Read: Chapters 1 and 2
Energy band diagram shows the bottom edge of conduction band, $E_c$, and top edge of valence band, $E_v$.

$E_c$ and $E_v$ are separated by the band gap energy, $E_g$. 
Temperature Effect on BandGap

How does the band gap change with temperature?
Measuring the Band Gap Energy by Light Absorption

- $E_g$ can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

<table>
<thead>
<tr>
<th>Material</th>
<th>PbTe</th>
<th>Ge</th>
<th>Si</th>
<th>GaAs</th>
<th>GaP</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_g$ (eV)</td>
<td>0.31</td>
<td>0.67</td>
<td>1.12</td>
<td>1.42</td>
<td>2.25</td>
<td>6.0</td>
</tr>
</tbody>
</table>
• Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
• Metal conduction band is half-filled.
• Semiconductors have lower $E_g$'s than insulators and can be doped.
Donor and Acceptor Levels in the Band Model

Conduction Band

Donor Level

Donor ionization energy

Acceptance Level

Acceptor ionization energy

Valence Band

Ionization energy of selected donors and acceptors in silicon

<table>
<thead>
<tr>
<th>Dopant</th>
<th>Donors</th>
<th>Acceptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sb</td>
<td>39</td>
<td>45</td>
</tr>
<tr>
<td>P</td>
<td>44</td>
<td>54</td>
</tr>
<tr>
<td>As</td>
<td>54</td>
<td>45</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>57</td>
</tr>
<tr>
<td>Al</td>
<td></td>
<td>160</td>
</tr>
<tr>
<td>In</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ionization energy, \( E_c - E_d \) or \( E_a - E_v \) (meV)

\[
E_{ion} = \frac{m_0 q^4}{8 \varepsilon_0^2 h^2} = 13.6 \text{ eV}
\]
Dopants and Free Carriers

Donors
- n-type
  - $T \rightarrow 0 \text{ K}$
  - Increasing $T$
  - Room temperature

Acceptors
- p-type
  - $T \rightarrow 0 \text{ K}$
  - Increasing $T$
  - Room temperature

Dopant ionization energy $\sim 50\text{meV}$ (very low).
**Effective Mass**

In an electric field, $E$, an electron or a hole accelerates.

$$a = \frac{-qE}{m_n}$$  \hspace{1cm} \text{electrons}

$$a = \frac{qE}{m_p}$$  \hspace{1cm} \text{holes}

Remember: $F = ma = -qE$

<table>
<thead>
<tr>
<th>Electron and hole effective masses</th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
<th>GaP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_n/m_0$</td>
<td>0.26</td>
<td>0.12</td>
<td>0.068</td>
<td>0.82</td>
</tr>
<tr>
<td>$m_p/m_0$</td>
<td>0.39</td>
<td>0.30</td>
<td>0.50</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Density of States

\[ g_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \left( \frac{1}{\text{eV} \cdot \text{cm}^3} \right) \]

\[ g_c(E) \equiv \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \]

\[ g_v(E) \equiv \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \]
Thermal Equilibrium

- No external forces applied:
  - electric field = 0
  - magnetic field = 0
  - mechanical stress = 0

- Thermal agitation \(\rightarrow\) electrons and holes exchange energy with the crystal lattice and each other

  \[\Rightarrow\text{Every energy state in the conduction and valence bands has a certain probability of being occupied by an electron}\]
There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy (vibrating atoms, etc.)
Fermi Function

Probability that an available state at energy $E$ is occupied:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$E_F$ is called the **Fermi energy** or the **Fermi level**

There is only one Fermi level in a system at equilibrium.

At $E=E_F$, $f(E)=1/2$
Boltzmann Approximation

If $E - E_F > 3kT$, $f(E) \approx e^{-(E-E_F)/kT}$

If $E_F - E > 3kT$, $f(E) \approx 1 - e^{E-E_F/kT}$

Assume the two extremes:

High Energy (Large E): $E-E_f \gg kT$, $f(E) \to 0$
Low Energy (Small E): $E-E_f \ll kT$, $f(E) \to 1$
Effect of T on f(E)

![Graph showing the effect of temperature on the function f(E). The graph includes curves for T=0K, T=100K, and T=400K.](image)

- **T=0K**: The curve is the highest and steeply decreases with increasing E-EF(eV).
- **T=100K**: The curve is slightly lower than T=0K and less steep.
- **T=400K**: The curve is the lowest and the least steep of the three.
Question

- If $f(E)$ is the probability of a state being occupied by an electron, what is the probability of a state being occupied by a hole?
Equilibrium Distribution of Carriers

- Obtain \( n(E) \) by multiplying \( g_c(E) \) and \( f(E) \)
- Obtain \( p(E) \) by multiplying \( g_v(E) \) and \( 1-f(E) \)

\( n \) = electron density : number of unbound electrons / cm\(^3\)
\( p \) = hole density : number of holes / cm\(^3\)
**Equilibrium Carrier Concentrations**

- Integrate $n(E)$ over all the energies in the conduction band to obtain $n$

\[ n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E) dE \]

- By using the Boltzmann approximation, and extending the integration limit to $\infty$, we obtain

\[ n = N_c e^{-(E_c - E_F)/kT} \text{ where } N_c = 2 \left( \frac{2\pi m^*_{n} k T}{\hbar^2} \right)^{3/2} \]

$N_c$ is called the **effective density of states** *(of the conduction band).*
Integrate $p(E)$ over all the energies in the valence band to obtain $p$

$$p = \int_{\text{bottom of valence band}}^{E_v} g_v(E)[1 - f(E)]dE$$

By using the Boltzmann approximation, and extending the integration limit to $-\infty$, we obtain

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{\hbar^2}\right)^{3/2}$$

$N_v$ is called the effective density of states of the valence band.
Intrinsic Semiconductor

- Extremely pure semiconductor sample containing an insignificant amount of impurity atoms.

\[ n = p = n_i \]

\( E_f \) lies in the middle of the band gap

<table>
<thead>
<tr>
<th>Material</th>
<th>Ge</th>
<th>Si</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_g ) (eV)</td>
<td>0.67</td>
<td>1.12</td>
<td>1.42</td>
</tr>
<tr>
<td>( n_i ) (1/cm³)</td>
<td>( 2 \times 10^{13} )</td>
<td>( 1 \times 10^{10} )</td>
<td>( 2 \times 10^6 )</td>
</tr>
</tbody>
</table>
n-type

intrinsic
Remember: the closer $E_f$ moves up to $E_c$, the larger $n$ is; the closer $E_f$ moves down to $E_v$, the larger $p$ is.
For Si, $N_c = 2.8 \times 10^{19} \text{cm}^{-3}$ and $N_v = 1.04 \times 10^{19} \text{cm}^{-3}$.
Example: The Fermi Level and Carrier Concentrations

Where is $E_f$ for $n = 10^{17}$ cm$^{-3}$? **Solution:**

\[
n = N_c e^{-\frac{(E_c - E_f)}{kT}}
\]

\[
E_c - E_f = kT \ln\left(\frac{N_c}{n}\right) = 0.026 \ln\left(\frac{2.8 \times 10^{19}}{10^{17}}\right) = 0.146 \text{ eV}
\]

---

![Diagram illustrating the Fermi level ($E_f$), conduction band ($E_c$), and valence band ($E_v$). The Fermi level is approximately 0.146 eV above the conduction band.]
The \( np \) Product and the Intrinsic Carrier Concentration

Multiply \( n = N_c e^{-(E_c-E_f)/kT} \) and \( p = N_v e^{-(E_f-E_v)/kT} \)

\[
np = N_c N_v e^{-(E_c-E_v)/kT} = N_c N_v e^{-E_g/kT}
\]

\[
np = n_i^2
\]

\[
n_i = \sqrt{N_c N_v} e^{-E_g/2kT}
\]

• In an intrinsic (undoped) semiconductor, \( n = p = n_i \).
EXAMPLE: Carrier Concentrations

**Question:** What is the hole concentration in an N-type semiconductor with $10^{15}$ cm$^{-3}$ of donors?

**Solution:** $n = 10^{15}$ cm$^{-3}$.

$$p = \frac{n_i^2}{n} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} = 10^5 \text{ cm}^{-3}$$

After increasing $T$ by 60 °C, $n$ remains the same at $10^{15}$ cm$^{-3}$ while $p$ increases by about a factor of 2300 because $n_i^2 \propto e^{-E_g/kT}$.

**Question:** What is $n$ if $p = 10^{17}$ cm$^{-3}$ in a P-type silicon wafer?

**Solution:**

$$n = \frac{n_i^2}{p} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}} = 10^3 \text{ cm}^{-3}$$
**EXAMPLE: Complete ionization of the dopant atoms**

\( N_d = 10^{17} \text{ cm}^{-3} \) and \( E_c - E_d = 45 \text{ meV} \). What fraction of the donors are not ionized?

**Solution:** First assume that all the donors are ionized.

\[
n = N_d = 10^{17} \text{ cm}^{-3} \implies E_f = E_c - 146 \text{ meV}
\]

\[
\frac{E_d}{E_c} \quad \frac{E_f}{E_c}
\]

Probability of non-ionization \( \approx \frac{1}{1 + e^{(E_d - E_f)/kT}} \)

\[
= \frac{1}{1 + e^{(146 - 45)/26\text{meV}}} = 0.02
\]

Therefore, it is reasonable to assume complete ionization, i.e., \( n = N_d \).
Doped Si and Charge

• What is the net charge of your Si when it is electron and hole doped?
Bond Model of Electrons and Holes (Intrinsic Si)

- Silicon crystal in a two-dimensional representation.

- When an electron breaks loose and becomes a conduction electron, a hole is also created.
• As (Arsenic), a Group V element, introduces conduction electrons and creates **N-type silicon**, and is called a **donor**.
• B (Boron), a Group III element, introduces holes and creates **P-type silicon**, and is called an **acceptor**.
• Donors and acceptors are known as dopants.
General Effects of Doping on $n$ and $p$

Charge neutrality: $n + N_a^- - p - N_d^+ = 0$

$N_a^-$: number of ionized acceptors /cm$^3$

$N_d^+$: number of ionized donors /cm$^3$

Assuming total ionization of acceptors and donors:

$n + N_a^- - p - N_d^+ = 0$

$N_a$ : number of ionized acceptors /cm$^3$

$N_d^+$ : number of ionized donors /cm$^3$
General Effects of Doping on $n$ and $p$

I. $N_d - N_a \gg n_i$ (i.e., N-type)

$$n = N_d - N_a$$
$$p = n_i^2 / n$$

If $N_d \gg N_a$, $n = N_d$ and $p = n_i^2 / N_d$

II. $N_a - N_d \gg n_i$ (i.e., P-type)

$$p = N_a - N_d$$
$$n = n_i^2 / p$$

If $N_a \gg N_d$, $p = N_a$ and $n = n_i^2 / N_a$
EXAMPLE: Dopant Compensation

What are $n$ and $p$ in Si with (a) $N_d = 6 \times 10^{16}$ cm$^{-3}$ and $N_a = 2 \times 10^{16}$ cm$^{-3}$ and (b) additional $6 \times 10^{16}$ cm$^{-3}$ of $N_a$?

(a) \[ n = N_d - N_a = 4 \times 10^{16} \text{ cm}^{-3} \]
\[ p = n_i^2 / n = 10^{20} / 4 \times 10^{16} = 2.5 \times 10^3 \text{ cm}^{-3} \]

(b) \[ N_a = 2 \times 10^{16} + 6 \times 10^{16} = 8 \times 10^{16} \text{ cm}^{-3} > N_d! \]
\[ p = N_a - N_d = 8 \times 10^{16} - 6 \times 10^{16} = 2 \times 10^{16} \text{ cm}^{-3} \]
\[ n = n_i^2 / p = 10^{20} / 2 \times 10^{16} = 5 \times 10^3 \text{ cm}^{-3} \]
Carrier Concentrations at Extremely High and Low Temperatures

\[ \ln n = \frac{1}{T} \]

- **intrinsic regime**
  - \( n = N_d \)

- **freeze-out regime**

**Temperature Ranges**
- **high temp.**
- **room temperature**
- **cryogenic temperature**

**Energy Levels**
- \( E_e \)
- \( E_D \)
- \( E_v \)

**Temperature Regions**
- **0 K**
- **Low T**
- **Moderate T**
- **High T**

- Negligible
- Dominant
Infrared Detector Based on Freeze-out

To image the black-body radiation emitted by tumors requires a photodetector that responds to $h\nu$’s around 0.1 eV. In doped Si operating in the freeze-out mode, conduction electrons are created when the infrared photons provide the energy to ionized the donor atoms.
Chapter Summary


\[ n = N_c e^{-(E_c - E_f)/kT} \]

\[ p = N_v e^{-(E_f - E_v)/kT} \]

\[ n = N_d - N_a \]

\[ p = N_a - N_d \]

\[ np = n_i^2 \]