Announcements

• HW9 is due now.
Question: how does this curve look for reverse bias?
PN Junction Review: Current Profile

\[ J_{\text{total}} \]

\[ J_{pN} \quad J_{nP} \]

N-side 0 P-side

\[ J_{\text{total}} \]

\[ J_{nN} \quad J_{pP} \]

N-side 0 P-side

x
Qualitative Solution - Definitions

\[ N_E = N_{AE} \]
\[ D_E = D_N \]
\[ \tau_E = \tau_n \]
\[ L_E = L_N \]
\[ n_{E0} = n_{p0} = n_i^2/N_E \]

\[ N_B = N_{DB} \]
\[ D_B = D_P \]
\[ \tau_B = \tau_p \]
\[ L_B = L_P \]
\[ p_{B0} = p_{n0} = n_i^2/N_B \]

\[ N_C = N_{AC} \]
\[ D_C = D_N \]
\[ \tau_C = \tau_n \]
\[ L_C = L_N \]
\[ n_{C0} = n_{p0} = n_i^2/N_C \]
Emitter Region Formulation

- Diffusion equation:
  \[ 0 = D_E \frac{d^2 \Delta n_E}{dx'^2} - \frac{\Delta n_E}{\tau_E} \]
- Boundary Conditions
  \[ \Delta n_E (x'' \to \infty) = 0 \]
  \[ \Delta n_E (x'' = 0) = n_{E0} (e^{qV_{EB}/kT} - 1) \]
Base Region Formulation

- Diffusion equation:
  \[ 0 = D_B \frac{d^2 \Delta n_B}{dx^2} - \frac{\Delta p_B}{\tau_B} \]

- Boundary Conditions
  \[ \Delta p_B(0) = p_{B0} \left( e^{qV_{EB}/kT} - 1 \right) \]
  \[ \Delta p_B(W) = p_{B0} \left( e^{qV_{CB}/kT} - 1 \right) \]
Collector Region Formulation

- Diffusion equation:
  \[ 0 = D_C \frac{d^2 \Delta n_C}{dx^2} - \frac{\Delta n_C}{\tau_C} \]

- Boundary Conditions
  \[ \Delta n_C(x' \to \infty) = 0 \]
  \[ \Delta n_C(x' = 0) = n_{C0} \left( e^{qV_{CB}/kT} - 1 \right) \]
Current Formulation

\[ I_{En} = -qAD_E \frac{d\Delta n_E}{dx''} \bigg|_{x''=0} \]

\[ I_{Ep} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=0} \]

\[ I_{Cp} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=W} \]

\[ I_{Cn} = qAD_C \frac{d\Delta n_C}{dx'} \bigg|_{x'=0} \]
Emitter Region Solution

• The solution of: \( 0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E} \)

is:
\[ \Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E} \]

• From the boundary conditions:
\[ \Delta n_E(x'' \to \infty) = 0 \]
\[ \Delta n_E(x'' = 0) = n_{E0}(e^{qV_{EB}/kT} - 1) \]

we have:
\[ \Delta n_E(x'') = n_{E0}(e^{qV_{EB}/kT} - 1)e^{-x''/L_E} \]

and:
\[ I_{En} = -qA \frac{D_E}{L_E} n_{E0}(e^{qV_{EB}/kT} - 1) \]
Collector Region Solution

• The solution of: \[ 0 = D_C \frac{d^2 \Delta n_C}{dx^2} - \frac{\Delta n_C}{\tau_C} \]
is:
  \[ \Delta n_C(x') = A_1 e^{-x'/L_C} + A_2 e^{x'/L_C} \]

• From the boundary conditions:
  \[ \Delta n_C(x' \to \infty) = 0 \]
  \[ \Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1) \]
we have:
  \[ \Delta n_C(x') = n_{C0}(e^{qV_{CB}/kT} - 1)e^{-x'/L_C} \]
and:
  \[ I_{Cn} = -qA \frac{D_C}{L_C} n_{C0}(e^{qV_{CB}/kT} - 1) \]
**Base Region Solution**

- The solution of: \( 0 = D_B \frac{d^2 \Delta n_B}{dx^2} - \frac{\Delta p_B}{\tau_B} \)

  is:

  \[
  \Delta p_B(x) = A_1 e^{-x/L_B} + A_2 e^{x/L_B}
  \]

- From the boundary conditions:

  \[
  \Delta p_B(0) = p_{B0} \left( e^{qV_{EB}/kT} - 1 \right)
  \]

  \[
  \Delta p_B(W) = p_{B0} \left( e^{qV_{CB}/kT} - 1 \right)
  \]

  we have:

  \[
  \Delta p_B(x) = p_{B0} \left( e^{qV_{EB}/kT} - 1 \right) \left( \frac{e^{(W-x)/L_B} - e^{-(w-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}} \right)
  \]

  \[
  + p_{B0} \left( e^{qV_{CB}/kT} - 1 \right) \left( \frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}} \right)
  \]
Base Region Solution (cont’d)

• Now, we know: \( \sinh(\xi) = \frac{e^\xi - e^{-\xi}}{2} \)

• Therefore, we can write:

\[
\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\frac{e^{(W-x)/L_B} - e^{-(w-x)/L_B}}{e^W/L_B - e^{-W/L_B}}\right)
+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^W/L_B - e^{-W/L_B}}\right)
\]

as:

\[
\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\frac{\sinh\left(\frac{(W-x)/L_B}{L_B}\right)}{\sinh\left(\frac{W}{L_B}\right)}
+ p_{B0}(e^{qV_{CB}/kT} - 1)\frac{\sinh\left(\frac{x/L_B}{L_B}\right)}{\sinh\left(\frac{W}{L_B}\right)}
\]
Base Region Solution (cont’d)

- Now, we know: \[ \cosh(\xi) = \frac{e^\xi + e^{-\xi}}{2} \]
- Therefore, we have:

\[
I_{Ep} = qA \frac{D_B}{L_B} p_{B0} \left[ \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{1}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right]
\]

and:

\[
I_{Cp} = qA \frac{D_B}{L_B} p_{B0} \left[ \frac{1}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right]
\]
Terminal Currents

- We know:

\[ I_{En} = -qA \frac{D_E}{L_E} n_{E0} (e^{qV_{EB}/kT} - 1) \]

\[ I_{Ep} = qA \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} (e^{qV_{EB}/kT} - 1) - \frac{1}{\sinh(W/L_B)} (e^{qV_{CB}/kT} - 1) \]

\[ I_{Cp} = qA \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} (e^{qV_{EB}/kT} - 1) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)} (e^{qV_{CB}/kT} - 1) \]

\[ I_{Cn} = -qA \frac{D_C}{L_C} n_{C0} (e^{qV_{CB}/kT} - 1) \]

- Therefore:

\[ I_E = qA \left[ \left( \frac{D_E}{L_E} n_{E0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) - \left( \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{CB}/kT} - 1) \right] \]

\[ I_C = qA \left[ \left( \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) - \left( \frac{D_C}{L_C} n_{C0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{CB}/kT} - 1) \right] \]

Question: What is I_B?
Simplification

• In real BJTs, we make $W \ll L_B$ so that we have a high gain. Then, we note:
  \[
  \sinh(\xi) \rightarrow \xi \cdots \xi \ll 1 \\
  \cosh(\xi) \rightarrow 1 + \frac{\xi^2}{2} \cdots \xi \ll 1
  \]

• So, we have:
  \[
  \Delta p_B(x) \approx p_{B0} \left( e^{qV_{EB}/kT} - 1 \right) \left( 1 - \frac{x}{W} \right) \\
  + p_{B0} \left( e^{qV_{CB}/kT} - 1 \right) \left( \frac{x}{W} \right)
  \]
Simplified Analysis

Consider the carrier distribution in a forward active pnp transistor

Question: why does the carrier concentration show linear dependence in the base?
Question:

• Plot carrier distribution profile for a BJT under cut-off.
Performance Parameters

\[ \gamma = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E}} \]

\[ \alpha_T = \frac{1}{1 + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]

\[ \alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]

\[ \beta_{dc} = \frac{1}{\frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]
**Base width modulation**

When the reverse bias applied to the C-B junction increases, the C-B depletion width increases and W decreases. Thus, the collector current, $I_C$ increases. This is also known as “Early Effect”. More prominent in narrow-base transistors.

![Graph showing base width modulation]

- $I_C$, $V_{EC}$
- Levels: 3 mA, 2 mA, 1 mA
Output resistance:

\[ r_0 \equiv \left( \frac{\partial I_C}{\partial V_{CE}} \right)^{-1} = \frac{V_A}{I_C} \]

A large \( V_A \) (i.e. a larger \( r_o \)) is desirable for voltage gain.
How can we reduce the base-width modulation effect?

Base-Width Modulation by Collector Voltage

(N+     P     N)
emitter    base    collector

$V_{CE}$

$V_{BE}$

$W_{B1}$
$W_{B2}$
$W_{B3}$

reduction of base width

$V_{CE1} < V_{CE2} < V_{CE3}$

(Depletion region in collector is not shown)

$V_{CE}$

$V_{BE}$

$n'$

$x$
The base-width modulation effect is reduced if we

(A) Increase the base width,
(B) Increase the base doping concentration, $N_B$, or
(C) Decrease the collector doping concentration, $N_C$.

Which of the above is the most acceptable action?