Semiconductor Fundamentals

Lecture 4

Reading: Chapter 3
Announcements

- 1st HW due Tuesday.
- New lecture room: 3106 Etcheverry, starting next Thursday (9/13)
- TA OHs will be held in Cory 353
What to memorize?

- Learn the concepts
- Understand the equations
- Only memorize the most important and fundamental equations
Drift

Electron and Hole Mobilities

- Drift is the motion caused by an electric field.
Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

- Electrons *drift* in the direction opposite to the electric field → current flows

  - Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as quasi-classical particles moving at a constant average *drift velocity* $v_d$
**Effective Mass**

In an electric field, $\varepsilon$, an electron or a hole accelerates.

\[ a = \frac{-q\varepsilon}{m_n} \quad \text{electrons} \]

\[ a = \frac{q\varepsilon}{m_p} \quad \text{holes} \]

Remember:

F = ma = -qE

**Electron and hole effective masses**

<table>
<thead>
<tr>
<th></th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
<th>GaP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_n/m_0$</td>
<td>0.26</td>
<td>0.12</td>
<td>0.068</td>
<td>0.82</td>
</tr>
<tr>
<td>$m_p/m_0$</td>
<td>0.39</td>
<td>0.30</td>
<td>0.50</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Electron Momentum

- With every collision, the electron loses momentum
  \[ m_n \cdot v_d \]

- Between collisions, the electron gains momentum
  \[ (-q)\varepsilon \tau_{mn} \]

Remember:
F=ma=mV/t = -qE

where \( \tau_{mn} = \) average time between scattering events
Electron and Hole Mobilities

\[ m_p v = q \varepsilon \tau_{mp} \]

\[ \nu = \frac{q \varepsilon \tau_{mp}}{m_p} \]

\[ \nu = \mu_p \varepsilon \]

\[ \mu_p = \frac{q \tau_{mp}}{m_p} \]

\[ \nu = -\mu_n \varepsilon \]

\[ \mu_n = \frac{q \tau_{mn}}{m_n} \]

- \( \mu_p \) is the hole mobility and \( \mu_n \) is the electron mobility
Electron and Hole Mobilities

\[ v = \mu \varepsilon; \quad \mu \text{ has the dimensions of } \frac{v}{\varepsilon} \quad \left[ \frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right]. \]

**Electron and hole mobilities of selected semiconductors**

<table>
<thead>
<tr>
<th>( \mu_n ) (cm(^2)/V( \cdot )s)</th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
<th>InAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>3900</td>
<td>8500</td>
<td>30000</td>
<td></td>
</tr>
<tr>
<td>( \mu_p ) (cm(^2)/V( \cdot )s)</td>
<td>470</td>
<td>1900</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?
Drift Velocity, Mean Free Time, Mean Free Path

**EXAMPLE:** Given $\mu_p = 470 \text{ cm}^2/\text{V} \cdot \text{s}$, what is the hole drift velocity at $\varepsilon = 10^3 \text{ V/cm}$? What is $\tau_{mp}$ and what is the distance traveled between collisions (called the **mean free path**)? Hint: When in doubt, use the MKS system of units.
EXAMPLE: Given $\mu_p = 470 \text{ cm}^2/\text{V}\cdot\text{s}$, what is the hole drift velocity at $\varepsilon = 10^3 \text{ V/cm}$? What is $\tau_{mp}$ and what is the distance traveled between collisions (called the mean free path)? Hint: When in doubt, use the MKS system of units.

Solution: $v = \mu_p \varepsilon = 470 \text{ cm}^2/\text{V}\cdot\text{s} \times 10^3 \text{ V/cm} = 4.7 \times 10^5 \text{ cm/s}$

$$\tau_{mp} = \frac{\mu_p m_p}{q} \frac{470 \text{ cm}^2/\text{V} \cdot \text{s}}{0.39 \times 9.1 \times 10^{-31} \text{ kg}/1.6 \times 10^{-19} \text{ C}}$$

$$= 0.047 \text{ m}^2/\text{V} \cdot \text{s} \times 2.2 \times 10^{-12} \text{ kg/C} = 1 \times 10^{-13} \text{s} = 0.1 \text{ ps}$$

mean free path $= \tau_{mh} v_{th} \sim 1 \times 10^{-13} \text{s} \times 2.2 \times 10^7 \text{ cm/s}$

$$= 2.2 \times 10^{-6} \text{ cm} = 220 \text{ Å} = 22 \text{ nm}$$

This is smaller than the typical dimensions of devices, but getting close.
Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:
1. Phonon Scattering
2. Impurity (Dopant) Ion Scattering

**Phonon scattering** mobility decreases when temperature rises:

\[
\mu_{\text{phonon}} \propto \tau_{\text{phonon}} \propto \frac{1}{\text{phonon density} \times \text{carrier thermal velocity}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}
\]

\[
\mu = q \tau/m \propto T
\]

\[
v_{th} \propto T^{1/2}
\]
Impurity (Dopant)-Ion Scattering or Coulombic Scattering

There is less change in the direction of travel if the electron zips by the ion at a higher speed.

\[ \mu_{\text{impurity}} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d} \]
Total Mobility

\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{impurity}}} \]

\[ \frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}} \]

Mobility (cm\(^2\) V\(^{-1}\) s\(^{-1}\))

Electrons

Holes

\[ N_a + N_d \text{ (cm}^{-3} \text{)} \]

Temperature Effect on Mobility

**Question:**
What $N_d$ will make $d\mu_n/dT = 0$ at room temperature?
**Velocity Saturation**

When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy. Therefore, the kinetic energy is capped and the velocity does not rise above a saturation velocity, $v_{sat}$, no matter how large $\varepsilon$ is.

*Velocity saturation* has a deleterious effect on device speed as we will see in the later chapters.
Current density $J_p = qpv$ A/cm$^2$ or C/cm$^2$·sec

**EXAMPLE:** If $p = 10^{15}\text{cm}^{-3}$ and $v = 10^4\text{cm/s}$, then

$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$

$= 1.6 \text{C/s} \cdot \text{cm}^2 = 1.6 \text{A/cm}^2$
Drift Current and Conductivity

Remember:
- Holes travel in the direction of the Electric field
- Electrons travel in the direction opposite to that of the E-field
Drift Current and Conductivity

\[ J_{p,\text{drift}} = qp\nu = qp\mu_p \varepsilon \]

\[ J_{n,\text{drift}} = -qn\nu = qn\mu_n \varepsilon \]

\[ J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = (qn\mu_n + qp\mu_p) \varepsilon = \sigma \varepsilon \]

\[ \therefore \text{conductivity of a semiconductor is } \sigma = qn\mu_n + qp\mu_p \]

\[ \therefore \text{resistivity of a semiconductor is } \rho = \frac{1}{\sigma} \]
Particles diffuse from a higher-concentration location to a lower-concentration location.
**Diffusion Current**

\[
J_{n,\text{diffusion}} = qD_n \frac{dn}{dx} \\
J_{p,\text{diffusion}} = -qD_p \frac{dp}{dx}
\]

\(D\) is called the diffusion constant. Signs explained:
Total Current – Review of Four Current Components

\[ J_{TOTAL} = J_n + J_p \]

\[ J_n = J_{n,drift} + J_{n,diff} = qn \mu_n \xi + qD_n \frac{dn}{dx} \]

\[ J_p = J_{p,drift} + J_{p,diff} = qp \mu_p \xi - qD_p \frac{dp}{dx} \]

\[ J_{TOTAL} = J_{n,drift} + J_{n,diff} + J_{p,drift} + J_{p,diff} \]
Band Diagram: Potential vs. Kinetic Energy

$E_c$ represents the electron potential energy:

$$\text{P.E.} = E_c - E_{\text{reference}}$$
Ec and Ev vary in the opposite direction from the voltage. That is, Ec and Ev are higher where the voltage is lower.

\[ P.E. = -qV \]
\[ E_c - E_{\text{reference}} = -qV \]

Variation in Ec with position is called band bending.
Electric Field $\mathcal{E}$

\[ \mathcal{E} = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} \]
Non-Uniformly-Doped Semiconductor

• The position of $E_F$ relative to the band edges is determined by the carrier concentrations, which is determined by the dopant concentrations.

• **In equilibrium, $E_F$ is constant**; therefore, the band energies vary with position:

\[ E_c(x) \]
\[ E_F \]
\[ E_V(x) \]
• In equilibrium, there is no net flow of electrons or holes

\[ J_N = 0 \quad \text{and} \quad J_P = 0 \]

⇒ The drift and diffusion current components must balance each other exactly. (A built-in electric field exists, such that the drift current exactly cancels out the diffusion current due to the concentration gradient.)

\[ J_N = qn \mu_n \varepsilon + qD_N \frac{dn}{dx} = 0 \]
Consider a piece of non-uniformly doped semiconductor.

\[ n = N_c e^{-(E_c - E_f)/kT} \]

\[ \frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx} \]

\[ = -\frac{n}{kT} \frac{dE_c}{dx} \]

\[ = -\frac{n}{kT} q \varepsilon \]
Einstein Relationship between $D$ and $\mu$

\[
\frac{dn}{dx} = -\frac{n}{kT} q \xi
\]

\[
J_n = q n \mu_n \xi + q D_n \frac{dn}{dx} = 0 \quad \text{at equilibrium.}
\]

\[
0 = q n \mu_n \xi - q n \frac{q D_n \xi}{kT}
\]

\[
D_n = \frac{kT}{q} \mu_n \quad \text{Similarly,} \quad D_p = \frac{kT}{q} \mu_p
\]

These are known as the Einstein relationship.
EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with \( \mu_p = 410 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1} \)?

Solution:

\[
D_p = \left( \frac{kT}{q} \right) \mu_p = (26 \text{ mV}) \cdot 410 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1} = 11 \text{ cm}^2/\text{s}
\]

Remember: \( kT/q = 26 \text{ mV at room temperature} \).