EECS130
Integrated Circuit Devices
Professor Ali Javey
9/20/2007

PN Junctions
Lecture 2

Reading: Chapter 5
Announcements

• For THIS WEEK ONLY, Prof. Javey's office hours will be held on Tuesday, Sept 18 3:30-4:30 pm and Wednesday, Sept 19 5-6 pm.

• Exam 1 (Oct 4) will cover Semiconductor Fundamentals, Fabrication, and PN junctions.

• HW2 is due…… right now.

• HW3 is now posted on the web.

• You can check your grades on bspace.

• Check out the group discussion board.
Review: Qualitative Electrostatics

Band diagram

Built in-potential

From $\varepsilon = -\frac{dV}{dx}$
Review: Poisson’s Equation

\[ \frac{d^2 V}{dx^2} = -\frac{d \varepsilon}{dx} = -\frac{\rho}{\varepsilon_s} \]

What can Poisson’s equation tell us?

\( \varepsilon_s \): semiconductor permittivity (~12\( \varepsilon_o \)) for Si
\( \rho \): charge density (C/cm\(^3\))
Reverse-Biased PN Junction

(a) Equilibrium \((V_A = 0)\)

(c) Reverse bias \((V_A < 0)\)
Forward Biased PN Junction

(a) Equilibrium ($V_A = 0$)

$E_c$

$E_F$

$E_i$

$E_v$

$I_N$  

$I_F$
Effect of Bias on Electrostatics

Note: simply replace $V_{bi}$ with $V_{bi}-V_A$
Capacitance-Voltage Characteristics

- Is $C_{dep}$ a good thing?
- What are three ways to reduce $C_{dep}$?
Capacitance-Voltage Characteristics

\[ \frac{1}{C_{dep}^2} = \frac{W_{dep}^2}{A^2 \varepsilon_s^2} = \frac{2(\phi_{bi} + V)}{qN\varepsilon_s A^2} \]

Slope = \( \frac{2}{qN\varepsilon_s A^2} \)

Capacitance data

Increasing reverse bias
A Zener diode is designed to operate in the breakdown mode.
**Reverse Bias Junction Breakdown**

A large reverse current flows when the voltage exceeds certain value. Not destructive unless power dissipation causes excessive heating.

For a p⁺n or pn⁺ diode: \( V_{BR} \propto N^{-1} \)

where \( V_{BR} \) is the breakdown voltage and \( N \) is the (bulk) doping on the lightly doped side.

Two processes:
- **Avalanching:** Dominant process in lightly doped diodes
- **Zener process:** More important in heavily doped diodes
Peak Electric Field

$$\left| \int \mathcal{E} \, dx \right| = \frac{1}{2} |\mathcal{E}(0)| W = V_{bi} - V_A$$

- For a one-sided junction: 
  $$W \approx \sqrt{\frac{2 \varepsilon_s}{qN} \left( V_{bi} - V_A \right)}$$

Therefore

$$\mathcal{E}(0) = \frac{2(V_{bi} - V_A)}{W} \approx \sqrt{\frac{2qN(V_{bi} - V_A)}{\varepsilon_s}}$$
• If $V_{\text{reverse}} = -V_A$ is so large such that the peak electric field exceeds a critical value $\mathcal{E}_{\text{crit}}$, then the junction will break down (large reverse current will flow)

$$\mathcal{E}_{\text{crit}} = \sqrt{\frac{2qN(V_{bi} + V_{BR})}{\varepsilon_s}}$$

• Thus, the reverse bias at which breakdown occurs is

$$V_{BR} = \frac{\varepsilon_s \mathcal{E}_{\text{crit}}^2}{2qN} - V_{bi}$$
Peak Electric Field

\[ \varepsilon_p = \varepsilon(0) = \left[ \frac{2qN}{\varepsilon_s} (\phi_{bi} + |V_r|) \right]^{1/2} \]
Quantum Mechanical Tunneling

Potential energy barrier

$E$

d
$x$
Tunneling Breakdown

Dominant breakdown cause when both sides of a junction are very heavily doped.

\[ V_B = \frac{\varepsilon_s \varepsilon_{crit}^2}{2qN} - \phi_{bi} \]

\[ \varepsilon_p = \varepsilon_{crit} \approx 10^6 \text{ V/cm} \]
Avalanche Breakdown

impact ionization

avalanche breakdown

\[ V_B = \frac{\varepsilon_s \varepsilon_{\text{crit}}}{2qN} \]

\[ V_B \propto \frac{1}{N} = \frac{1}{N_a} + \frac{1}{N_d} \]
Empirical Observations of $V_{BR}$

- $V_{BR}$ decreases with increasing $N$
- $V_{BR}$ decreases with decreasing $E_G$
Question:

What happens to $V_{BR}$ of the avalanche breakdown as $T$ is increased?
Carrier Concentrations at Equilibrium

\[ p_{p0} \quad n_{n0} \]

\[ p_{p0}(-x_p) = N_A \]
\[ n_{p0}(-x_p) = \frac{n_i^2}{N_A} \]
\[ p_{n0}(x_n) = \frac{n_i^2}{N_D} \]

\[ n_{n0}(x_n) = N_D \]
When an external voltage is applied, the minority carrier concentration at the edge of the depletion layer will change. If a forward voltage ($V_A$=positive) is applied, the barrier will be lower and carrier injection (diffusion part) will increase. The minority carrier concentration at the edge of the depletion layer will increase.

If a reverse voltage ($V_A$ = negative) is applied, the barrier for carrier injection (diffusion part) will increase, and the minority carrier concentration at the edge of the depletion layer will decrease.

The drift of minority carriers across the junction does not change much with applied voltage. Why?

At $V_A = 0$, the carrier injection and the drift of minority carriers cancel each other such that an equilibrium conc. is maintained.

If “low-level-injection” condition is assumed, then the majority carrier concentration will not change under any of the above conditions.
Carrier concentration profile under bias

\[ p_n = p_{n0} + \Delta p_n(x) \]

\[ n_p = n_{p0} + \Delta n_p(x) \]
The Junction Law

pn product at the edge of the depletion when junction is biased:

\[ pn = n_i^2 e^{qV_A / kT} \]
Excess Carrier Concentrations at $-x_p$, $x_n$

**p-side**

\[
\begin{align*}
p_p(-x_p) &= N_A \\
n_p(-x_p) &= \frac{n_i^2 e^{qV_A/kT}}{N_A} \\
 &= n_{p0} e^{qV_A/kT}
\end{align*}
\]

\[
\Delta n_p(-x_p) = \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right)
\]

**n-side**

\[
\begin{align*}
n_n(x_n) &= N_D \\
p_n(x_n) &= \frac{n_i^2 e^{qV_A/kT}}{N_D} \\
 &= p_{n0} e^{qV_A/kT}
\end{align*}
\]

\[
\Delta p_n(x_n) = \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right)
\]
EXAMPLE: Carrier Injection

A PN junction has \( N_a = 10^{19}\text{cm}^{-3} \) and \( N_d = 10^{16}\text{cm}^{-3} \). The applied voltage is 0.6 V.

**Question:** What are the minority carrier concentrations at the depletion-region edges?

**Solution:**

\[
n(0) = n_{p0} e^{qV/kT} = 10 \times e^{0.6/0.026} = 10^{11}\text{ cm}^{-3}
\]

\[
p(0) = p_{N0} e^{qV/kT} = 10^4 \times e^{0.6/0.026} = 10^{14}\text{ cm}^{-3}
\]

**Question:** What are the excess minority carrier concentrations?

**Solution:**

\[
n'(0) = n(0) - n_{p0} = 10^{11} - 10 = 10^{11}\text{ cm}^{-3}
\]

\[
p'(0) = p(0) - p_{N0} = 10^{14} - 10^4 = 10^{14}\text{ cm}^{-3}
\]
Current density in PN junctions

• Remember the minority carrier diffusion equation from chapter 3?

\[
\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L
\]

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Steady state: \( \frac{\partial \Delta n}{\partial t} = 0 \)  
Only thermal/direct R-G: \( G_L = 0 \)
Excess Carrier Distribution

- From the minority carrier diffusion equation:
  \[
  \frac{d^2 \Delta p_n}{dx^2} = \frac{\Delta p_n}{D_p \tau_p} = \frac{\Delta p_n}{L_p^2}
  \]

- We have the following boundary conditions:
  \[
  \Delta p_n(x_n) = p_{no}(e^{qV_A/kT} - 1) \quad \Delta p_n(\infty) \to 0
  \]

- For simplicity, we will develop a new coordinate system:

NEW: \hspace{1cm} 0 \hspace{1cm} \rightarrow x' \hspace{1cm} 0 \hspace{1cm} \rightarrow x''

- Then, the solution is of the form:
  \[
  \Delta p_n(x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}
  \]
\[ \Delta p_n (x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p} \]

From the \( x = \infty \) boundary condition, \( A_1 = 0 \).
From the \( x = x_n \) boundary condition, \( A_2 = p_{no} (e^{qV_A/kT} - 1) \)

Therefore, \( \Delta p_n (x') = p_{no} (e^{qV_A/kT} - 1) e^{-x'/L_p}, \ x' > 0 \)

Similarly, we can derive
\[ \Delta n_p (x'') = n_{po} (e^{qV_A/kT} - 1) e^{-x''/L_n}, \ x'' > 0 \]
• Current density $J = J_n(x) + J_p(x)$

$$J_n(x) = q\mu_n n \mathcal{E} + qD_n \frac{dn}{dx} = q\mu_n n \mathcal{E} + qD_n \frac{d(\Delta n)}{dx}$$

$$J_p(x) = q\mu_p p \mathcal{E} - qD_p \frac{dp}{dx} = q\mu_p p \mathcal{E} - qD_p \frac{d(\Delta p)}{dx}$$

• $J$ is constant throughout the diode, but $J_n(x)$ and $J_p(x)$ vary with position
pn Diode I-V Characteristic

**p-side:** \[ J_n = -qD_n \frac{d\Delta n_p(x'')}{dx''} = q \frac{D_n}{L_n} n_{p0} (e^{qV_A/kT} - 1)e^{-x''/L_n} \]

**n-side:** \[ J_p = -qD_p \frac{d\Delta p_n(x')}{dx'} = q \frac{D_p}{L_p} p_{n0} (e^{qV_A/kT} - 1)e^{-x'/L_p} \]

\[ J = J_n \bigg|_{x=-x_p} + J_p \bigg|_{x=x_n} = J_n \bigg|_{x'=0} + J_p \bigg|_{x'=0} \]

\[ J = qn_i^2 \left[ \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right] (e^{qV_A/kT} - 1) \]

**Assumption:** No R-G in the depletion layer: \( J_n \) and \( J_p \) are constant in depletion layer