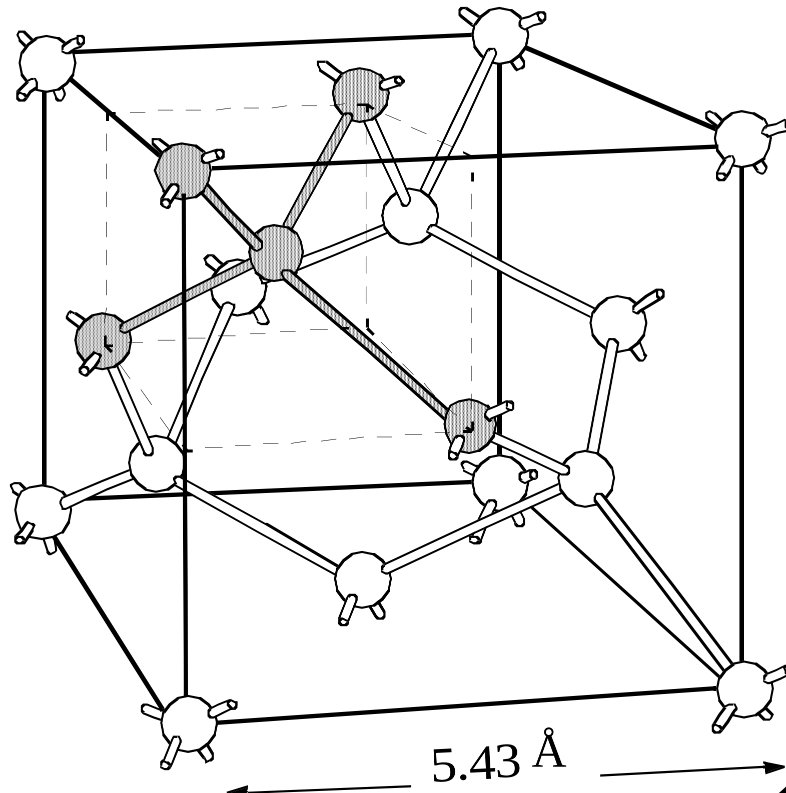
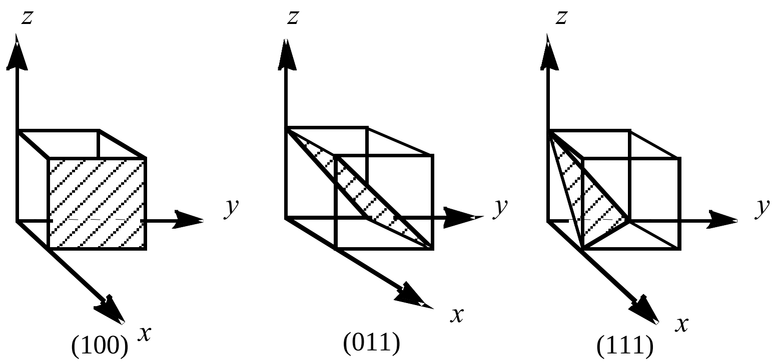
# *Chapter 1 Electrons and Holes in Semiconductors*

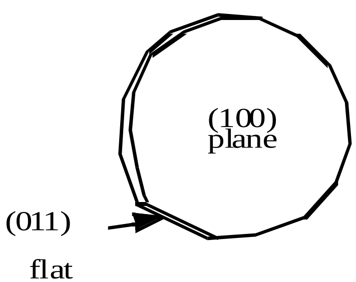
## *1.1 Silicon Crystal Structure*

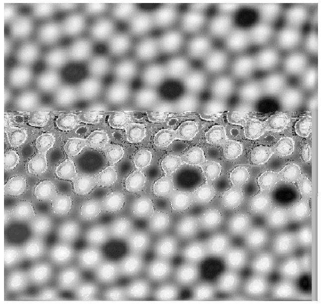
- ***Unit cell*** of silicon crystal is cubic.
- ***Each Si atom has 4 nearest neighbors.***



# *Silicon Wafers and Crystal Planes*



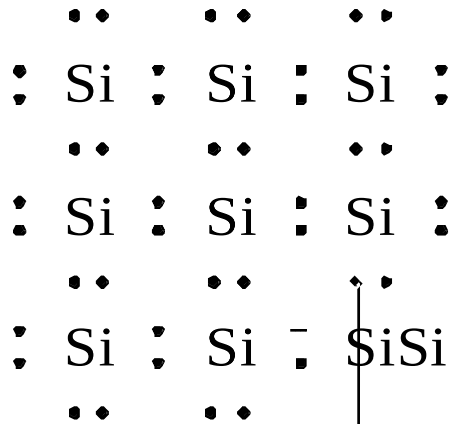



Si (111) plane

- The standard notation for crystal planes is based on the cubic unit cell.
- Silicon wafers are usually cut along the (100) plane with a flat or notch to help orient the wafer during IC fabrication.

# *1.2 Bond Model of Electrons and Holes*



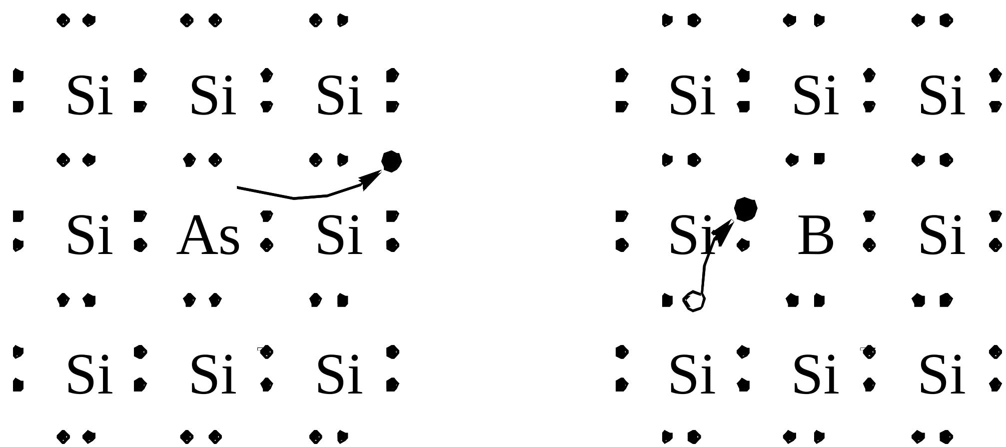

- Silicon crystal in a two-dimensional representation.

Si Si

Si Si Si

Si Si Si

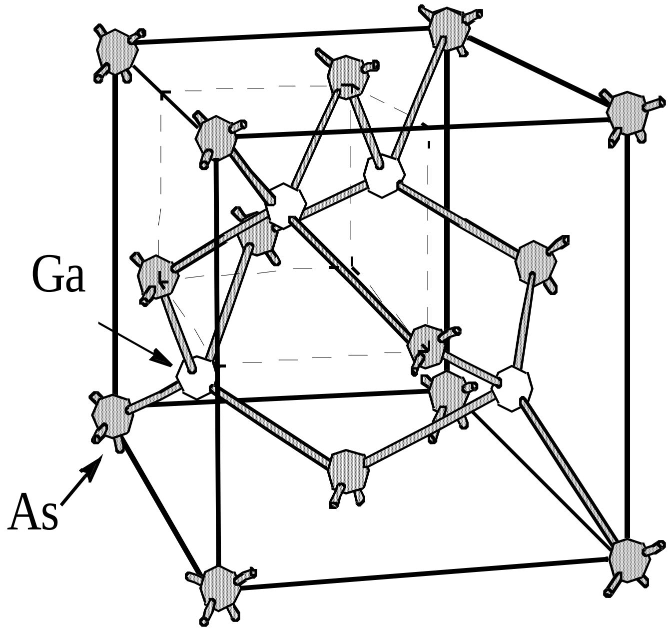
# *Dopants in Silicon*

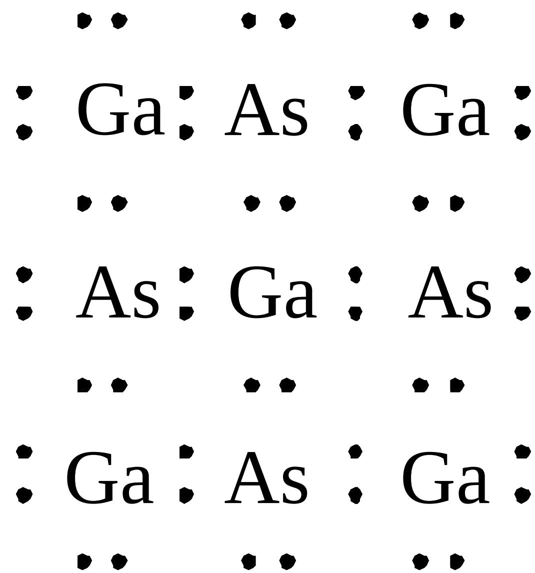



- As, a Group V element, introduces conduction electrons and creates ***N-type silicon,*** and is called a ***donor.***
- B, a Group III element, introduces holes and creates ***P-type silicon***, and is called an ***acceptor***.
- Donors and acceptors are known as dopants. Dopant ionization energy ~50meV (very low).

Hydrogen: $$E_{ion} = \frac{m_0 q^4}{8\varepsilon_0^2 h^2} = 13.6 \text{ eV}$$

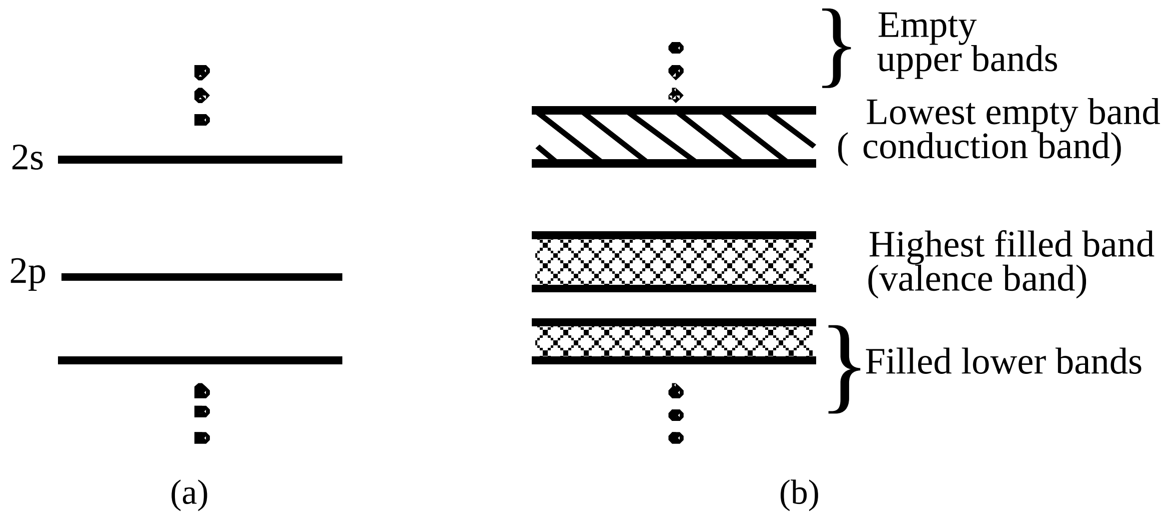
# *GaAs, III-V Compound Semiconductors, and Their Dopants*



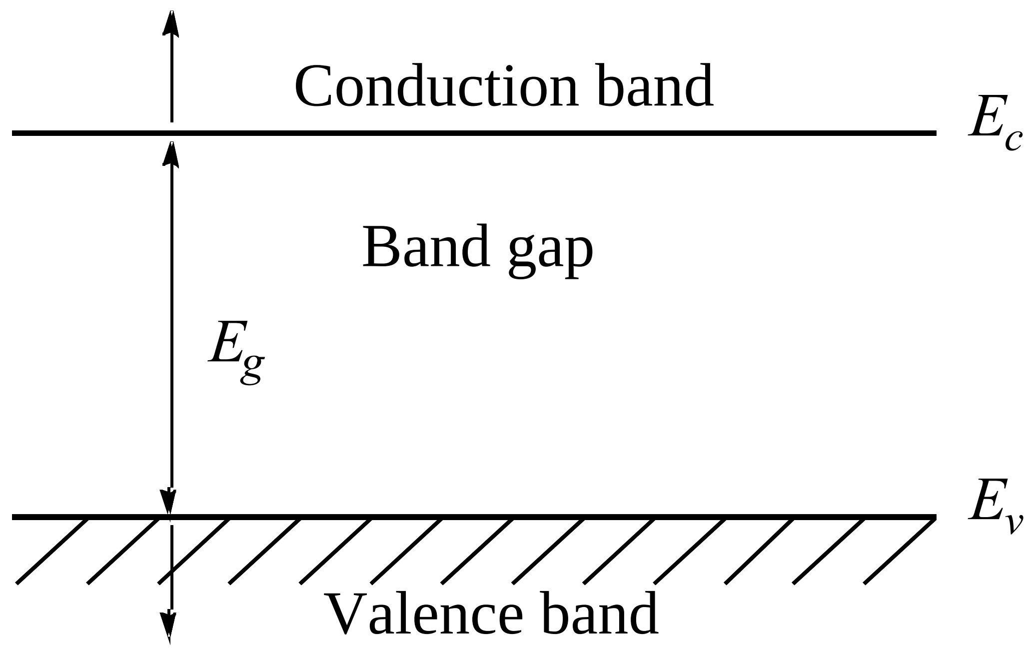

- GaAs has the same crystal structure as Si.
- GaAs, GaP, GaN are III-V compound semiconductors, important for optoelectronics.
- Which group of elements are candidates for donors? acceptors?

# 1.3 Energy Band Model



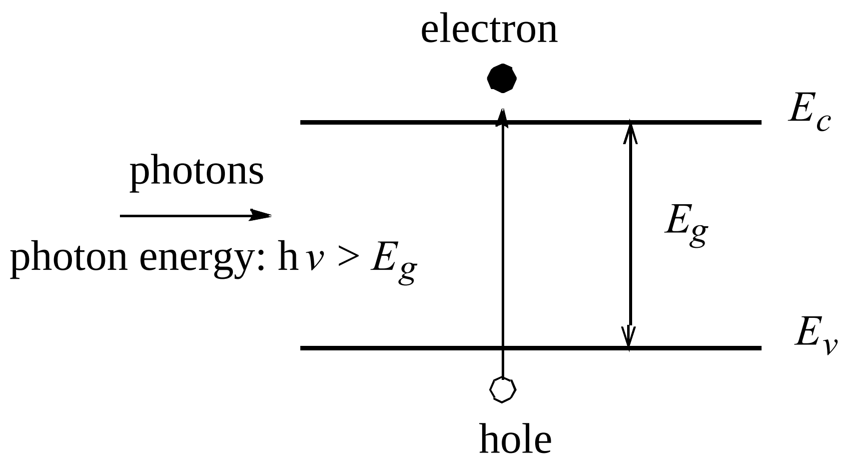

- Energy states of Si atom (a) expand into energy bands of Si crystal (b).
- The lower bands are filled and higher bands are empty in a semiconductor.
- The highest filled band is the ***valence band.***
- The lowest empty band is the ***conduction band*** .

# 1.3.1 Energy Band Diagram




- ***Energy band diagram*** shows the bottom edge of conduction band, $E_c$ , and top edge of valence band, $E_v$ .
- $E_c$ and $E_v$ are separated by the ***band gap energy*, $\boldsymbol{E_g}$** .

# *Measuring the Band Gap Energy by Light Absorption*

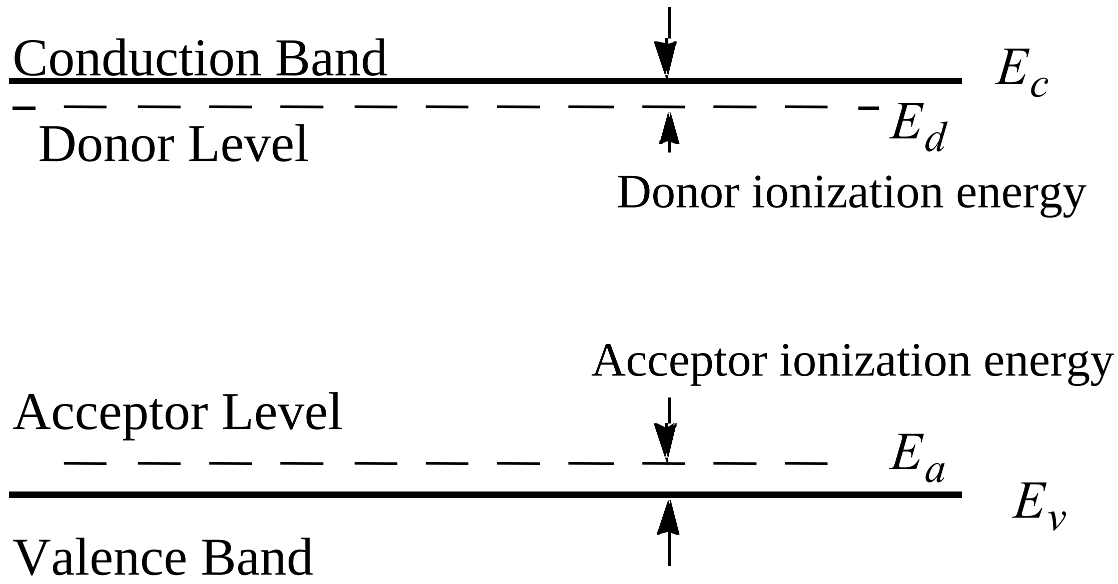



• $E_g$ can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

**Bandgap energies of selected semiconductors**

| Semiconductor | PbTe | Ge | Si | GaAs | GaP | Diamond |
|---|---|---|---|---|---|---|
| $E_g$ (eV) | 0.31 | 0.67 | 1.12 | 1.42 | 2.25 | 6.0 |

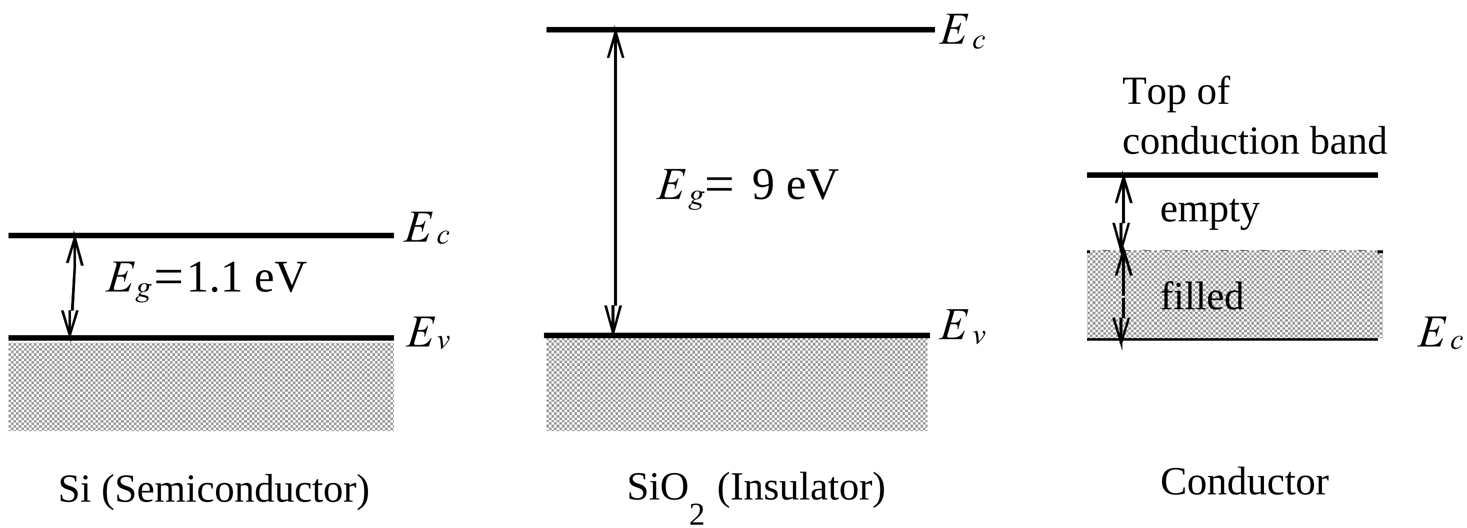
## 1.3.2 Donor and Acceptor Levels in the Band Model




**Ionization energy of selected donors and acceptors in silicon**

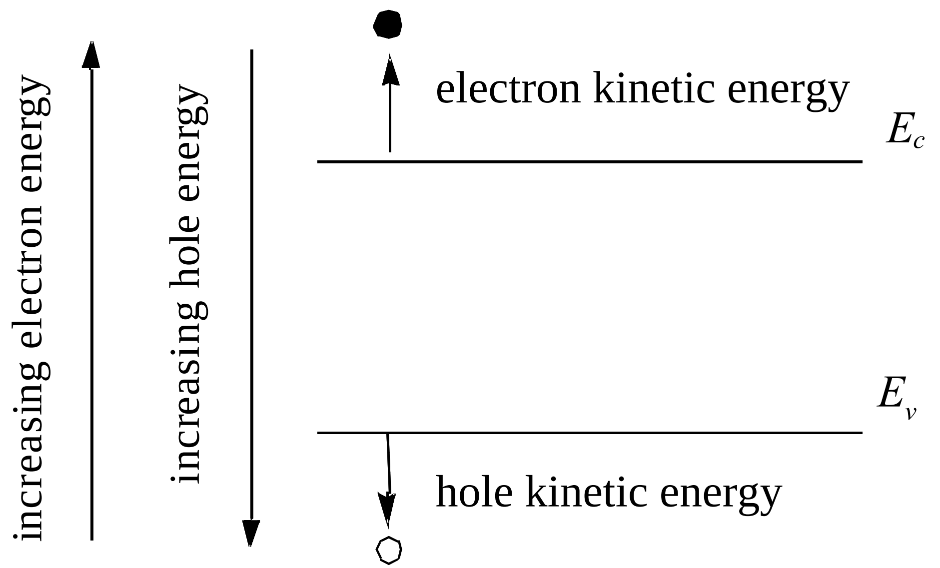
| | Donors | | | Acceptors | | |
|---|---|---|---|---|---|---|
| Dopant | Sb | P | As | B | Al | In |
| Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV) | 39 | 44 | 54 | 45 | 57 | 160 |

# *1.4 Semiconductors, Insulators, and Conductors*




- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower $E_g$'s than insulators and can be doped.

# *1.5 Electrons and Holes*




- Both electrons and holes tend to seek their lowest energy positions.
- Electrons tend to fall in the energy band diagram.
- Holes float up like bubbles in water.

## 1.5.1 Effective Mass

In an electric field, $\mathcal{E}$, an electron or a hole accelerates.

$$a = \frac{-q\mathcal{E}}{m_n} \qquad \text{electrons}$$

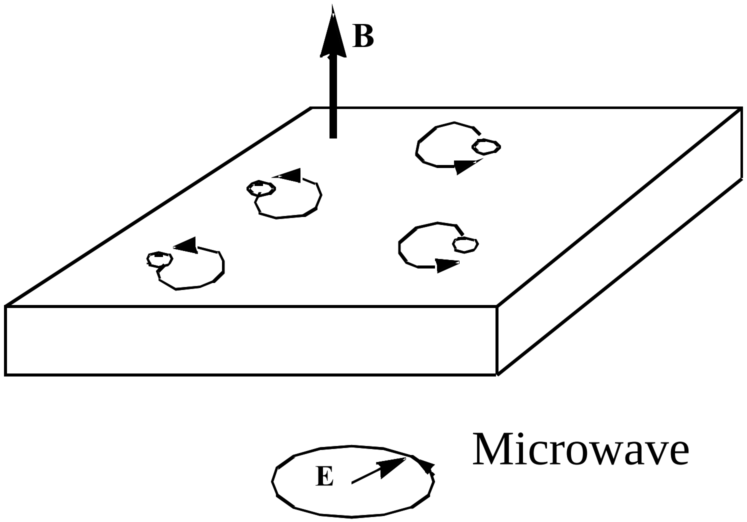
$$a = \frac{q\mathcal{E}}{m_p} \qquad \text{holes}$$

**Electron and hole effective masses**

| | **Si** | **Ge** | **GaAs** | **GaP** |
|---|---|---|---|---|
| $m_n/m_0$ | 0.26 | 0.12 | 0.068 | 0.82 |
| $m_p/m_0$ | 0.39 | 0.30 | 0.50 | 0.60 |

# 1.5.2 How to Measure the Effective Mass

Cyclotron Resonance
Technique




Centripetal force =
Lorentzian force

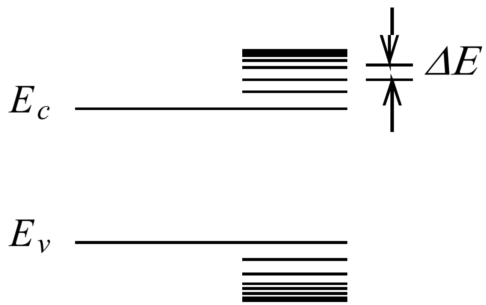
$$\frac{m_n v^2}{r} = qvB$$

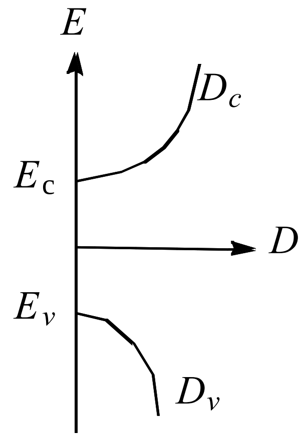
$$v = \frac{qBr}{m_n}$$

$$f_{cr} = \frac{v}{2\pi r} = \frac{qB}{2\pi m_n}$$

$f_{cr}$ is the Cyclotron resonance frequency. It is independent of $v$ and $r$. Electrons strongly absorb microwaves of that frequency. By measuring $f_{cr}$, $m_n$ can be found.

# 1.6 Density of States



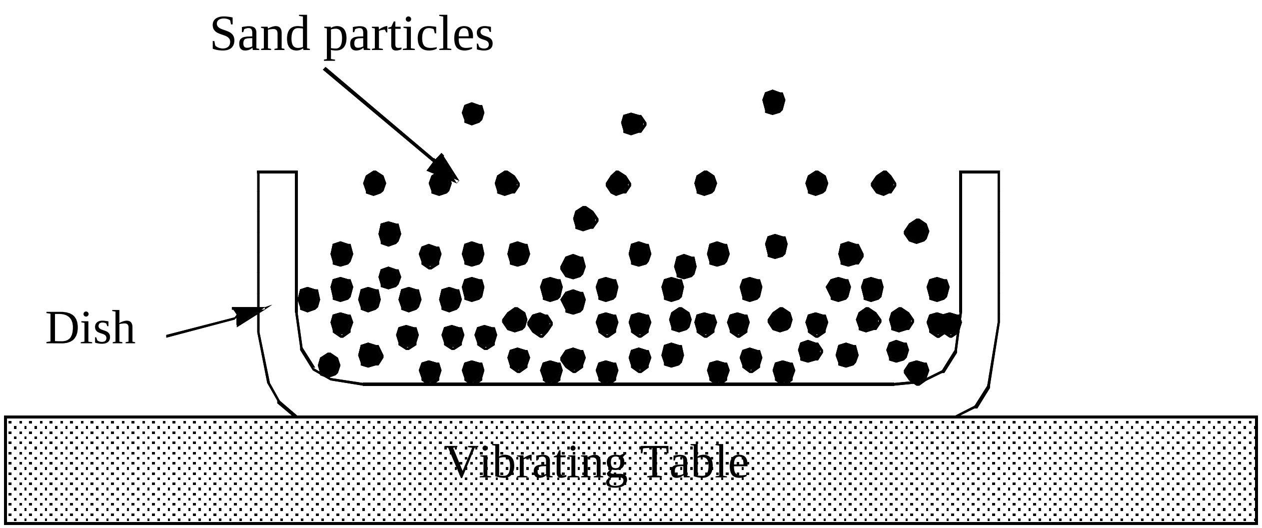

$$D_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \quad \left(\frac{1}{\text{eV} \cdot \text{cm}^3}\right)$$

$$D_c(E) \equiv \frac{8\pi m_n \sqrt{2m_n (E - E_c)}}{h^3}$$

$$D_v(E) \equiv \frac{8\pi m_p \sqrt{2m_p (E_v - E)}}{h^3}$$

# *1.7 Thermal Equilibrium*

## 1.7.1 An Analogy for Thermal Equilibrium




- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy (vibrating atoms, etc.)

# 1.7.2 Fermi Function–The Probability of an Energy State Being Occupied by an Electron

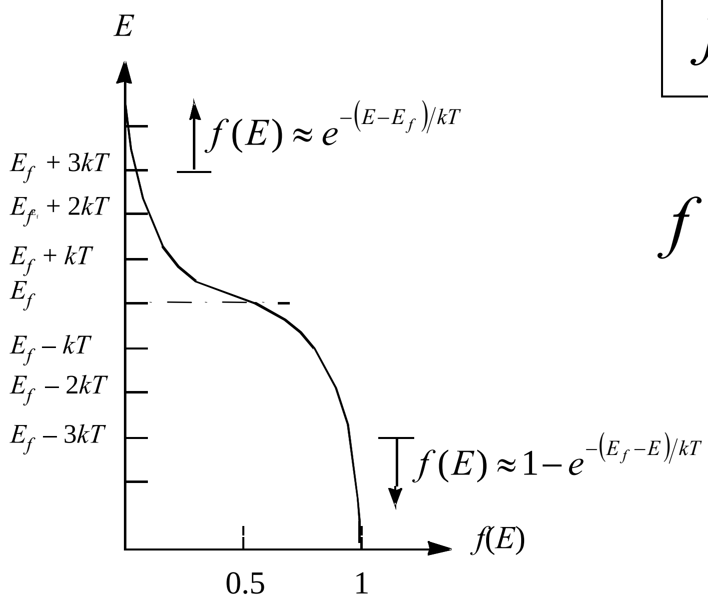
$$f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$$

$E_f$ is called the ***Fermi energy*** or the ***Fermi level.***

Boltzmann approximation:

$$\boxed{f(E) \approx e^{-\left(E-E_f\right)/kT}} \qquad E - E_f >> kT$$

$$f(E) \approx 1 - e^{-\left(E_f-E\right)/kT} \qquad E - E_f << -kT$$




***Remember: there is only one Fermi-level in a system at equilibrium.***

# *1.8 Electron and Hole Concentrations*

## 1.8.1 Derivation of *n* and *p* from *D*(*E*) and *f*(*E*)

$$n = \int_{E_c}^{\text{top of conduction band}} f(E) D_c(E) dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c}\, e^{-(E - E_f)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{-(E_c - E_f)/kT} \int_0^{\infty} \sqrt{E - E_c}\, e^{-(E - E_c)/kT} dE$$

# *Electron and Hole Concentrations*

$$\boxed{n = N_c e^{-(E_c - E_f)/kT}}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

$N_c$ is called the ***effective density of states (of the conduction band)*** .

$$\boxed{p = N_v e^{-(E_f - E_v)/kT}}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

$N_v$ is called the ***effective density of states of the valence band.***

**Remember: the closer $E_f$ moves up to $N_c$, the larger $n$ is; the closer $E_f$ moves down to $N_v$ , the larger $p$ is.**

**For Si, $N_c$ = 2.8×10$^{19}$ cm$^{-3}$ and $N_v$ = 1.04×10$^{19}$ cm$^{-3}$.**

## 1.8.2 The Fermi Level and Carrier Concentrations

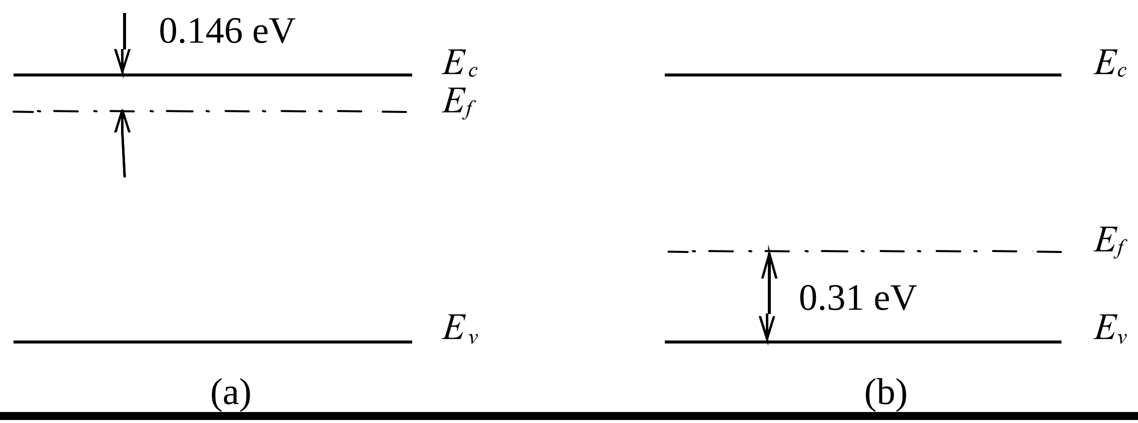
*Where is $E_f$ for $n = 10^{17}$ cm$^{-3}$? And for $p = 10^{14}$ cm$^{-3}$?*

***Solution**: (a)* $n = N_c e^{-(E_c - E_f)/kT}$

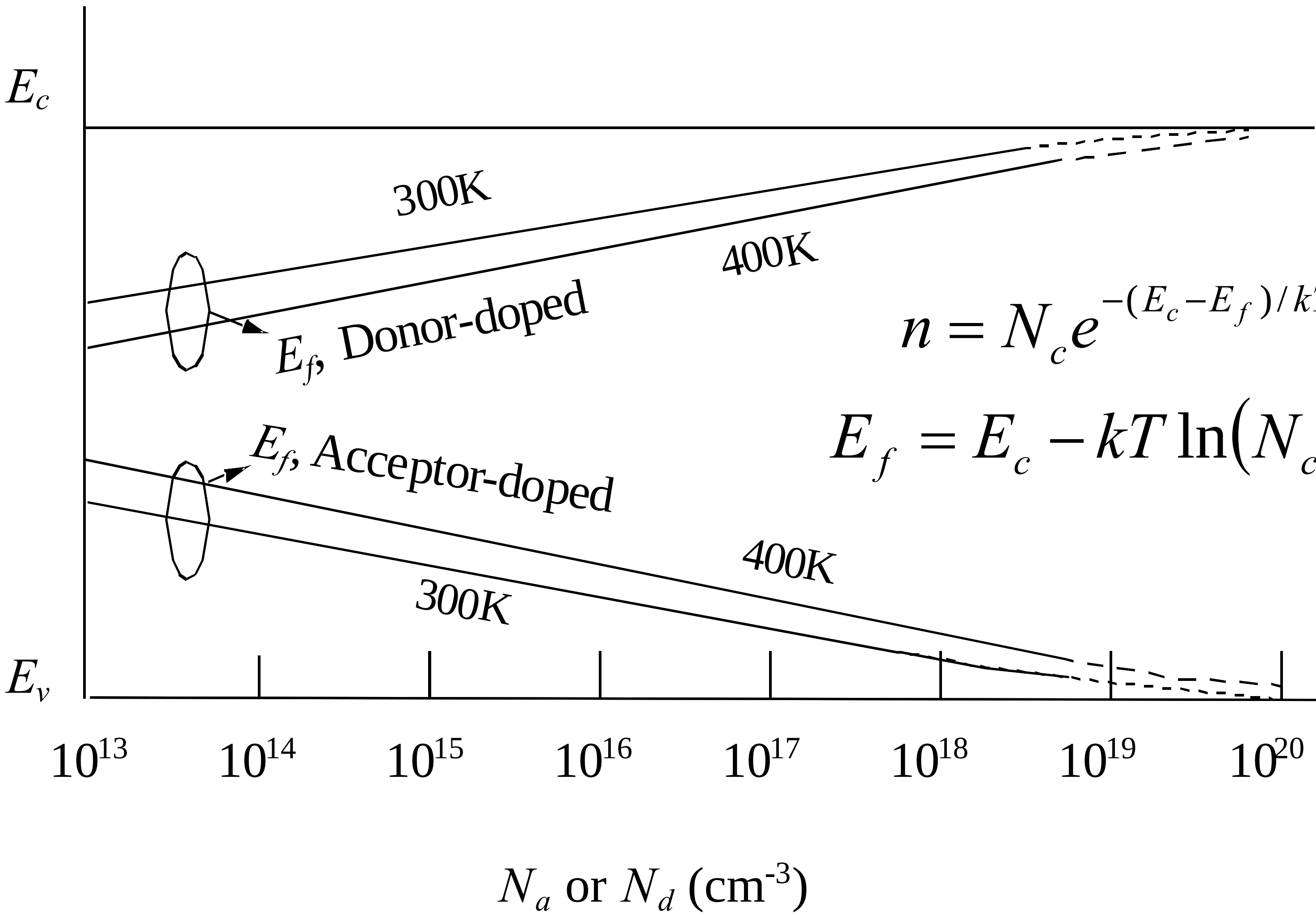
$$E_c - E_f = kT\ln(N_c/n) = 0.026\ln\left(2.8\times10^{19}/10^{17}\right) = 0.146\,\text{eV}$$

*(b) For $p = 10^{14}$cm$^{-3}$, from Eq.(1.8.8),*

$$E_f - E_v = kT\ln(N_v/p) = 0.026\ln\left(1.04\times10^{19}/10^{14}\right) = 0.31\,\text{eV}$$



# 1.8.2 The Fermi Level and Carrier Concentrations




$$n = N_c e^{-(E_c - E_f)/kT}$$

$$E_f = E_c - kT \ln(N_c / n)$$

## 1.8.3 The *np* Product and the Intrinsic Carrier Concentration

Multiply $n = N_c e^{-(E_c - E_f)/kT}$ and $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$\boxed{np = n_i^{\,2}}$$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$

- In an intrinsic (undoped) semiconductor, $n = p = n_i$ .
- $n_i$ is the ***intrinsic carrier concentration***, ~$10^{10}$ $cm^{-3}$ for Si.

# *EXAMPLE: Carrier Concentrations*

***Question:*** *What is the hole concentration in an N-type semiconductor with $10^{15}$ cm$^{-3}$ of donors?*

***Solution:*** *$n = 10^{15}$ cm$^{-3}$.*

$$p = \frac{n_i^{\,2}}{n} \approx \frac{10^{20}\,\text{cm}^{-3}}{10^{15}\,\text{cm}^{-3}} = 10^{5}\,\text{cm}^{-3}$$

*After increasing T by 60℃, n remains the same at $10^{15}$ cm$^{-3}$ while p increases by about a factor of 2300 because $n_i^{\,2} \propto e^{-E_g/kT}$.*

***Question:*** *What is n if $p = 10^{17}$cm$^{-3}$ in a P-type silicon wafer?*

***Solution:***

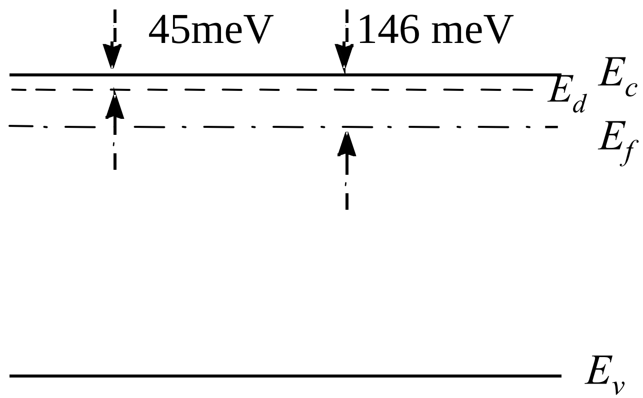
$$n = \frac{n_i^{\,2}}{p} \approx \frac{10^{20}\,\text{cm}^{-3}}{10^{17}\,\text{cm}^{-3}} = 10^{3}\,\text{cm}^{-3}$$

# *EXAMPLE: Complete ionization of the dopant atoms*

*$N_d = 10^{17}$ cm$^{-3}$. What fraction of the donors are not ionized?*

***Solution:*** *First assume that all the donors **are** ionized.*

$$n = N_d = 10^{17}\,\text{cm}^{-3} \Rightarrow E_f = E_c - 146\,\text{meV}$$




$$\text{Probability of non-ionization} \approx \frac{1}{1+e^{(E_d-E_f)/kT}} = \frac{1}{1+e^{((146-45)\text{meV})/26\text{meV}}} = 0.02$$

*Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.*

# *1.9 General Effects of Doping on n and p*

Charge neutrality: $n + N_a = p + N_d$

$$np = n_i^{\,2}$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2}\right)^2 + n_i^{\,2}\right]^{1/2}$$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2}\right)^2 + n_i^{\,2}\right]^{1/2}$$

# *1.9 General Effects of Doping on n and p*

**I.** $N_d - N_a >> n_i$ (i.e., N-type)

$$\boxed{\begin{aligned} n &= N_d - N_a \\ p &= n_i^2 / n \end{aligned}}$$

If $N_d >> N_a$ , $n = N_d$ and $p = n_i^2 / N_d$

**II.** $N_a - N_d >> n_i$ (i.e., P-type)

$$\boxed{\begin{aligned} p &= N_a - N_d \\ n &= n_i^2 / p \end{aligned}}$$

If $N_a >> N_d$ , $p = N_a$ and $n = n_i^2 / N_a$

# *EXAMPLE: Dopant Compensation*

*What are n and p in Si with (a) $N_d = 6\times10^{16}$ cm$^{-3}$ and $N_a = 2\times10^{16}$ cm$^{-3}$ and (b) additional $6\times10^{16}$ cm$^{-3}$ of $N_a$?*

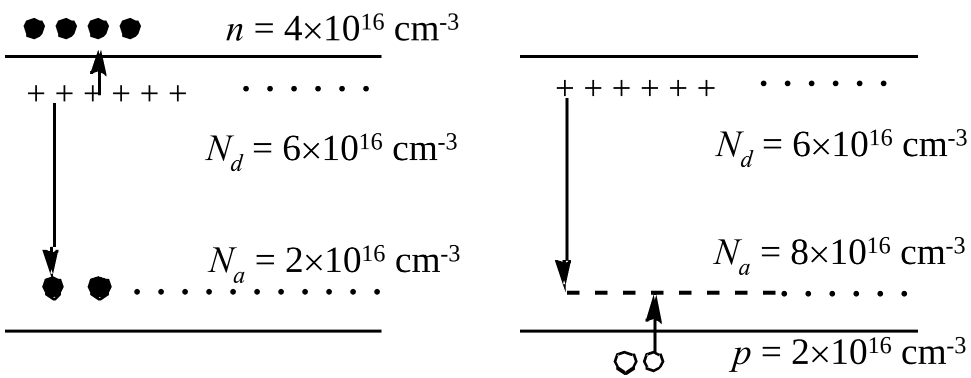
(a) $n = N_d - N_a = 4\times10^{16}\,\text{cm}^{-3}$

$p = n_i^{\,2} / n = 10^{20} / 4\times10^{16} = 2.5\times10^{3}\,\text{cm}^{-3}$

(b) $N_a = 2\times10^{16} + 6\times10^{16} = 8\times10^{16}\ \text{cm}^{-3} > N_d$!

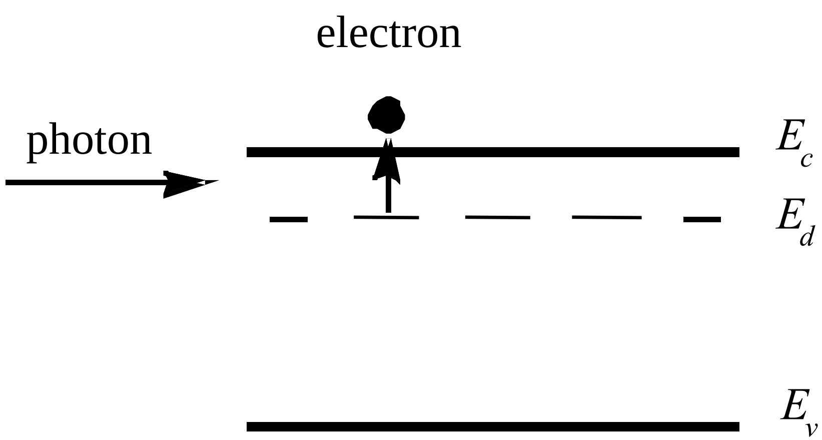
$p = N_a - N_d = 8\times10^{16} - 6\times10^{16} = 2\times10^{16}\,\text{cm}^{-3}$

$n = n_i^{\,2} / p = 10^{20} / 2\times10^{16} = 5\times10^{3}\,\text{cm}^{-3}$

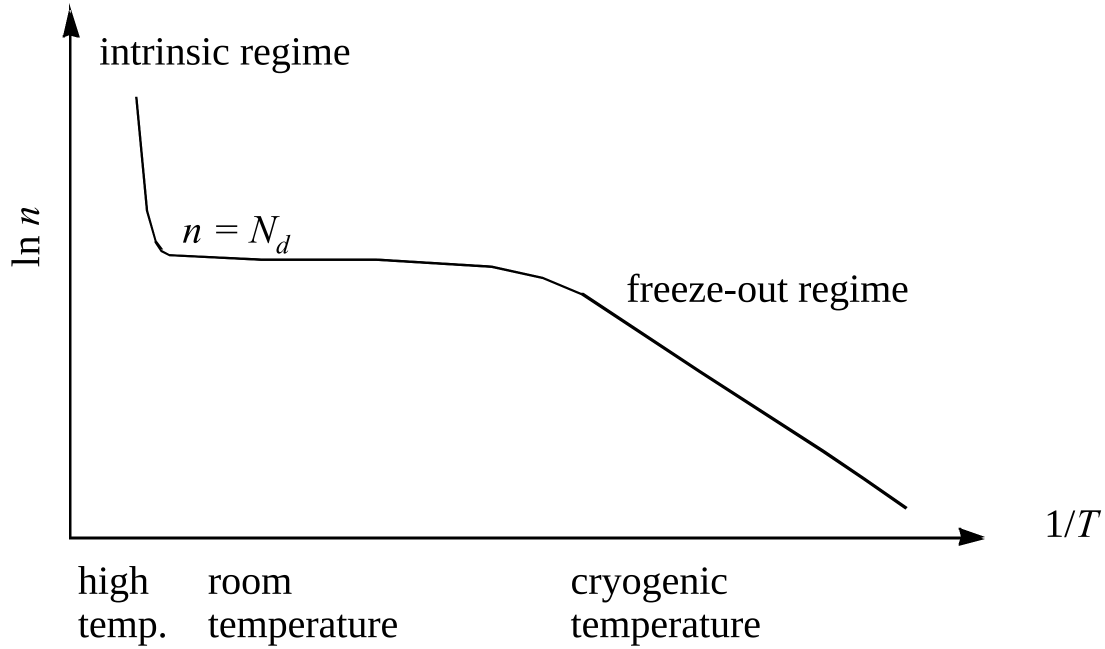


# *Infrared Detector Based on Freeze-out*

To image the black-body radiation emitted by tumors requires a photodetector that responds to $h\nu$'s around 0.1 eV. In doped Si operating in the freeze-out mode, conduction electrons are created when the infrared photons provide the energy to ionized the donor atoms.



# 1.10 Carrier Concentrations at Extremely High and Low Temperatures




$$\text{high T:} \quad n = p = n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

$$\text{low T:} \quad n = \left[\frac{N_c N_d}{2}\right]^{1/2} e^{-(E_c - E_d)/2kT}$$

# 1.11 Chapter Summary

Energy band diagram. Acceptor. Donor. $m_n$, $m_p$. Fermi function. $E_f$.

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^{\,2}$$