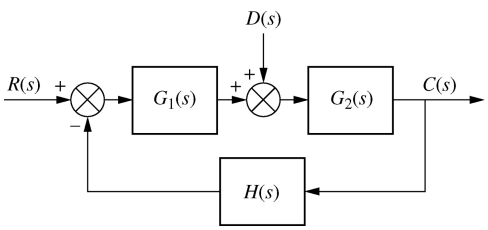
	value	angle
zero z_c		°
pole p_c		°
total	—	°

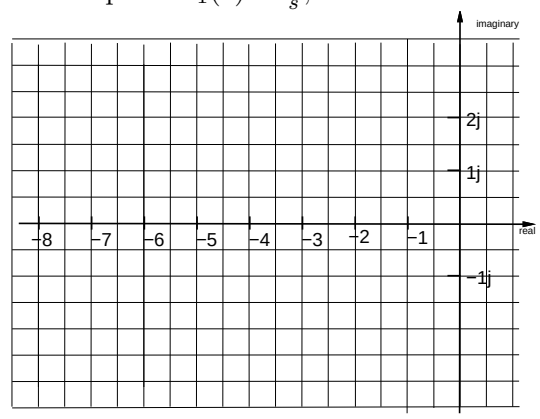
[10 pts] c. For the determined lead network, sketch the root locus, considering real-axis segments, real-axis asymptote intercept, asymptote angles, and locus near p, p^*.



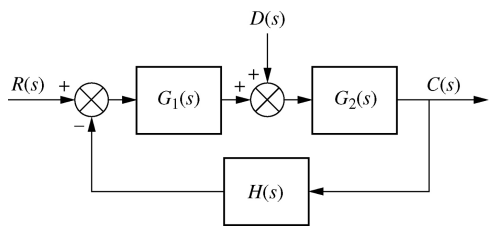
Problem 5. Non-unity gain Compensation (17 pts)

[2 pts] a. Let $D(s) = 0$ (no disturbance). Given plant with open loop transfer function $G_2(s) = \frac{1}{s+4}$. With $H(s) = 1$ and compensator in the feedforward path $G_1(s) = \frac{k}{s}$, sketch the root locus below.





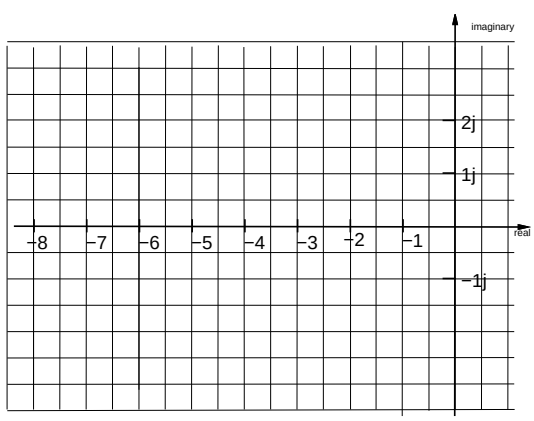
Let $D(s) = 0$, $G_1(s) = k$, $G_2(s) = \frac{1}{s+4}$, and controller in the feedback path $H(s) = \frac{1}{s}$.



[4 pts] b. Determine the closed loop transfer function for this system $T(s)$:

$T(s) = \frac{C(s)}{R(s)} \quad = \quad$ _____

[6 pts] c. Sketch the root locus for $T(s)$ for $0 \leq k < \infty$.



[5 pts] d. Compare the step responses for the systems in part a) and part b). Which controller can better track a step input? Briefly explain why in the space below.

page for scratch work