In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

\[
\begin{align*}
\tan^{-1} \frac{4}{1} &= 5.7^\circ \\
\tan^{-1} \frac{1}{3} &= 11.3^\circ \\
\tan^{-1} \frac{7}{4} &= 14^\circ \\
\tan^{-1} \frac{1}{\sqrt{3}} &= 18.4^\circ \\
\tan^{-1} \frac{11}{2} &= 26.6^\circ \\
\tan^{-1} \frac{1}{\sqrt{3}} &= 30^\circ \\
\tan^{-1} 1 &= 45^\circ \\
\tan^{-1} \sqrt{3} &= 60^\circ \\
\sin 30^\circ &= \frac{1}{2} \\
\cos 30^\circ &= \frac{\sqrt{3}}{2}
\end{align*}
\]

\[
\begin{align*}
20 \log_{10} 1 &= 0 dB \\
20 \log_{10} 2 &= 6 dB \\
\pi &\approx 3.14 \\
20 \log_{10} \sqrt{2} &= 3 dB \\
20 \log_{10} \frac{1}{2} &= -6 dB \\
2\pi &\approx 6.28 \\
20 \log_{10} 5 &= 20 dB - 6 dB = 14 dB \\
20 \log_{10} \sqrt{10} &= 10 dB \\
\pi/2 &\approx 1.57 \\
\frac{1}{e} &\approx 0.37 \\
\sqrt{10} &\approx 3.164 \\
\pi/4 &\approx 0.79 \\
\frac{1}{e^2} &\approx 0.14 \\
\sqrt{2} &\approx 1.41 \\
\sqrt{3} &\approx 1.73 \\
\frac{1}{e^3} &\approx 0.05 \\
\frac{1}{\sqrt{2}} &\approx 0.71 \\
\frac{1}{\sqrt{3}} &\approx 0.58
\end{align*}
\]
Problem 1 (18 pts)

Each part is independent.

[3 pts] a) Consider a single-input single-output system with input $u$ and output $y$ shown in the block diagram below. Assuming zero initial conditions, find the differential equation relating $u(t)$ and $y(t)$.

$$\frac{d^3 y(t)}{d t^3} = $$

[8 pts] b) A system with input $u$ and state $x$ is described by the differential equation

$$\dot{x} = \frac{1}{(x - 4)^2} + 10(u - x)^3$$

Linearize the system about $x = 2, u = 2$ and express in the form $\dot{x} = A\delta x + B\delta u$.

$A =$

$B =$

[7 pts] c) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to force and current to velocity. Let $f_i = R_i$ for $i = 1, 2, 3$, $\dot{x}_i = i_i$ for $i = 1, 2$, $C_i = \frac{1}{K_i}$, for $i = 1, 2, 3$, $L_1 = M_1, L_2 = M_2$. 

![Diagram of equivalent electrical circuit]
Problem 2 Steady State Error (19 pts)

[7 pts] a) For the system below, let \( H(s) = 1 \), \( G_1(s) = \frac{k(s+10)}{s} \), and \( G_2(s) = \frac{1}{s+4} \).

For \( d(t) = 0 \), and \( r(t) = tu(t) \) a unit ramp, determine the static error constant, \( K_v \). \( K_v = \)______

[7 pts] b) For the system below, let \( H(s) = 1 \), \( G_1(s) = \frac{k(s+10)}{s} \), and \( G_2(s) = \frac{1}{s+4} \).

For \( d(t) = tu(t) \), a unit ramp, and \( r(t) = 0 \), find the steady state expression for \( c(t) \) for large \( t \).

\( c(t) = \)______

[5 pts] c) For the system below, let \( H(s) = \frac{N_H}{D_H} \), \( G_1(s) = \frac{N_{G1}}{D_{G1}} \), and \( G_2(s) = \frac{N_{G2}}{D_{G2}} \).

Assume \( d(t) = 0 \)

Let \( y(t) = r(t) - c(t) \). Find \( \frac{Y(s)}{R(s)} \).

\( \frac{Y(s)}{R(s)} = \)______

\[ \text{Diagram of the system} \]
Problem 3. Root Locus Plotting (23 pts)

Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s - 1)}{(s^2 + 2s + 2)(s + 4)}$$

For the root locus $(1 + kG(s) = 0)$ with $k > 0$:

[1 pts] a) Determine the number of branches of the root locus = __________

[2 pts] b) Determine the locus of poles on the real axis ________________

[2 pts] c) Determine the angles for each asymptote:____________________

[3 pts] d) determine the real axis intercept for the asymptotes $s =$ ______

[6 pts] e) Determine the angle of departure for the root locus for the pole at $s = -1 + j =$ ______

[5 pts] f) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)

[4 pts] g) Mark the point on the root locus where the system is first unstable. Estimate the minimum value of $k > 0$ for which the closed loop system would be unstable. $k =$ ____
Problem 4. Root Locus Compensation (23 pts)

Given open loop transfer function \( G(s) \):

\[
G(s) = G_1(s)G_3(s) = G_1(s)\frac{1}{(s + 2)^3(s + 2 + 5\sqrt{3})}
\]

where \( G_3(s) \) is the open-loop plant, and \( G_1(s) \) is a lead compensation of the form \( G_1(s) = k\frac{s+z_c}{s+p_c} \).

The closed loop system, using unity gain feedback and the lead controller, should have a pair of poles at \( p = -2 \pm j\sqrt{3} \).

[4 pts] a. Show that for closed loop poles \( p \), that the angle contribution contribution from \( G_3(p) \) is \( \approx -280^\circ \).

[9 pts] b. Find a lead network pole \( p_c \) and zero location \( z_c \) such that \( p \) is approximately on the root locus, within \( \pm 10 \) degrees.

<table>
<thead>
<tr>
<th></th>
<th>value</th>
<th>angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero ( z_c )</td>
<td></td>
<td>( ^\circ )</td>
</tr>
<tr>
<td>pole ( p_c )</td>
<td></td>
<td>( ^\circ )</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>( ^\circ )</td>
</tr>
</tbody>
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[10 pts] c. For the determined lead network, sketch the root locus, considering real-axis segments, real-axis asymptote intercept, asymptote angles, and locus near \( p, p^* \).
Problem 5. Non-unity gain Compensation (17 pts)

[2 pts] a. Let $D(s) = 0$ (no disturbance). Given plant with open loop transfer function $G_2(s) = \frac{1}{s+4}$.
With $H(s) = 1$ and compensator in the feedforward path $G_1(s) = \frac{k}{s}$, sketch the root locus below.

Let $D(s) = 0$, $G_1(s) = k$, $G_2(s) = \frac{1}{s+4}$, and controller in the feedback path $H(s) = \frac{1}{s}$.

[4 pts] b. Determine the closed loop transfer function for this system $T(s)$:

$$T(s) = \frac{C(s)}{R(s)} =$$

[6 pts] c. Sketch the root locus for $T(s)$ for $0 \leq k < \infty$.

[5 pts] d. Compare the step responses for the systems in part a) and part b). Which controller can better track a step input? Briefly explain why in the space below.
page for scratch work