Lecture #4

ANNOUNCEMENTS

• Prof. King will not hold office hours this week, but will hold an extra office hour next Mo (2/3) from 11AM-12:30PM

• Quiz #1 will be given at the beginning of class on Th 2/6
  – covers material in Chapters 1 & 2 (HW#1 & HW#2)
  – closed book; one page of notes allowed

OUTLINE

– Drift (Chapter 3.1)
  » carrier motion
  » mobility
  » resistivity

Nondegenerately Doped Semiconductor

• Recall that the expressions for $n$ and $p$ were derived using the Boltzmann approximation, i.e. we assumed

\[ E_v + 3kT \leq E_F \leq E_c - 3kT \]

The semiconductor is said to be nondegenerately doped in this case.
Degenerately Doped Semiconductor

- If a semiconductor is very heavily doped, the Boltzmann approximation is not valid.

In Si at $T=300K$: $E_c - E_F < 3kT$ if $N_D > 1.6 \times 10^{18}$ cm$^{-3}$

$E_F - E_v < 3kT$ if $N_A > 9.1 \times 10^{17}$ cm$^{-3}$

The semiconductor is said to be degenerately doped in this case.

$$E_v + 3kT \leq E_F \leq E_c - 3kT$$

Band Gap Narrowing

- If the dopant concentration is a significant fraction of the silicon atomic density, the energy-band structure is perturbed

  $\rightarrow$ the band gap is reduced by $\Delta E_G$

$$N = 10^{18} \text{ cm}^{-3}:$$

$$N = 10^{19} \text{ cm}^{-3}:$$
Free Carriers in Semiconductors

- Three primary types of carrier action occur inside a semiconductor:
  - drift
  - diffusion
  - recombination-generation

Electrons as Moving Particles

\[ F = (-q)E = m_0 a \quad F = (-q)E = m_n^* a \]

where

\( m_n^* \) is the electron effective mass
Carrier Effective Mass

In an electric field, $\varepsilon$, an electron or a hole accelerates:

$$a = -\frac{q \varepsilon}{m_n} \quad \text{electrons}$$

$$a = \frac{q \varepsilon}{m_p} \quad \text{holes}$$

Electron and hole conductivity effective masses:

<table>
<thead>
<tr>
<th></th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_n/m_0$</td>
<td>0.26</td>
<td>0.12</td>
<td>0.068</td>
</tr>
<tr>
<td>$m_p/m_0$</td>
<td>0.39</td>
<td>0.30</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Thermal Velocity

Average electron or hole kinetic energy $\frac{3}{2}kT = \frac{1}{2}m^*v_{th}^2$

$$v_{th} = \sqrt{\frac{3kT}{m^*}} = \sqrt{\frac{3 \times 0.026 \text{ eV} \times (1.6 \times 10^{-19} \text{ J/eV})}{0.26 \times 9.1 \times 10^{-31} \text{ kg}}}$$

$$= 2.3 \times 10^5 \text{ m/s} = 2.3 \times 10^7 \text{ cm/s}$$
Carrier Scattering

- Mobile electrons and atoms in the Si lattice are always in random thermal motion.
  - Electrons make frequent collisions with the vibrating atoms
    - “lattice scattering” or “phonon scattering”
      - increases with increasing temperature
  - Average velocity of thermal motion for electrons: \( \sim 10^7 \text{ cm/s @ 300K} \)

- Other scattering mechanisms:
  - deflection by ionized impurity atoms
  - deflection due to Coulombic force between carriers
    - “carrier-carrier scattering”
    - only significant at high carrier concentrations

- The net current in any direction is zero, if no electric field is applied.

Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

- Electrons \textit{drift} in the direction opposite to the electric field \( \rightarrow \) current flows

  - Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as quasi-classical particles moving at a constant average \textit{drift velocity} \( v_d \).
Electron Momentum

- With every collision, the electron loses momentum
  \( m_n \ast v_d \)

- Between collisions, the electron gains momentum
  \((-q) \varepsilon \tau_{mn}\)

  where \( \tau_{mn} = \text{average time between scattering events} \)

Carrier Mobility

\[ m_n \ast v_d = (-q) \varepsilon \tau_{mn} \]

\[ |v_d| = q \varepsilon \tau_{mn} / m_n \ast = \mu_n \varepsilon \]

- \( \mu_n \equiv [q \tau_{mn} / m_n \ast] \) is the electron mobility

Similarly, for holes:

\[ |v_d| = q \varepsilon \tau_{mp} / m_p \ast = \mu_p \varepsilon \]

- \( \mu_p \equiv [q \tau_{mp} / m_p \ast] \) is the hole mobility
Electron and Hole Mobilities

\[ \mu \text{ has the dimensions of } \frac{v}{E} : \quad \left[ \frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right] \]

Electron and hole mobilities of selected intrinsic semiconductors (T=300K)

<table>
<thead>
<tr>
<th></th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
<th>InAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_n ) (cm(^2)/V( \cdot )s)</td>
<td>1400</td>
<td>3900</td>
<td>8500</td>
<td>30000</td>
</tr>
<tr>
<td>( \mu_p ) (cm(^2)/V( \cdot )s)</td>
<td>470</td>
<td>1900</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

Example: Drift Velocity Calculation

Find the hole drift velocity in an intrinsic Si sample for \( \mathcal{E} = 10^3 \text{ V/cm} \). What is \( \tau_{mp} \) and what is the distance traveled between collisions?

Solution:
**Mobility Dependence on Doping**

\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{impurity}}} \]

\[ \frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}} \]

**Drift Current**

\( \nu_d \ t \ A = \text{volume from which all holes cross plane in time } t \)

\( p \ \nu_d \ t \ A = \# \ of \ holes \ crossing \ plane \ in \ time \ t \)

\( q \ p \ \nu_d \ t \ A = \text{charge crossing plane in time } t \)

\( q \ p \ \nu_d \ A = \text{charge crossing plane per unit time} = \text{hole current} \)

⇒ Hole current per unit area \( J = q \ p \ \nu_d \)
Conductivity and Resistivity

\[ J_{n,\text{drift}} = -qv = qn\mu_n \]

\[ J_{p,\text{drift}} = qp = qp\mu_p \]

\[ J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = \sigma = (qn\mu_n + qp\mu_p) \]

Conductivity of a semiconductor is \( \sigma = qn\mu_n + qp\mu_p \)

Resistivity \( \rho = 1 / \sigma \) (Unit: ohm-cm)

Resistivity Dependence on Doping

For n-type mat'l:
\[ \rho \equiv \frac{1}{qn\mu_n} \]

For p-type mat'l:
\[ \rho \equiv \frac{1}{qp\mu_p} \]

Note: This plot does not apply for compensated material!
Electrical Resistance

where \( \rho \) is the resistivity

\[
R \equiv \frac{V}{I} = \rho \frac{L}{Wt}
\]

(Unit: ohms)

Example

Consider a Si sample doped with \( 10^{16}/\text{cm}^3 \) Boron. What is its resistivity?

Answer:

\[ N_A = 10^{16}/\text{cm}^3, \quad N_D = 0 \quad (N_A >> N_D \rightarrow \text{p-type}) \]

\[ \rightarrow \rho \approx 10^{16}/\text{cm}^3 \quad \text{and} \quad n \approx 10^4/\text{cm}^3 \]

\[
\rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qp\mu_p}
\]

\[
= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \, \Omega \cdot \text{cm}
\]
Example: Dopant Compensation

Consider the same Si sample, doped additionally with $10^{17}/\text{cm}^3$ Arsenic. What is its resistivity?

**Answer:**

\[ N_A = 10^{16}/\text{cm}^3, \quad N_D = 10^{17}/\text{cm}^3 \quad (N_D \gg N_A \rightarrow \text{n-type}) \]

\[ \rightarrow n \approx 9 \times 10^{16}/\text{cm}^3 \quad \text{and} \quad p \approx 1.1 \times 10^3/\text{cm}^3 \]

\[ \rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qn\mu_n} \]

\[ = \left[(1.6 \times 10^{-19})(9 \times 10^{16})(600)\right]^{-1} = 0.12 \, \Omega \, \text{cm} \]

Summary

- Electrons and holes moving under the influence of an electric field can be modelled as quasi-classical particles with average drift velocity
  \[ |v_d| = \mu \, \mathcal{E} \]

- The conductivity of a semiconductor is dependent on the carrier concentrations and mobilities
  \[ \sigma = qn\mu_n + qp\mu_p \]

- Resistivity
  \[ \rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p} \]