Chapter 2 Motion and Recombination of Electrons and Holes

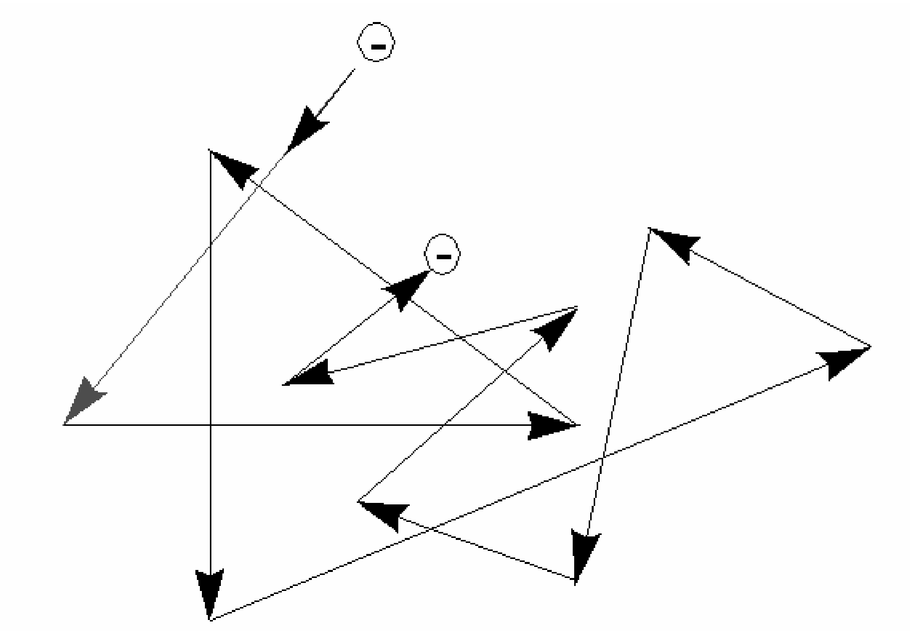
2.1 Thermal Energy and Thermal Velocity

Average electron or hole kinetic energy $= \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$

$$v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23}\ \mathrm{JK}^{-1} \times 300\ \mathrm{K}}{0.26 \times 9.1 \times 10^{-31}\ \mathrm{kg}}}$$

$$= 2.3 \times 10^5\ \mathrm{m/s} = 2.3 \times 10^7\ \mathrm{cm/s}$$

2.1 Thermal Motion



- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero.
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$

2.2.1 Electron and Hole Mobilities

$$m_p v = q \mathcal{E} \tau_{mp}$$

$$v = \frac{q \mathcal{E} \tau_{mp}}{m_p}$$

$$\boxed{\begin{gathered} v = \mu_p \mathcal{E} \\ \mu_p = \frac{q \tau_{mp}}{m_p} \end{gathered}} \qquad \boxed{\begin{gathered} v = -\mu_n \mathcal{E} \\ \mu_n = \frac{q \tau_{mn}}{m_n} \end{gathered}}$$

- μ_p is the hole mobility and μ_n is the electron mobility

2.2.1 Electron and Hole Mobilities

$$v = \mu \mathcal{E}\ ; \quad \mu \text{ has the dimensions of } v/\mathcal{E} \quad \left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right].$$

Electron and hole mobilities of selected semiconductors

	Si	Ge	GaAs	InAs
μ_n (cm^2/V·s)	1400	3900	8500	30000
μ_p (cm^2/V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

Drift Velocity, Mean Free Time, Mean Free Path

EXAMPLE: *Given $\mu_p = 470$ cm^2/V·s, what is the hole drift velocity at $\mathcal{E} = 10^3$ V/cm? What is τ_{mp} and what is the distance traveled between collisions (called the **mean free path**)? Hint: When in doubt, use the MKS system of units.*

Solution: $v = \mu_p \mathcal{E} = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 10^3\ \text{V/cm} = 4.7\times 10^5\ \text{cm/s}$

$$\tau_{mp} = \mu_p m_p/q = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 0.39 \times 9.1\times10^{-31}\ \text{kg}/1.6\times10^{-19}\ \text{C}$$

$$= 0.047\ \text{m}^2/\text{V}\cdot\text{s} \times 2.2\times10^{-12}\ \text{kg/C} = 1\times10^{-13}\text{s} = 0.1\ \text{ps}$$

$$\textit{mean free path} = \tau_{mh} v_{th} \sim 1\times 10^{-13}\ \text{s} \times 2.2\times10^{7}\ \text{cm/s}$$

$$= 2.2\times10^{-6}\ \text{cm} = 220\ \text{Å} = 22\ \text{nm}$$

This is smaller than the typical dimensions of devices, but getting close.

2.2.2 Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:

1. Phonon Scattering
2. Impurity (Dopant) Ion Scattering

Phonon scattering mobility decreases when temperature rises:

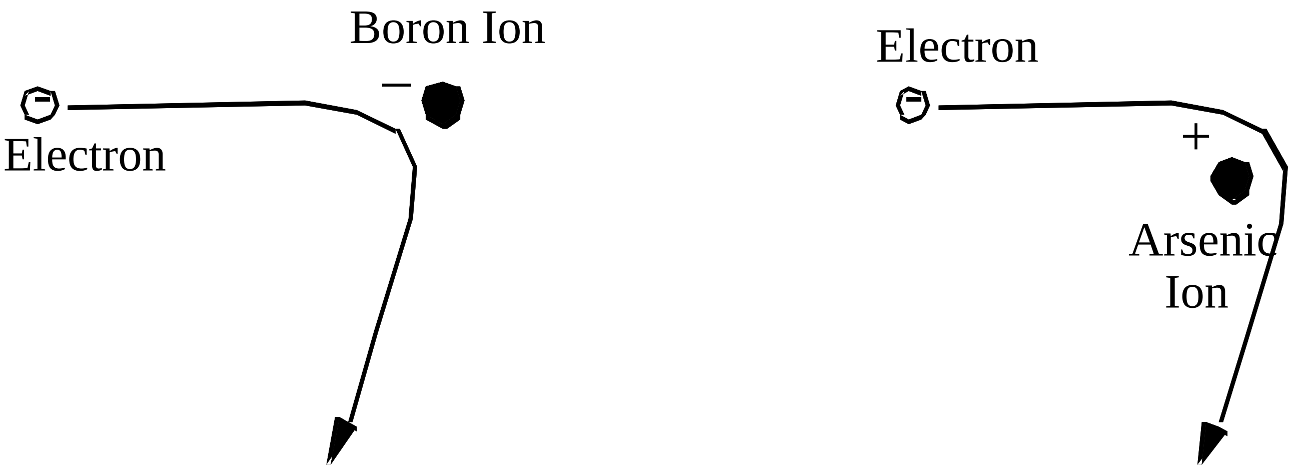
$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{phonon\ \ density \times carrier\ \ thermal\ \ velocity} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$$\mu = q\tau/m$$

$$\propto T$$

$$v_{th} \propto T^{1/2}$$

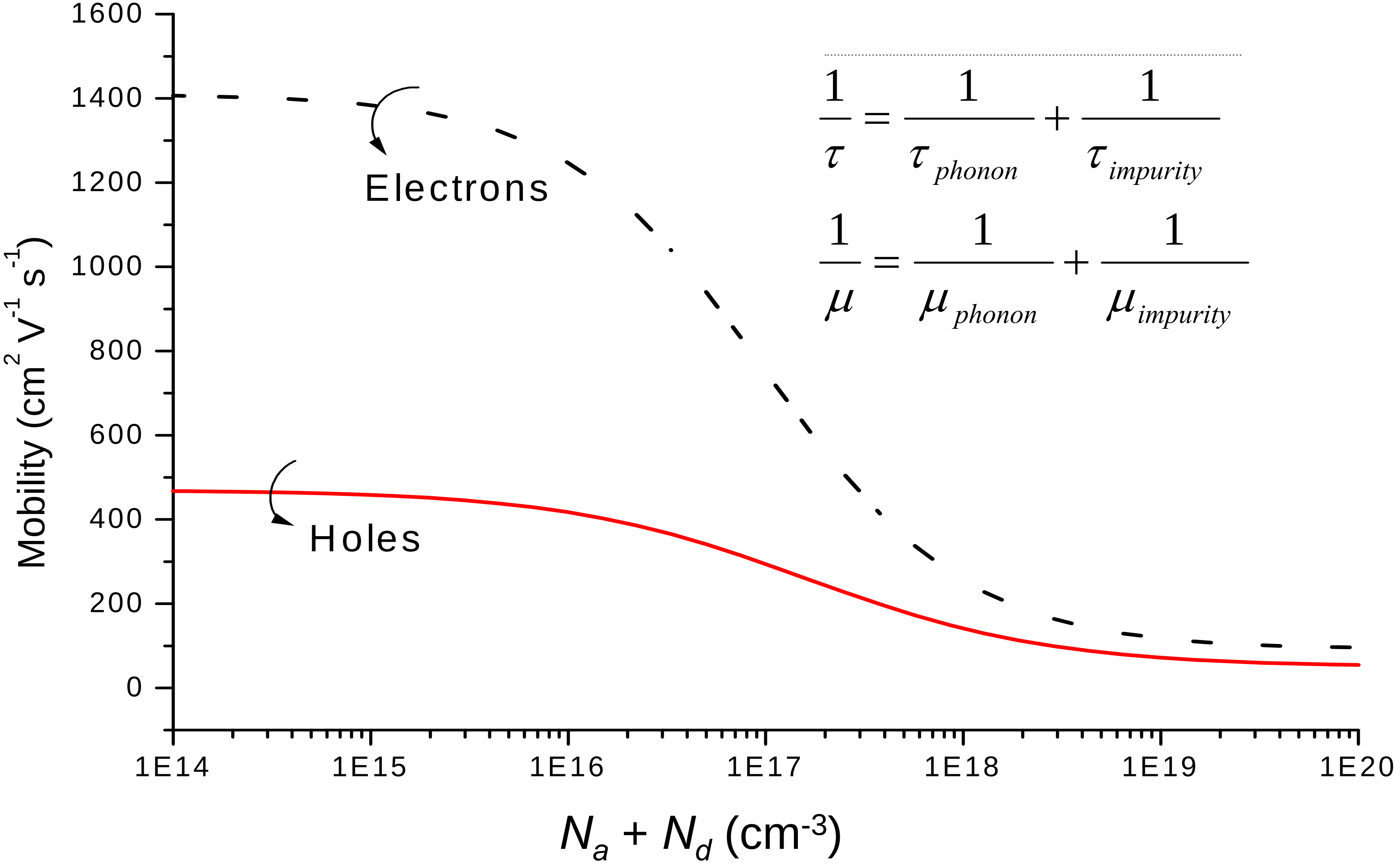
Impurity (Dopant)-Ion Scattering or Coulombic Scattering



There is less change in the direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

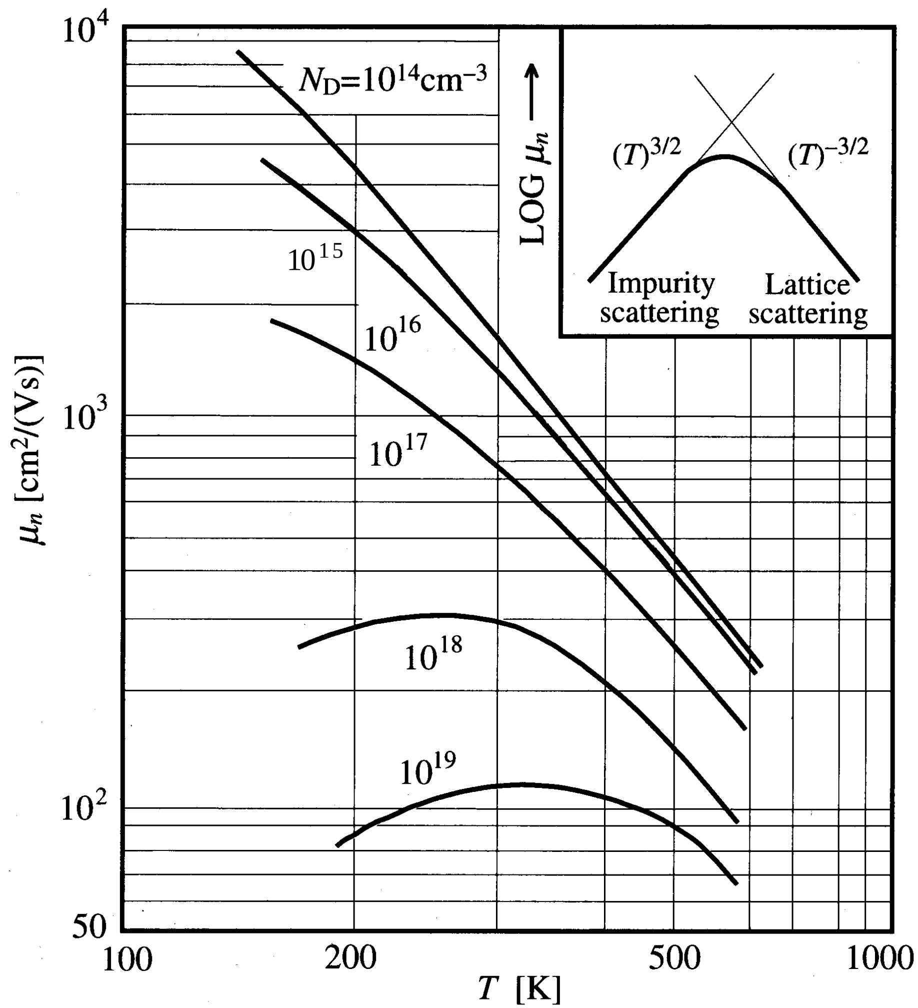
Total Mobility



$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

Temperature Effect on Mobility



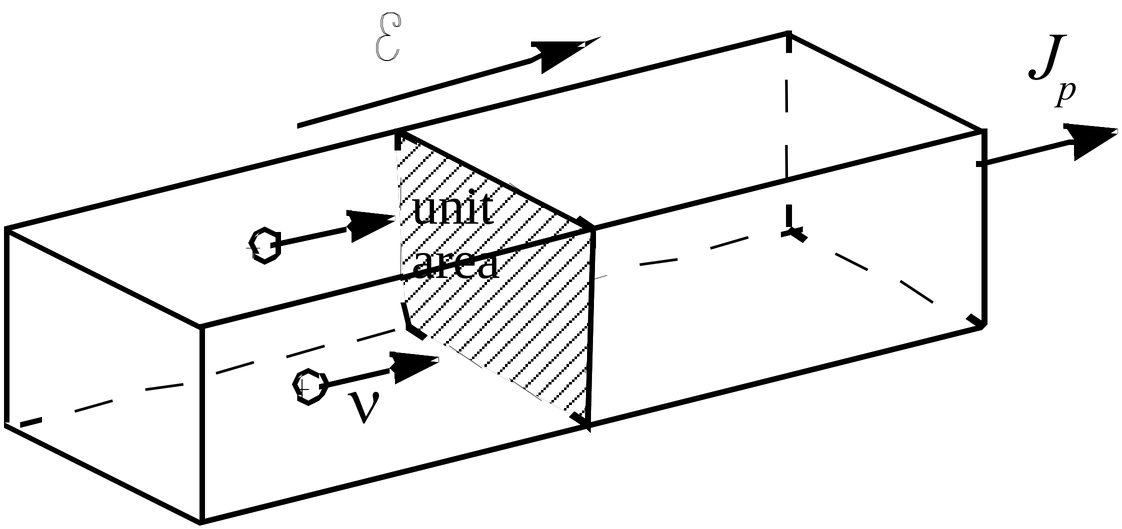
Question:
What N_d will make $d\mu_n/dT = 0$ at room temperature?

Velocity Saturation

When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy. Therefore, the kinetic energy is capped and the velocity does not rise above a saturation velocity, v_{sat}, no matter how large $\mathcal{E}$ is.

Velocity saturation has a deleterious effect on device speed as shown in Ch. 6.

2.2.3 Drift Current and Conductivity



Current density $\boxed{J_p = qpv}$ A/cm^2 or C/cm$^2\cdot$sec

EXAMPLE: If $p = 10^{15}$cm^{-3} and $v = 10^4$ cm/s, then

$$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$$

$$= 1.6\ \text{C/s}\cdot\text{cm}^2 = 1.6\ \text{A/cm}^2$$

2.2.3 Drift Current and Conductivity

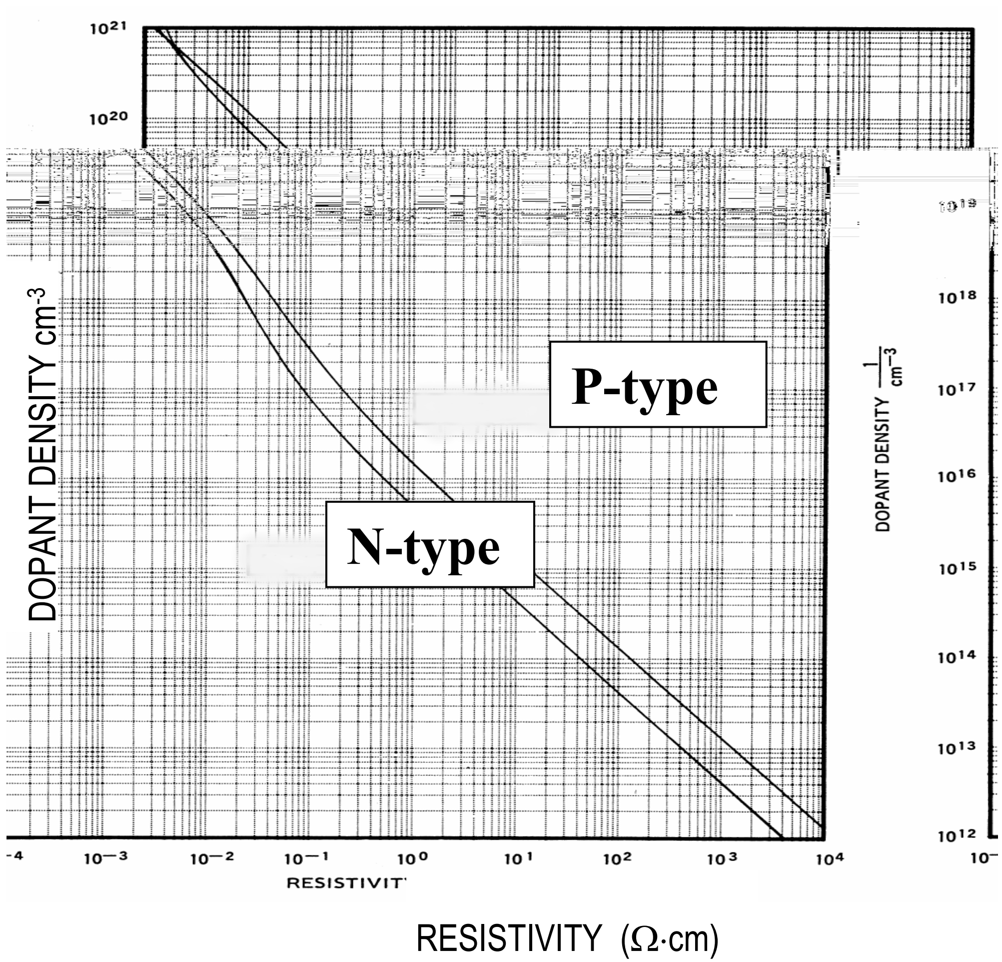
$$J_{p,drift} = qpv = qp\mu_p\mathcal{E}$$

$$J_{n,drift} = -qnv = qn\mu_n\mathcal{E}$$

$$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma\mathcal{E} = (qn\mu_n + qp\mu_p)\mathcal{E}$$

$$\therefore \ \textit{conductivity of a semiconductor is } \sigma = qn\mu_n + qp\mu_p$$

Relationship between Resistivity and Dopant Density



$$\rho = 1/\sigma$$

EXAMPLE: Temperature Dependence of Resistance

(a) What is the resistivity (ρ) of silicon doped with 10^{17}cm^{-3} of arsenic?

(b) What is the resistance (R) of a piece of this silicon material 1μm long and 0.1 μm^2 in cross-sectional area?

Solution:

(a) Using the N-type curve in the previous figure, we find that ρ = 0.084 Ω-cm.

(b) $R = \rho L/A = 0.084\ \Omega\text{-cm} \times 1\ \mu\text{m} / 0.1\ \mu\text{m}^2$

$= 0.084\ \Omega\text{-cm} \times 10^{-4}\ \text{cm}/ 10^{-10}\ \text{cm}^2$

$= 8.4 \times 10^{-4}$

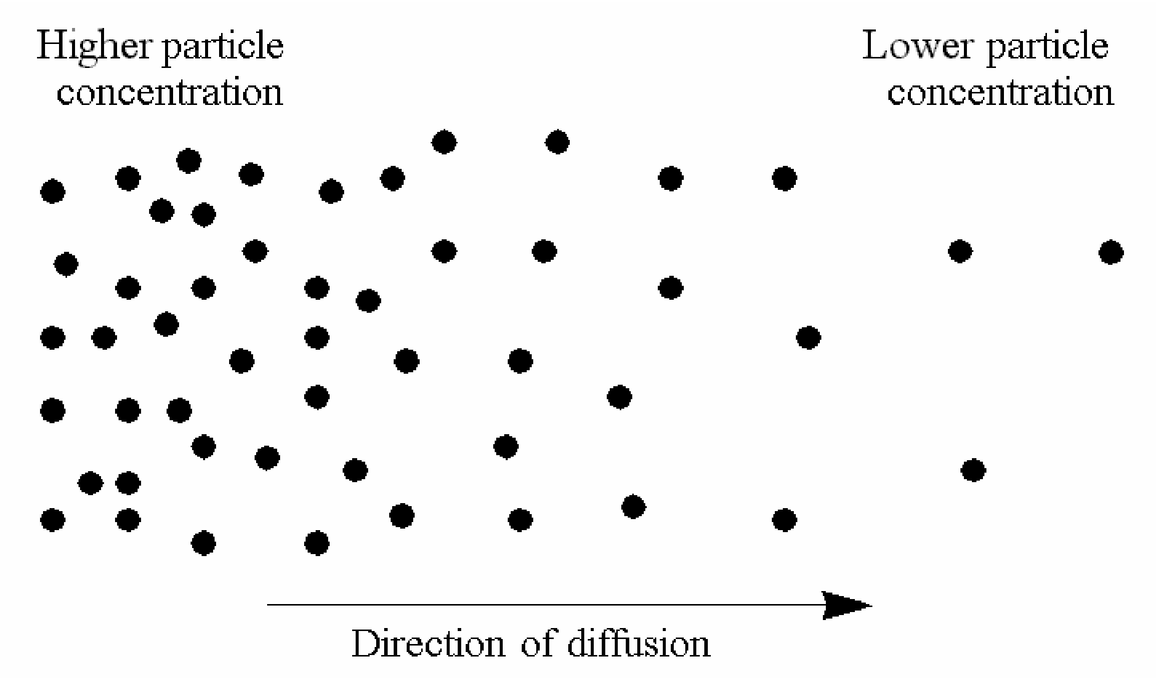
EXAMPLE: Temperature Dependence of Resistance

By what factor will R increase or decrease from T=300 K to T=400 K?

Solution: *The temperature dependent factor in σ (and therefore ρ) is μ_n. From the mobility vs. temperature curve for 10^{17} cm^{-3}, we find that μ_n decreases from 770 at 300K to 400 at 400K. As a result, R* ***increases*** *by*

$$\frac{770}{400} = 1.93$$

2.3 Diffusion Current

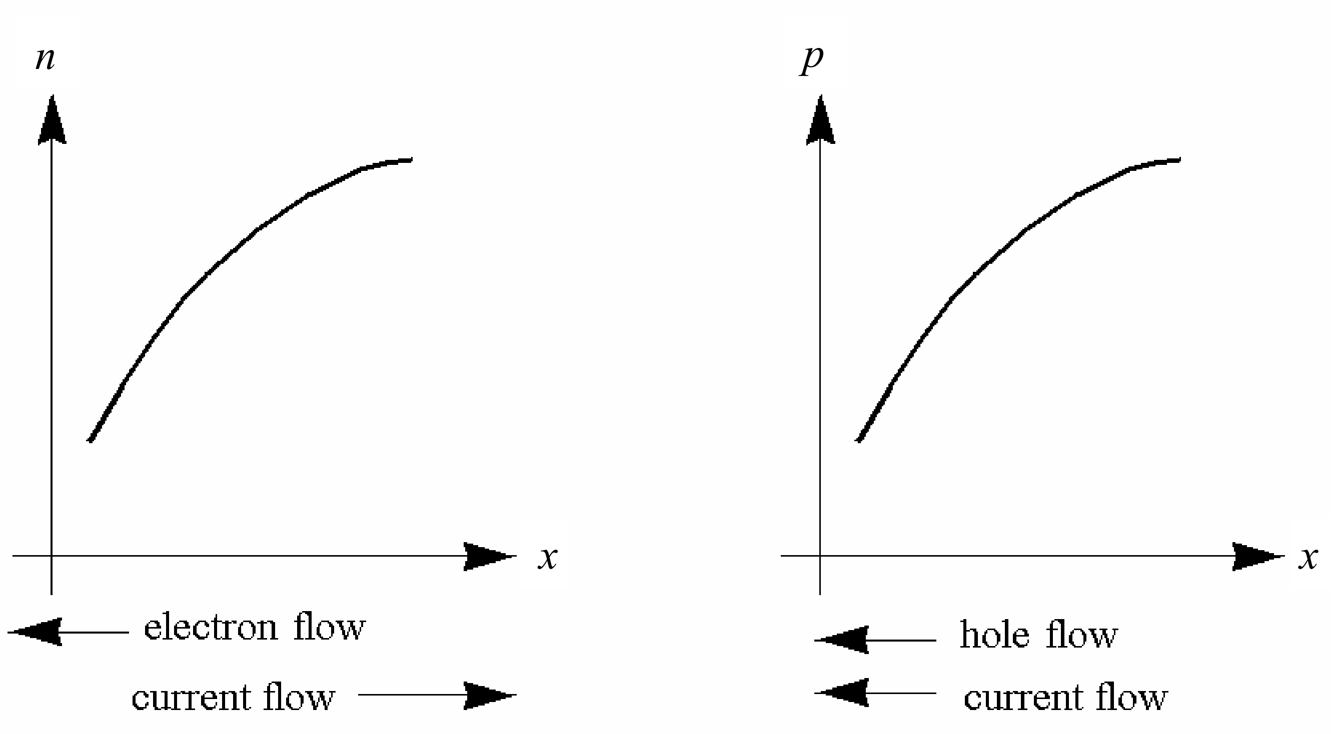


Particles diffuse from a higher-concentration location to a lower-concentration location.

2.3 Diffusion Current

$$J_{n,diffusion} = qD_n \frac{dn}{dx} \qquad J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

D is called the diffusion constant. Signs explained:



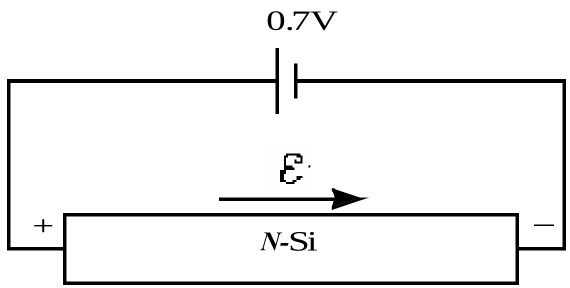
Total Current – Review of Four Current Components

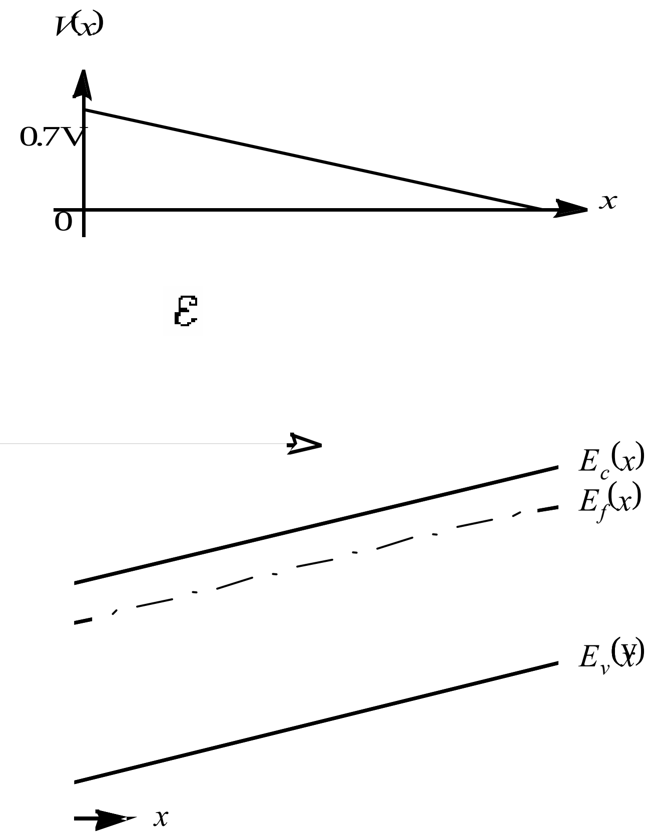
$$\boxed{J_{TOTAL} = J_n + J_p}$$

$$J_n = J_{n,drift} + J_{n,diffusion} = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diffusion} = qp\mu_p \mathcal{E} - qD_p \frac{dp}{dx}$$

2.4 Relation Between the Energy Band Diagram and V, $\mathcal{E}$



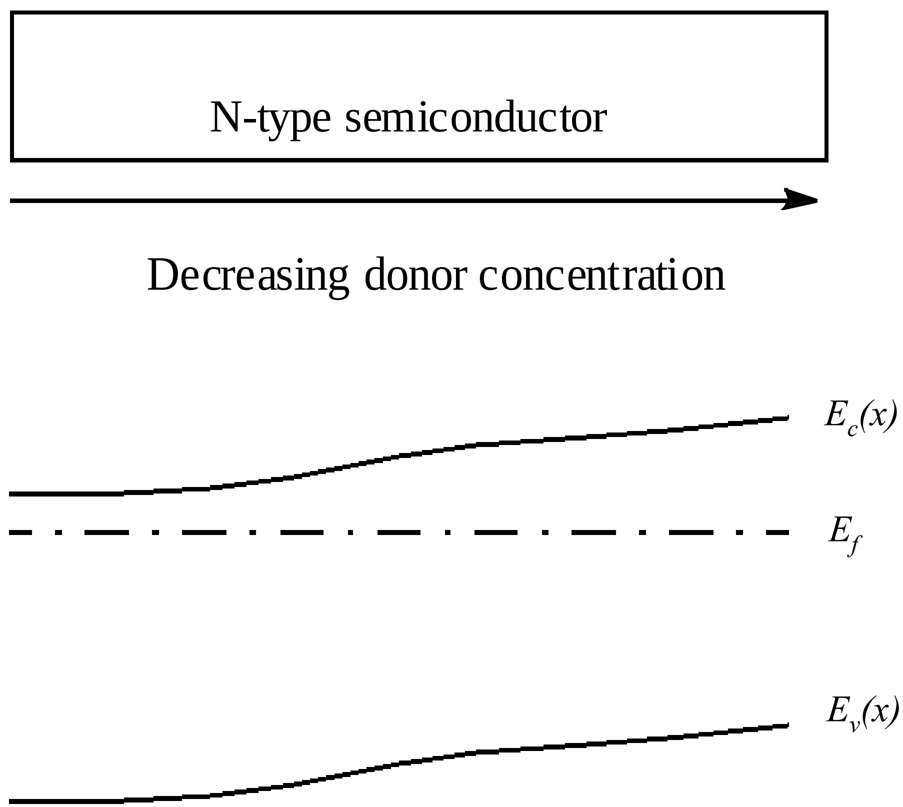


E_c and E_v vary in the opposite direction from the voltage. That is, E_c and E_v are higher where the voltage is lower.

$$\mathcal{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$

2.5 Einstein Relationship between D and μ

Consider a piece of non-uniformly doped semiconductor.



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q \mathcal{E}$$

2.5 Einstein Relationship between D and μ

$$\frac{dn}{dx} = -\frac{n}{kT} q \mathcal{E}$$

$$J_n = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx} = 0 \quad \text{at equilibrium.}$$

$$0 = qn\mu_n \mathcal{E} - qn \frac{qD_n}{kT} \mathcal{E}$$

$$\boxed{D_n = \frac{kT}{q} \mu_n} \quad \text{Similarly,} \quad \boxed{D_p = \frac{kT}{q} \mu_p}$$

These are known as the <u>Einstein relationship</u>.

EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with $\mu_p = 410\ cm^2\ V^{-1}s^{-1}$?

Solution:

$$D_p = \left(\frac{kT}{q}\right)\mu_p = (26\,\mathrm{mV})\cdot 410\,\mathrm{cm}^2\mathrm{V}^{-1}\mathrm{s}^{-1} = 11\,\mathrm{cm}^2/\mathrm{s}$$

Remember: kT/q = 26 mV at room temperature.

2.6 Electron-Hole Recombination

The equilibrium carrier concentrations are denoted with n_0 and p_0. The total electron and hole concentrations can be different from n_0 and p_0 . These differences are called the ***excess carrier concentrations*** n' and p'.

$$\boxed{\begin{aligned} n &\equiv n_0 + n' \\ p &\equiv p_0 + p' \end{aligned}}$$

Charge Neutrality

Charge neutrality is satisfied at equilibrium ($n' = p' = 0$). When a non-zero n' is present, an equal p' may be assumed to be present to maintain charge equality and vice-versa. If charge neutrality is not satisfied, then the net charge will attract or repel the (majority) carriers through the drift current until neutrality is restored.

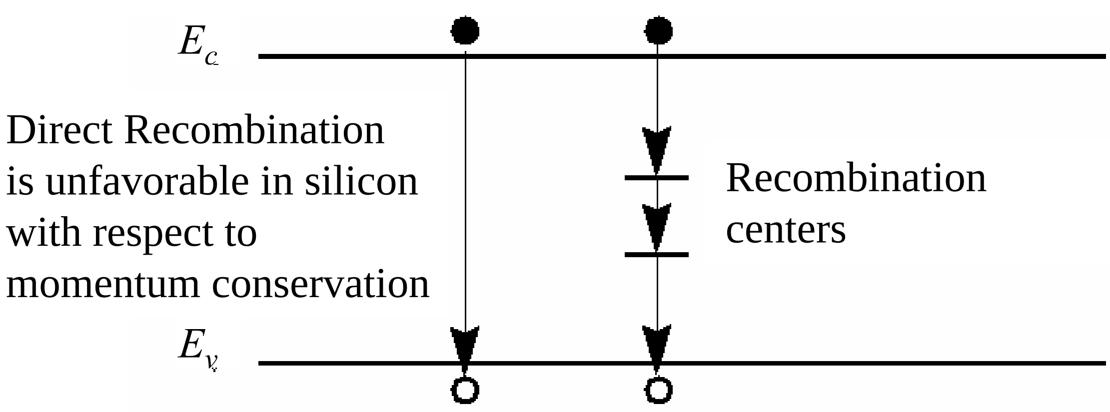
$$\boxed{n' = p'}$$

Recombination Lifetime

Assume light generates n' and p'. If the light is suddenly turned off, n' and p' decay with time until they become zero. The process of decay is called ***recombination.*** The time constant of decay is the ***recombination time*** or ***carrier lifetime,*** τ. Recombination is nature's way of restoring equilibrium ($n' = p' = 0$).

Recombination Lifetime

τ ranges from 1ns to 1ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt. These ***deep traps*** capture electrons or holes to facilitate recombination and are called ***recombination centers.***



Rate of recombination ($s^{-1}cm^{-3}$)

$$\frac{dn'}{dt} = -\frac{n'}{\tau}$$

$$n' = p'$$

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} = \frac{dp'}{dt}$$

EXAMPLE: Photoconductors

A bar of Si is doped with boron at 10^{15}cm^{-3}. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{20}/s·cm^3. The recombination lifetime is 10μs. What are (a) p_0, (b) n_0, (c) p', (d) n', (e) p, (f) n, and (g) the np product?

EXAMPLE: Photoconductors

Solution:

(a) What is p_0?

$$p_0 = N_a = 10^{15}\ \mathrm{cm^{-3}}$$

(b) What is n_0 ?

$$n_0 = n_i^2/p_0 = 10^5\ \mathrm{cm^{-3}}$$

(c) What is p'?

In steady-state, the rate of generation is equal to the rate of recombination.

$$10^{20}/\mathrm{s\text{-}cm^3} = p'/\tau$$

$$\therefore\ p' = 10^{20}/\mathrm{s\text{-}cm^3} \cdot 10^{-5}\mathrm{s} = 10^{15}\ \mathrm{cm^{-3}}$$

EXAMPLE: Photoconductors

(d) What is n′?

$n' = p' = 10^{15}\ \text{cm}^{-3}$

(e) What is p?

$p = p_0 + p' = 10^{15}\text{cm}^{-3} + 10^{15}\text{cm}^{-3} = 2\times10^{15}\text{cm}^{-3}$

(f) What is n?

$n = n_0 + n' = 10^{5}\text{cm}^{-3} + 10^{15}\text{cm}^{-3} \sim 10^{15}\text{cm}^{-3}$ *since* $n_0 << n'$

(g) What is np?

$np \sim 2\times10^{15}\text{cm}^{-3} \cdot 10^{15}\text{cm}^{-3} = 2\times10^{30}\ \text{cm}^{-6} >> n_i^2 = 10^{20}\ \text{cm}^{-6}$.

The np product can be very different from n_i^2.

2.7 Thermal Generation

If n' is negative, then there are fewer electrons than the equilibrium value.

As a result, there is a net rate of ***thermal generation*** at the rate of $|n'|/\tau$.

2.8 Quasi-equilibrium and Quasi-Fermi Levels

Whenever $n' = p' \neq 0$, $np \neq n_i^2$. However, we would like to preserve and use the relations:

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

But these equations lead to $np = n_i^2$. The solution is to introduce two ***quasi-Fermi levels*** E_{fn} and E_{fp} such that

$$n = N_c e^{-(E_c - E_{fn})/kT}$$

$$p = N_v e^{-(E_{fp} - E_v)/kT}$$

Even when electrons and holes are not at equilibrium, *within each group* the carriers are usually at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.

EXAMPLE: Quasi-Fermi Levels

Consider a Si sample with $N_d = 10^{17}\,cm^{-3}$ *and* $n' = p' = 10^{15}\,cm^{-3}$.

(a) Find E_f.

$$n = N_d = 10^{17}\ \mathrm{cm^{-3}} = N_c \exp[-(E_c - E_f)/kT]$$

$$\therefore\ E_c - E_f = 0.15\ \mathrm{eV}. \quad (E_f \textit{ is below } E_c \textit{ by 0.15 eV.})$$

Now assume $n' = p' = 10^{15}\ cm^{-3}$.

(b) Find E_{fn} *and* E_{fp}.

Note: n′ and p′ are much less than the majority carrier concentration. This condition is called ***low-level injection.***

EXAMPLE: Quasi-Fermi Levels

$$n = 1.01\times10^{17}\text{cm}^{-3} = N_c e^{-(E_c - E_{fn})/kT}$$

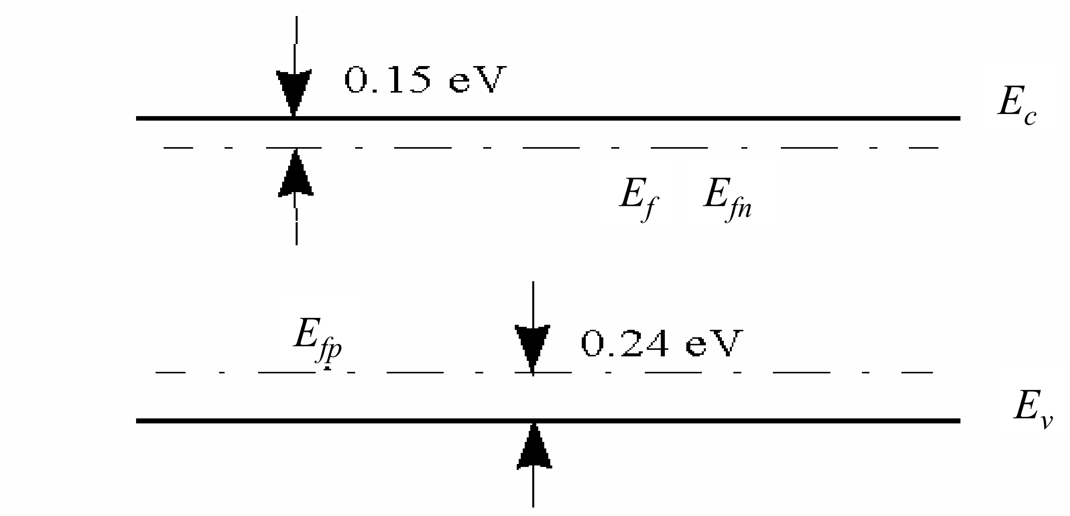
$$\therefore\ E_c - E_{fn} = kT \times \ln(N_c/1.01\times10^{17}\text{cm}^{-3})$$
$$= 26\ \text{meV} \times \ln(2.8\times10^{19}\text{cm}^{-3}/1.01\times10^{17}\text{cm}^{-3})$$
$$= 0.15\ \text{eV}$$

E_{fn} is nearly identical to E_f because $n \approx n_0$.

EXAMPLE: Quasi-Fermi Levels

$$p = 10^{15}\ \text{cm}^{-3} = N_v e^{-(E_{fp}-E_v)/kT}$$

$$\therefore E_{fp} - E_v = kT \times \ln(N_v/10^{15}\text{cm}^{-3})$$
$$= 26\ \text{meV} \times \ln(1.04\times10^{19}\text{cm}^{-3}/10^{15}\text{cm}^{-3})$$
$$= 0.24\ \text{eV}$$



2.9 Chapter Summary

$$v_p = \mu_p \mathcal{E}$$

$$v_n = \mu_n \mathcal{E}$$

$$J_{p,drift} = qp\mu_p \mathcal{E}$$

$$J_{n,drift} = qn\mu_n \mathcal{E}$$

$$J_{n,diffusion} = qD_n \frac{dn}{dx}$$

$$J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

$$D_n = \frac{kT}{q}\mu_n$$

$$D_p = \frac{kT}{q}\mu_p$$

2.9 Chapter Summary

τ is the recombination lifetime.

n' and p' are the ***excess carrier concentrations***.

$$\boxed{\begin{aligned} n &= n_0 + n' \\ p &= p_0 + p' \end{aligned}}$$

Charge neutrality requires $n' = p'$.

$$\boxed{\textit{rate of recombination} = n'/\tau = p'/\tau}$$

E_{fn} and E_{fp} are the quasi-Fermi levels of electrons and holes.

$$\boxed{\begin{aligned} n &= N_c e^{-(E_c - E_{fn})/kT} \\ p &= N_v e^{-(E_{fp} - E_v)/kT} \end{aligned}}$$