Announcements

- 1st HW due..... right now....
- 2nd HW due next Tuesday
- Professor Javey’s OH for tomorrow is cancelled. He will have an extra OH next week.
Band Diagram: Potential vs. Kinetic Energy

$E_c$ represents the electron potential energy:

$$P.E. = E_c - E_{\text{reference}}$$
$E_c$ and $E_v$ vary in the opposite direction from the voltage. That is, $E_c$ and $E_v$ are higher where the voltage is lower.

\[
P.E. = -qV
\]

\[
E_c - E_{\text{reference}} = -qV
\]
Electric Field $\mathcal{E}$

\[
\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx}
\]
Non-Uniformly-Doped Semiconductor

- The position of $E_F$ relative to the band edges is determined by the carrier concentrations, which is determined by the dopant concentrations.

- In equilibrium, $E_F$ is constant; therefore, the band energies vary with position:

  \[ E_c(x) \]
  \[ E_F \]
  \[ E_v(x) \]
• In equilibrium, there is no net flow of electrons or holes

\[ J_N = 0 \quad \text{and} \quad J_p = 0 \]

⇒ The drift and diffusion current components must balance each other exactly. (A built-in electric field exists, such that the drift current exactly cancels out the diffusion current due to the concentration gradient.)

\[ J_N = qn\mu_n \xi + qD_N \frac{dn}{dx} = 0 \]
Einstein Relationship between D and $\mu$

Consider a piece of non-uniformly doped semiconductor.

\[
n = N_c e^{-(E_c - E_f)/kT}
\]

\[
\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}
\]

\[
= -\frac{n}{kT} \frac{dE_c}{dx}
\]

\[
= -\frac{n}{kT} q \mathcal{E}
\]
**Einstein Relationship between \(D\) and \(\mu\)**

\[
\frac{dn}{dx} = -\frac{n}{kT} q^E
\]

\[
J_n = qn\mu_n^E + qD_n \frac{dn}{dx} = 0 \quad \text{at equilibrium.}
\]

\[
0 = qn\mu_n^E - qn \frac{qD_n^E}{kT}
\]

\[
D_n = \frac{kT}{q} \mu_n
\]

Similarly,

\[
D_p = \frac{kT}{q} \mu_p
\]

These are known as the **Einstein relationship**.
EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with $\mu_p = 410 \text{ cm}^2 \text{ V}^{-1}\text{s}^{-1}$?

Solution:

$$D_p = \left( \frac{kT}{q} \right) \mu_p = (26 \text{ mV}) \cdot 410 \text{ cm}^2 \text{V}^{-1}\text{s}^{-1} = 11 \text{ cm}^2/\text{s}$$

Remember: $kT/q = 26 \text{ mV at room temperature.}$
Generation and Recombination

• Generation:

• Recombination:

  • Recombination and Generation processes act to change the carrier concentrations, and thereby indirectly affect current flow.
Generation Processes

**Band-to-Band**
- Thermal energy or Light
- \( E_c \) \( E_v \)

**R-G Center**
- Thermal energy
- \( E_c \) \( E_v \) \( E_T \)

**Impact Ionization**
Recombination Processes

Direct

R-G Center

Auger

Photon (Light)

$E_T$

Thermal energy

$E_c$

$E_v$
Excess Carriers and Charge Neutrality

Excess

\[ n \equiv n_0 + \Delta n \]
\[ p \equiv p_0 + \Delta p \]

Charge neutrality:
\[ \Delta n = \Delta p \]

If not neutral, then built in field causes drift until neutrality is achieved
Recombination Lifetime

Assume light generates $\Delta n$ and $\Delta p$. If the light is suddenly turned off, $\Delta n$ and $\Delta p$ decay with time until they become zero. The process of decay is called recombination. The time constant of decay is the recombination time or carrier lifetime, $\tau$. Recombination is nature’s way of restoring equilibrium ($\Delta n = \Delta p = 0$).
τ ranges from 1 ns to 1 ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt. These *deep traps* capture electrons or holes to facilitate recombination and are called *recombination centers*.
Rate of recombination \((s^{-1}cm^{-3})\)

Consider recombination only.

\[ \frac{dn}{dt} = -\frac{\Delta n}{\tau} \]

\[ \Delta n = \Delta p \]

\[ \frac{dn}{dt} = -\frac{\Delta n}{\tau} = -\frac{\Delta p}{\tau} = \frac{dp}{dt} \]
A bar of Si is doped with boron at $10^{15}\text{cm}^{-3}$. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of $10^{20}/\text{s}\cdot\text{cm}^3$. The recombination lifetime is $10\mu\text{s}$. What are (a) $p_0$, (b) $n_0$, (c) $\Delta p$, (d) $\Delta n$, (e) $p$, (f) $n$, and (g) the $np$ product?
EXAMPLE: Photoconductors

Solution:

(a) What is $p_0$?
\[ p_0 = N_a = 10^{15} \text{ cm}^{-3} \]

(b) What is $n_0$?
\[ n_0 = n_i^2/p_0 = 10^5 \text{ cm}^{-3} \]

(c) What is $\Delta p$?
In steady-state, the rate of generation is equal to the rate of recombination.
\[ 10^{20}/\text{s-cm}^3 = \Delta p/\tau \]
\[ \therefore \Delta p = 10^{20}/\text{s-cm}^3 \cdot 10^{-5}\text{s} = 10^{15} \text{ cm}^{-3} \]
EXAMPLE: Photoconductors

(d) What is $\Delta n$?
$\Delta n = \Delta p = 10^{15} \text{ cm}^{-3}$

(e) What is $p$?
$p = p_0 + \Delta p = 10^{15} \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} = 2 \times 10^{15} \text{ cm}^{-3}$

(f) What is $n$?
$n = n_0 + \Delta n = 10^5 \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} \sim 10^{15} \text{ cm}^{-3}$ since $n_0 << \Delta n$

(g) What is $np$?
$np \sim 2 \times 10^{15} \text{ cm}^{-3} \cdot 10^{15} \text{ cm}^{-3} = 2 \times 10^{30} \text{ cm}^{-6} \gg n_i^2 = 10^{20} \text{ cm}^{-6}$. The $np$ product can be very different from $n_i^2$. 
**Quasi-equilibrium and Quasi-Fermi Levels**

Whenever $\Delta n = \Delta p \neq 0$, $np \neq n_i^2$. However, we would like to preserve and use the relations:

\[
    n = N_c e^{-(E_c - E_f) / kT}
\]

\[
    p = N_v e^{-(E_f - E_v) / kT}
\]

But these equations lead to $np = n_i^2$. The solution is to introduce two *quasi-Fermi levels* $E_{fn}$ and $E_{fp}$ such that

\[
    n = N_c e^{-(E_c - E_{fn}) / kT}
\]

\[
    p = N_v e^{-(E_{fp} - E_v) / kT}
\]

Even when electrons and holes are not at equilibrium, *within each group* the carriers are usually at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.
**Example: Quasi-Fermi Levels**

Consider a Si sample with $N_D = 10^{17} \text{ cm}^{-3}$ and $\Delta n = \Delta p = 10^{14} \text{ cm}^{-3}$.

(a) Find $n$:

$$ n = n_0 + \Delta n = N_D + \Delta n \approx 10^{17} \text{ cm}^{-3} $$

(b) Find $p$:

$$ p = p_0 + \Delta p = (n_i^2 / N_D) + \Delta p \approx 10^{14} \text{ cm}^{-3} $$

(c) Find the np product:

$$ np \approx 10^{17} \times 10^{14} = 10^{31} \text{ cm}^{-6} \gg n_i^2 $$
(d) Find \( F_N \):
\[
n = 10^{17} \text{ cm}^{-3} = N_c e^{-(E_c - F_N)/kT}
\]

\[
E_c - F_N = kT \times \ln(N_c/10^{17})
\]
\[
= 0.026 \text{ eV} \times \ln(2.8 \times 10^{19}/10^{17})
\]
\[
= 0.15 \text{ eV}
\]

(e) Find \( F_p \):
\[
p = 10^{14} \text{ cm}^{-3} = N_v e^{-(F_p - E_v)/kT}
\]

\[
F_p - E_v = kT \times \ln(N_v/10^{17})
\]
\[
= 0.026 \text{ eV} \times \ln(10^{19}/10^{14})
\]
\[
= 0.30 \text{ eV}
\]
Chapter Summary

\[ \nu_p = \mu_p \xi \]
\[ \nu_n = -\mu_n \xi \]
\[ J_{p,drift} = q\mu_p \xi \]
\[ J_{n,drift} = q\mu_n \xi \]
\[ J_{n,diffusion} = qD_n \frac{dn}{dx} \]
\[ J_{p,diffusion} = -qD_p \frac{dp}{dx} \]

\[ D_n = \frac{kT}{q} \mu_n \]
\[ D_p = \frac{kT}{q} \mu_p \]
Chapter Summary

\( \tau \) is the recombination lifetime.

\( n' \) and \( p' \) are the excess carrier concentrations.

\[
\begin{align*}
n &= n_0 + \Delta n \\
p &= p_0 + \Delta p
\end{align*}
\]

Charge neutrality requires \( \Delta n = \Delta p \).

\[
\text{rate of recombination} = \frac{\Delta n}{\tau} = \frac{\Delta p}{\tau}
\]

\( E_{fn} \) and \( E_{fp} \) are the quasi-Fermi levels of electrons and holes.

\[
\begin{align*}
n &= N_c e^{-(E_c - E_{fn})/kT} \\
p &= N_v e^{-(E_{fp} - E_v)/kT}
\end{align*}
\]