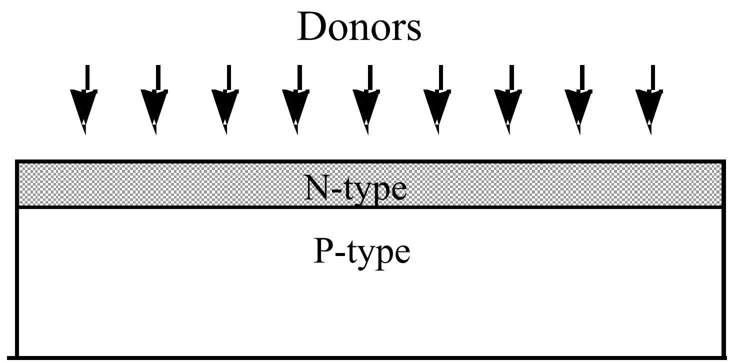
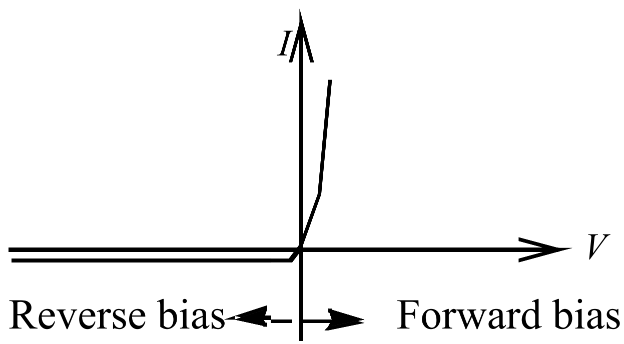
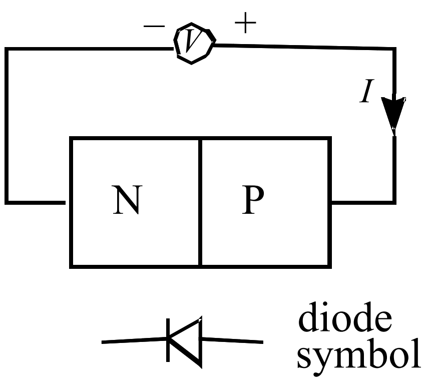
Chapter 4   PN Junctions

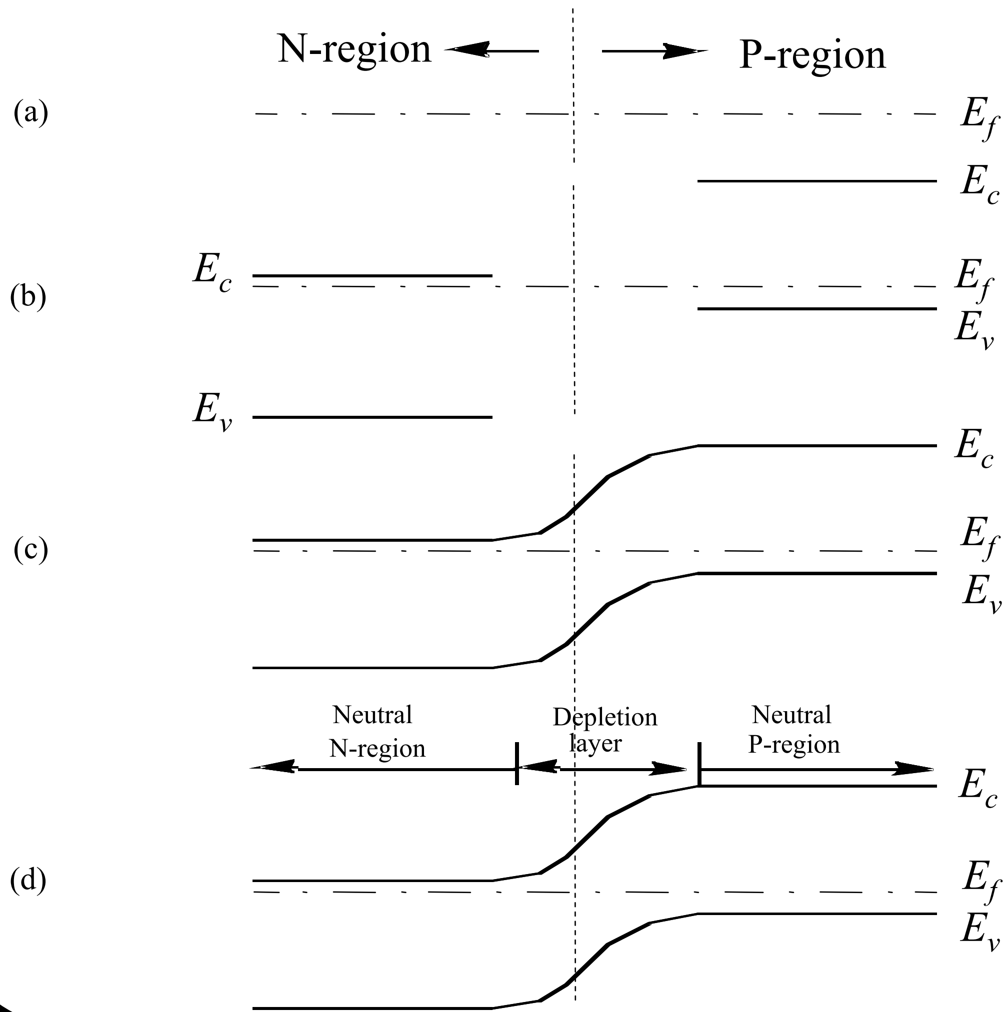






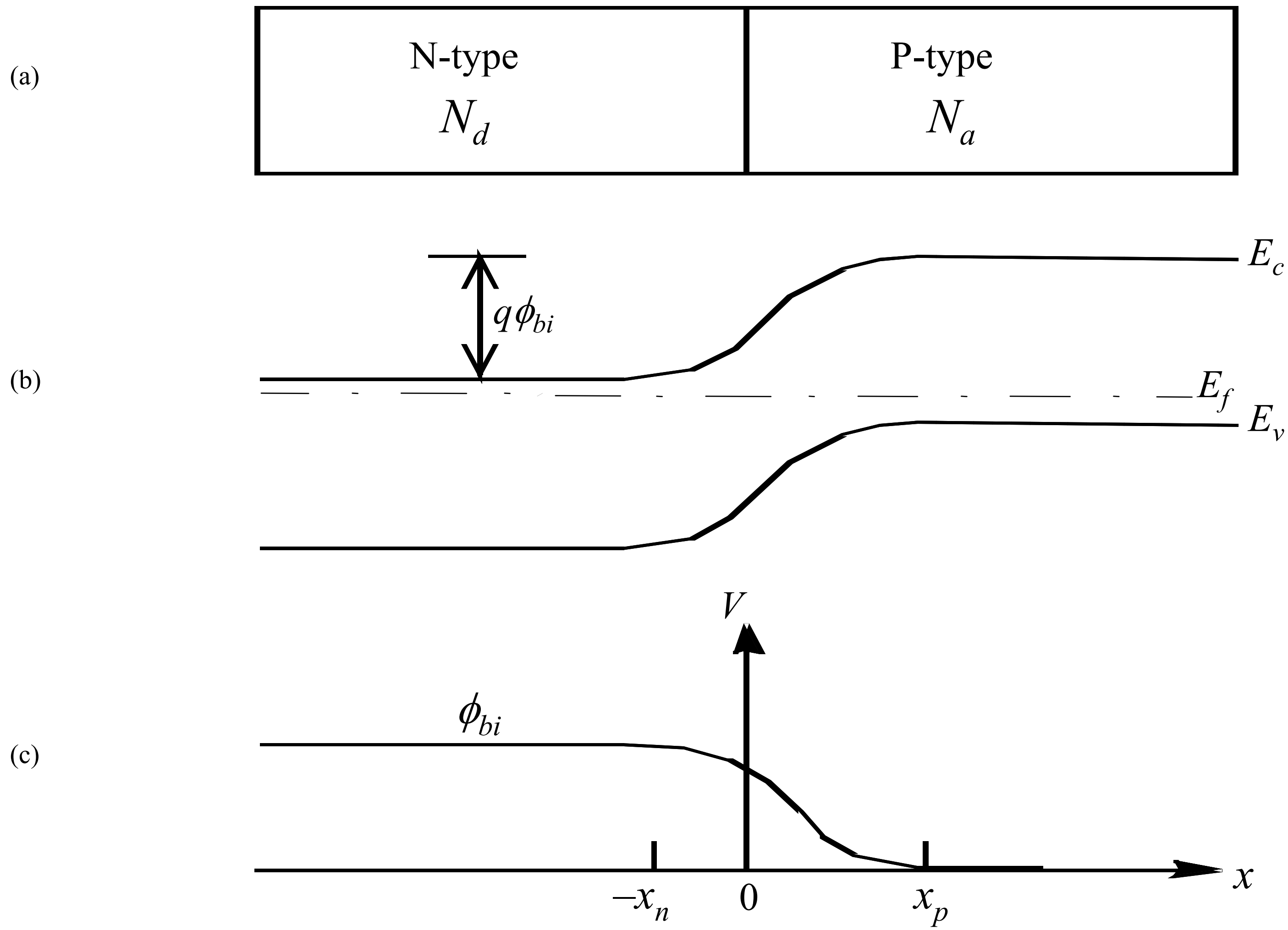
A PN junction is present in every semiconductor device.

4.1.1 Energy Band Diagram and Depletion Layer of a PN Junction



A depletion layer exists at the PN junction. $n \approx 0$ and $p \approx 0$ in the depletion layer.

4.1.2 Built-in Potential



Can the built-in potential be measured with a voltmeter?

Thermal Couple and Thermoelectric Generator

The total built-in voltage in a closed circuit is zero, therefore it cannot be read.

Thermal Couple:
When the junction of two wires, such as W and Pt, is placed in a furnace, a non-zero voltage can be read with a voltmeter between the two cold ends.

Thermoelectric Generator:
In addition to voltage, significant electric power can also be extracted. Heat-to-electricity conversion efficiency can be optimized by using exotic semiconductors of P and N types.

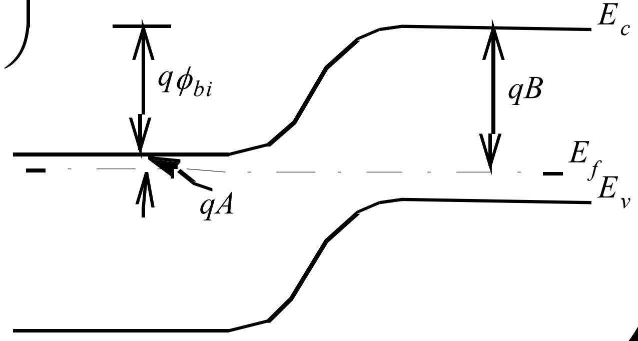
4.1.2 Built-in Potential

N-region $\quad n = N_d = N_c e^{-qA/kT} \Rightarrow A = \dfrac{kT}{q} \ln \dfrac{N_c}{N_d}$

P-region $\quad n = \dfrac{n_i^2}{N_a} = N_c e^{-qB/kT} \Rightarrow B = \dfrac{kT}{q} \ln \dfrac{N_c N_a}{n_i^2}$

$$\phi_{bi} = B - A = \frac{kT}{q}\left(\ln \frac{N_c N_a}{n_i^2} - \ln \frac{N_c}{N_d} \right)$$

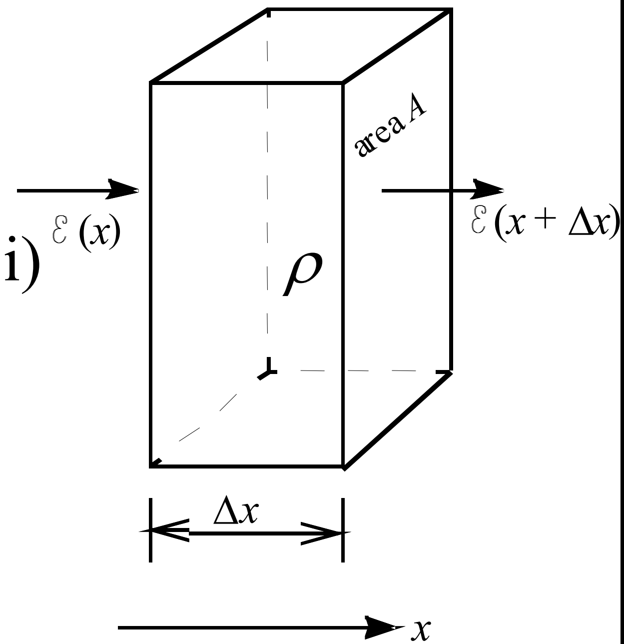
$$\boxed{\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$



4.1.3 Poisson's Equation

Gauss's Law:

$$\varepsilon_s \mathcal{E}(x+\Delta x)A - \varepsilon_s \mathcal{E}(x)A = \rho \Delta x A$$



ε_s: semiconductor permittivity (~ 12 for Si)

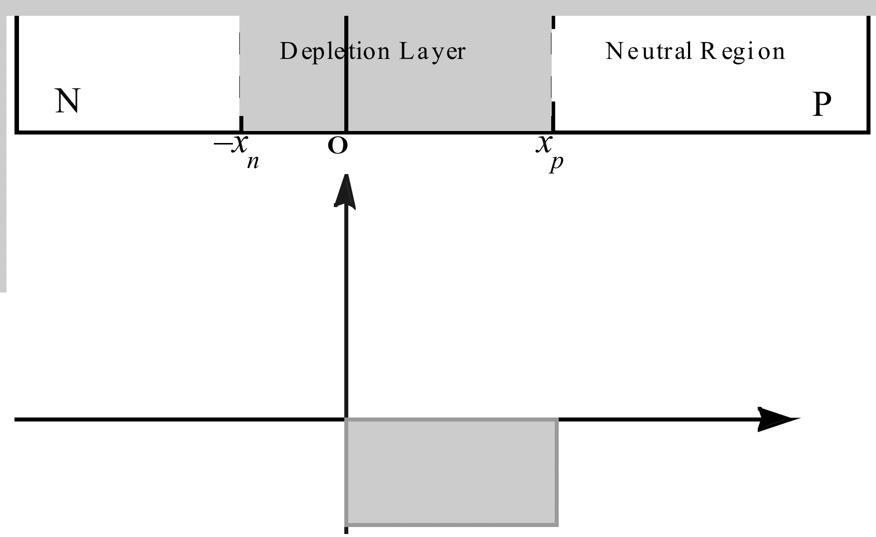
ρ: charge density (C/cm^3)

$$\frac{\mathcal{E}(x+\Delta x) - \mathcal{E}(x)}{\Delta x} = \frac{\rho}{\varepsilon_s}$$

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_s}$$

$$\boxed{\frac{d^2V}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho}{\varepsilon_s}}$$

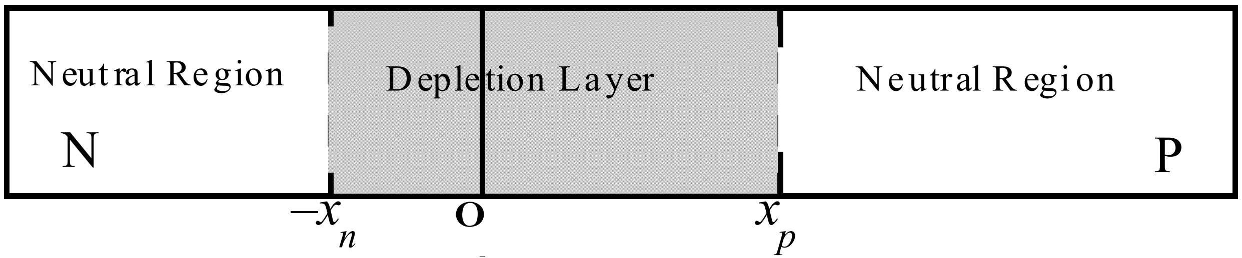
→ ***What can Poisson's equation tell us?***



On the ***P-side*** of the depletion layer, $\rho = -qN_a$

On the ***N-side***, $\rho = qN_d$

4.2.1 Field and Potential in the Depletion Layer



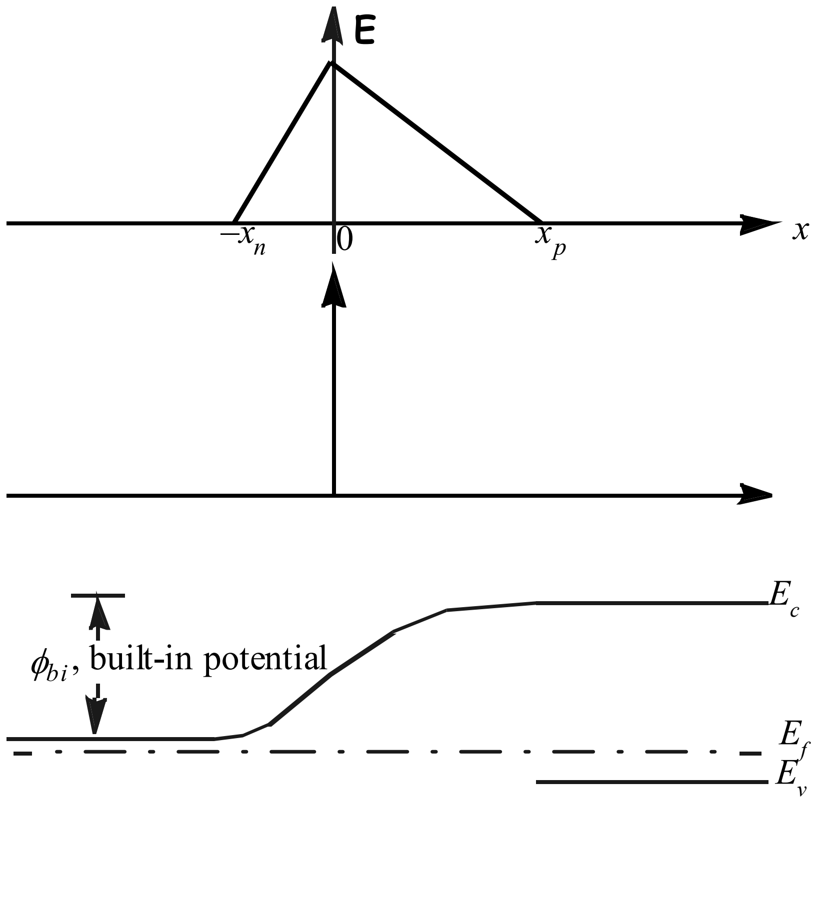
The electric field is continuous at $x = 0$.

$$\boxed{N_a x_p = N_d x_n}$$

Which side of the junction is depleted more?

A one-sided junction is called a ***N⁺P junction*** or ***P⁺N junction***

4.2.1 Field and Potential in the Depletion Layer



On the P-side,

$$V(x) = \frac{qN_a}{2\varepsilon_s}(x_p - x)^2$$

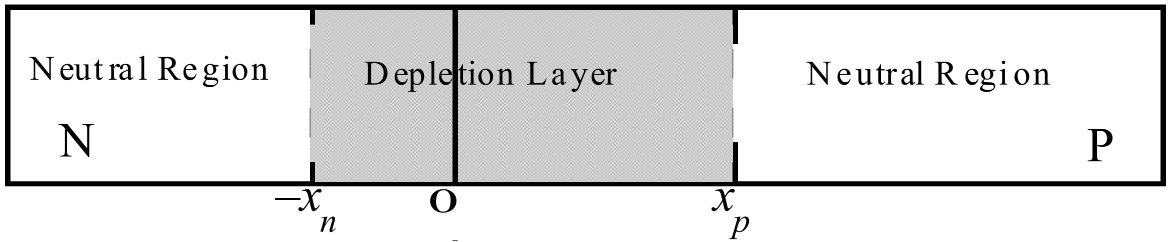
Arbitrarily choose the voltage at $x = x_p$ as $V = 0$.

On the N-side,

$$V(x) = D - \frac{qN_d}{2\varepsilon_s}(x + x_n)^2$$

$$= \phi_{bi} - \frac{qN_d}{2\varepsilon_s}(x + x_n)^2$$

4.2.2 Depletion-Layer Width



V is continuous at $x = 0$

$$x_n + x_p = W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}$$

If $N_a >> N_d$, as in a P$^+$N junction,

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}}} \approx x_n \quad \longrightarrow \quad \text{What about a N}^+\text{P junction?}$$

$$x_p = x_n N_a / N_d \cong 0$$

$$W_{dep} = \sqrt{2\varepsilon_s \phi_{bi} / qN} \quad \text{where} \quad \frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{\textit{lighter dopant density}}$$

EXAMPLE: *A P^+N junction has $N_a=10^{20}$ cm^{-3} and $N_d = 10^{17}$cm^{-3}. What is a) its built in potential, b) W_{dep} , c) x_n , and d) x_p ?*

Solution:

a)

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2} = 0.026\text{V} \ln \frac{10^{20} \times 10^{17}\,\text{cm}^{-6}}{10^{20}\,\text{cm}^{-6}} \approx 1\,\text{V}$$

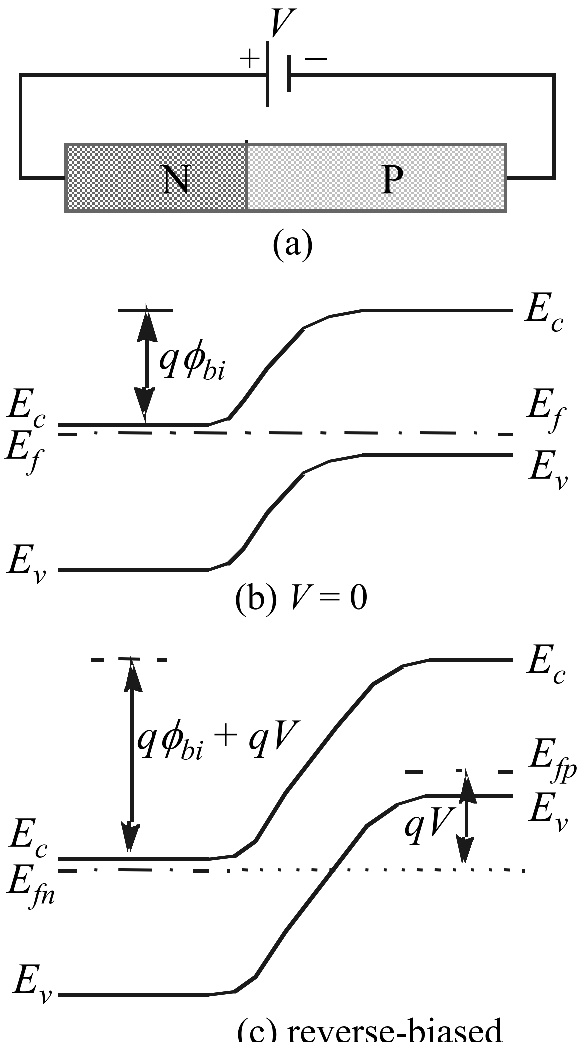
b)

$$W_{dep} \approx \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}} = \left(\frac{2\times 12\times 8.85\times 10^{-14} \times 1}{1.6\times 10^{-19} \times 10^{17}} \right)^{1/2} = 0.12\,\mu\text{m}$$

c) $\approx \quad = 0.12\,\mu\text{m}$

d)

4.3 Reverse-Biased PN Junction

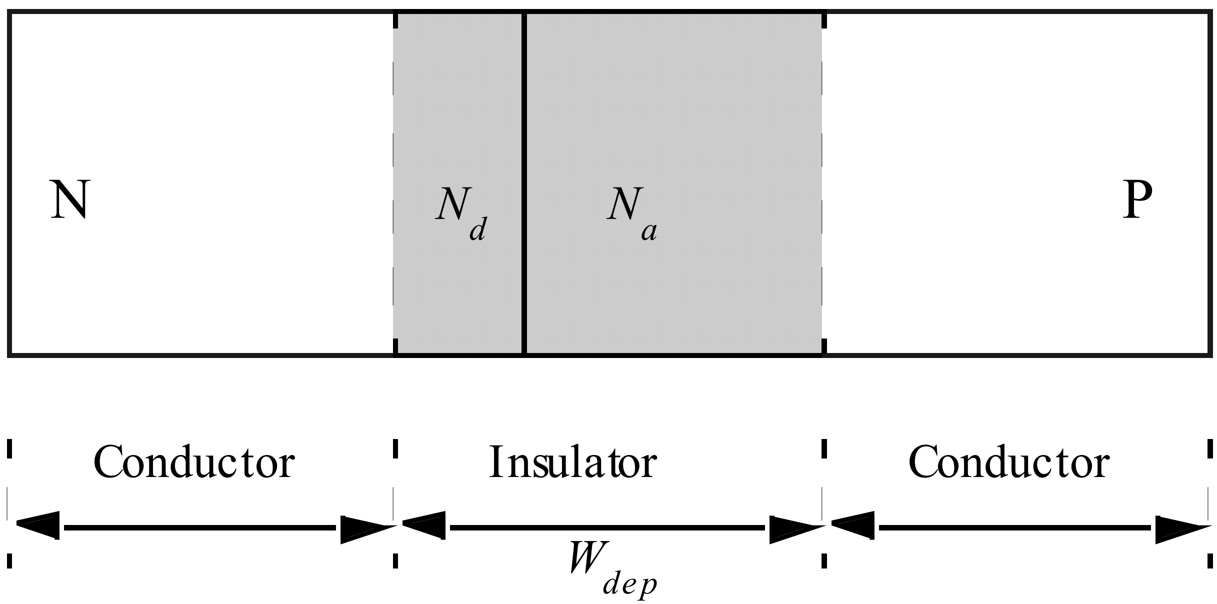


$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} + |V_r|)}{qN}} = \sqrt{\frac{2\varepsilon_s \cdot potential\ barrier}{qN}}}$$

$$\frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{lighter\ dopant\ density}$$

- ***Does the depletion layer widen or shrink with increasing reverse bias?***

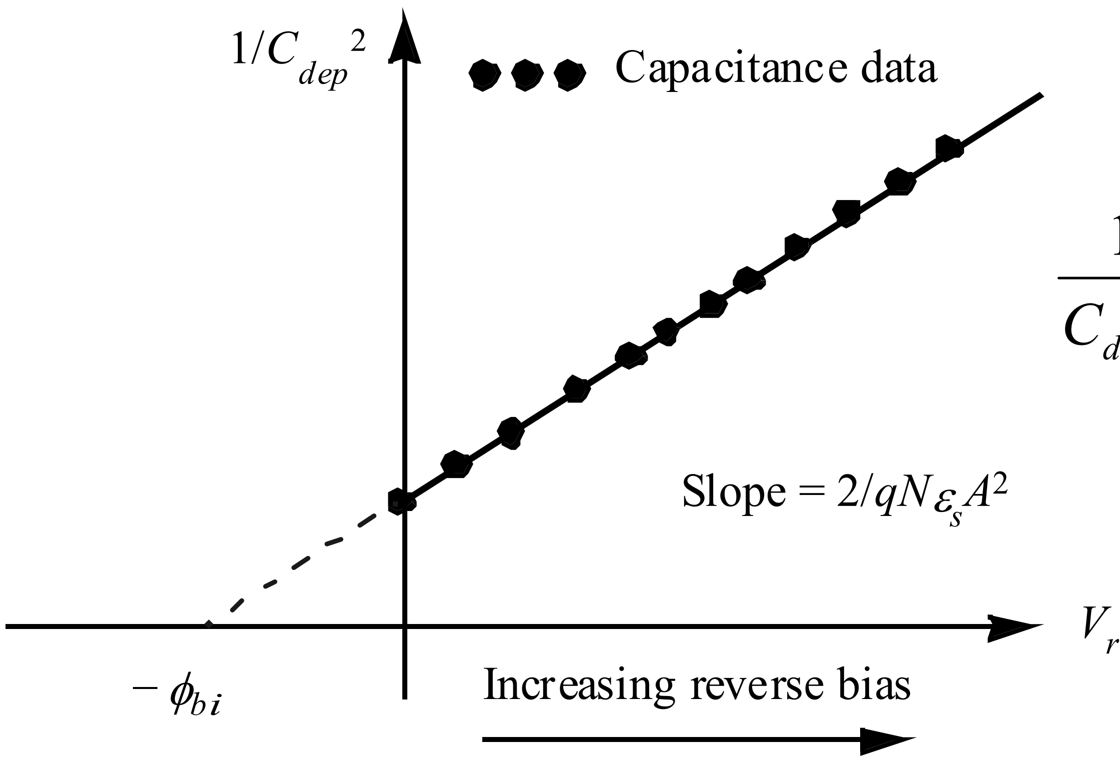
4.4 Capacitance-Voltage Characteristics



$$C_{dep} = A\frac{\varepsilon_s}{W_{dep}}$$

- Is C_{dep} a good thing?
- What are three ways to reduce C_{dep}?

4.4 Capacitance-Voltage Characteristics



$$\frac{1}{C_{dep}^2} = \frac{W_{dep}^2}{A^2 \varepsilon_s^2} = \frac{2(\phi_{bi} + V)}{qN\varepsilon_S A^2}$$

- Is there a way to obtain both N_a and N_d of a P^+N junction?

EXAMPLE: *If the slope of the line in the previous slide is $2x10^{23}$ F^{-2} V^{-1}, the intercept is 0.84V, and A is 1 μm^2, find the lighter and heavier doping concentrations N_l and N_h.*

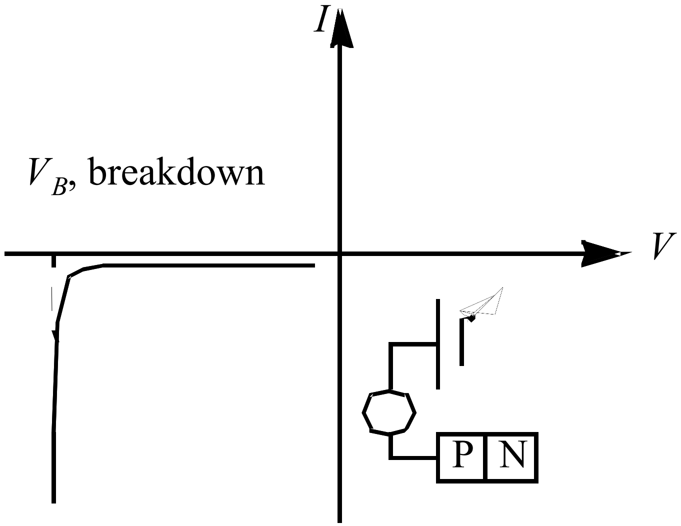
Solution:

$$N_l = 2/(slope \times q\varepsilon_s A^2)$$
$$= 2/(2\times10^{23}\times1.6\times10^{-19}\times12\times8.85\times10^{-14}\times10^{-8}\,\mathrm{cm}^2)$$
$$= 6\times10^{15}\ \mathrm{cm}^{-3}$$

$$\phi_{bi} = \frac{kT}{q}\ln\frac{N_h N_l}{n_i^2} \quad\Rightarrow\quad N_h = \frac{n_i^2}{N_l}e^{\frac{q\phi_{bi}}{kT}} = \frac{10^{20}}{6\times10^{15}}e^{\frac{0.84}{0.026}} = 1.8\times10^{18}\ \mathrm{cm}^{-3}$$

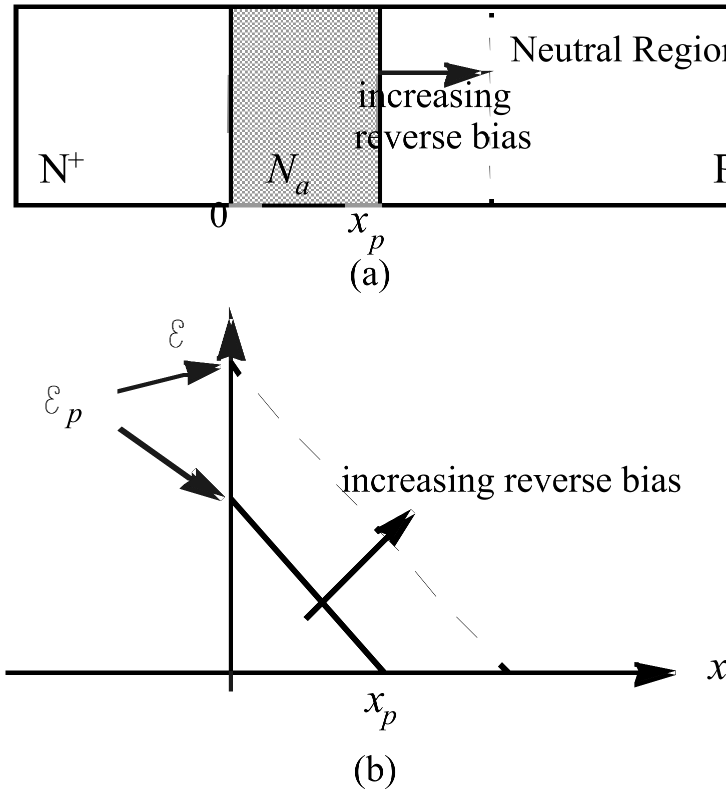
- *Is this an accurate way to determine N_l? N_h?*

4.5 Junction Breakdown



A ***Zener diode*** is designed to operate in the breakdown mode.

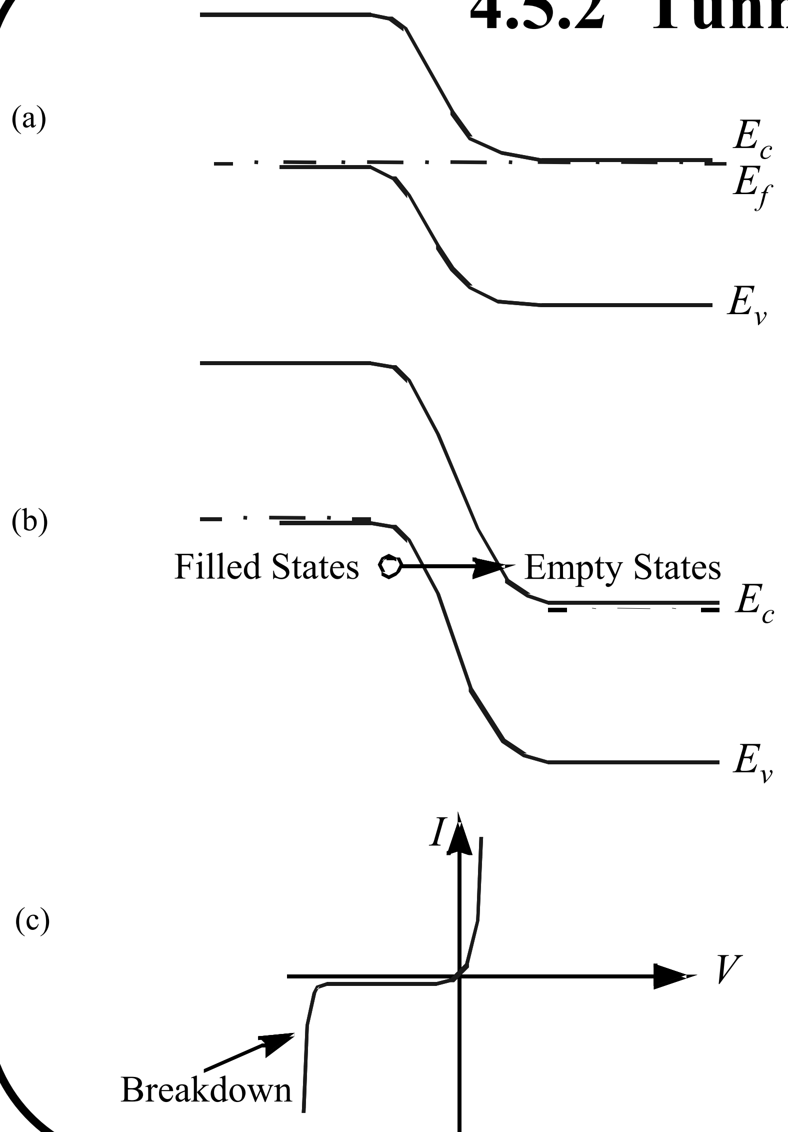
4.5.1 Peak Electric Field



$$\mathcal{E}_p = \mathcal{E}(0) = \left[\frac{2qN}{\varepsilon_s} (\phi_{bi} + |V_r|) \right]^{1/2}$$

- What kind of junction is shown in the figure?

4.5.2 Tunneling Breakdown

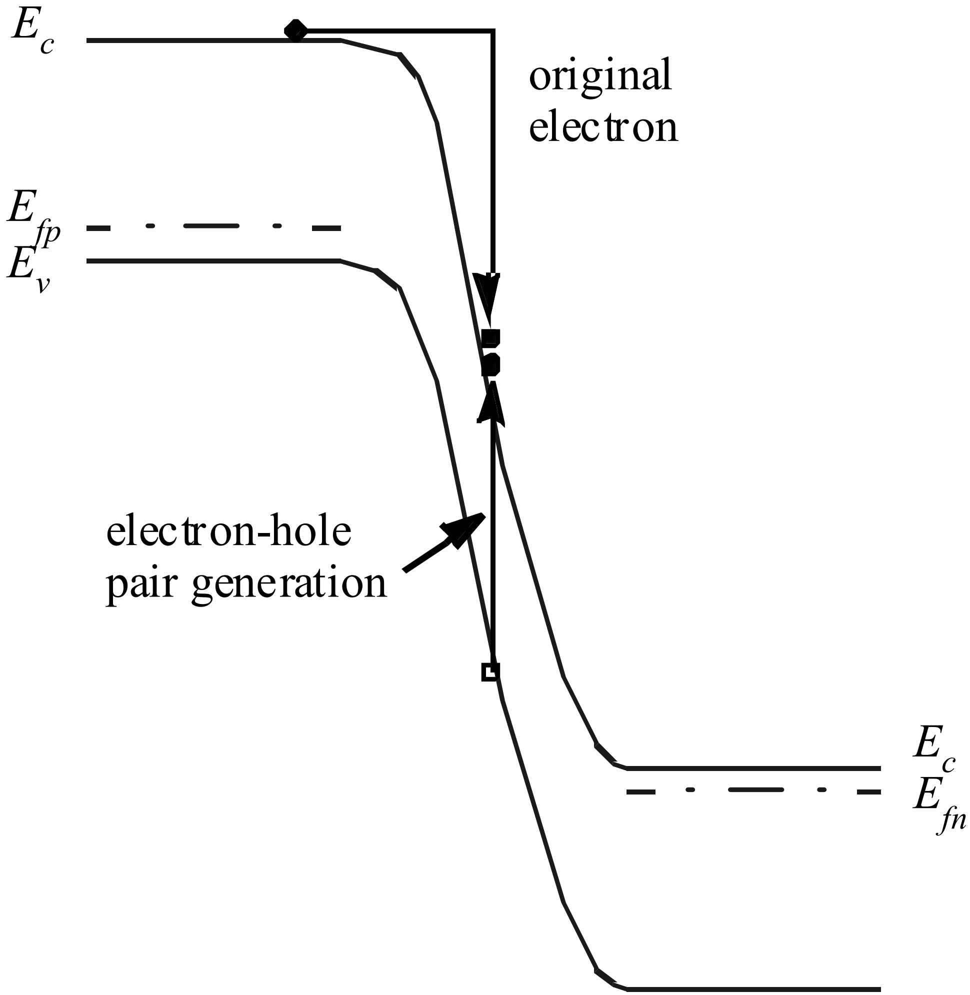


Dominant breakdown cause when both sides of a junction are very heavily doped.

$$V_B = \frac{\varepsilon_s \mathcal{E}_{crit}^{\;2}}{2qN} - \phi_{bi}$$

$$\mathcal{E}_p = \mathcal{E}_{crit} \approx 10^6 \text{ V/cm}$$

4.5.3 Avalanche Breakdown



impact ionization

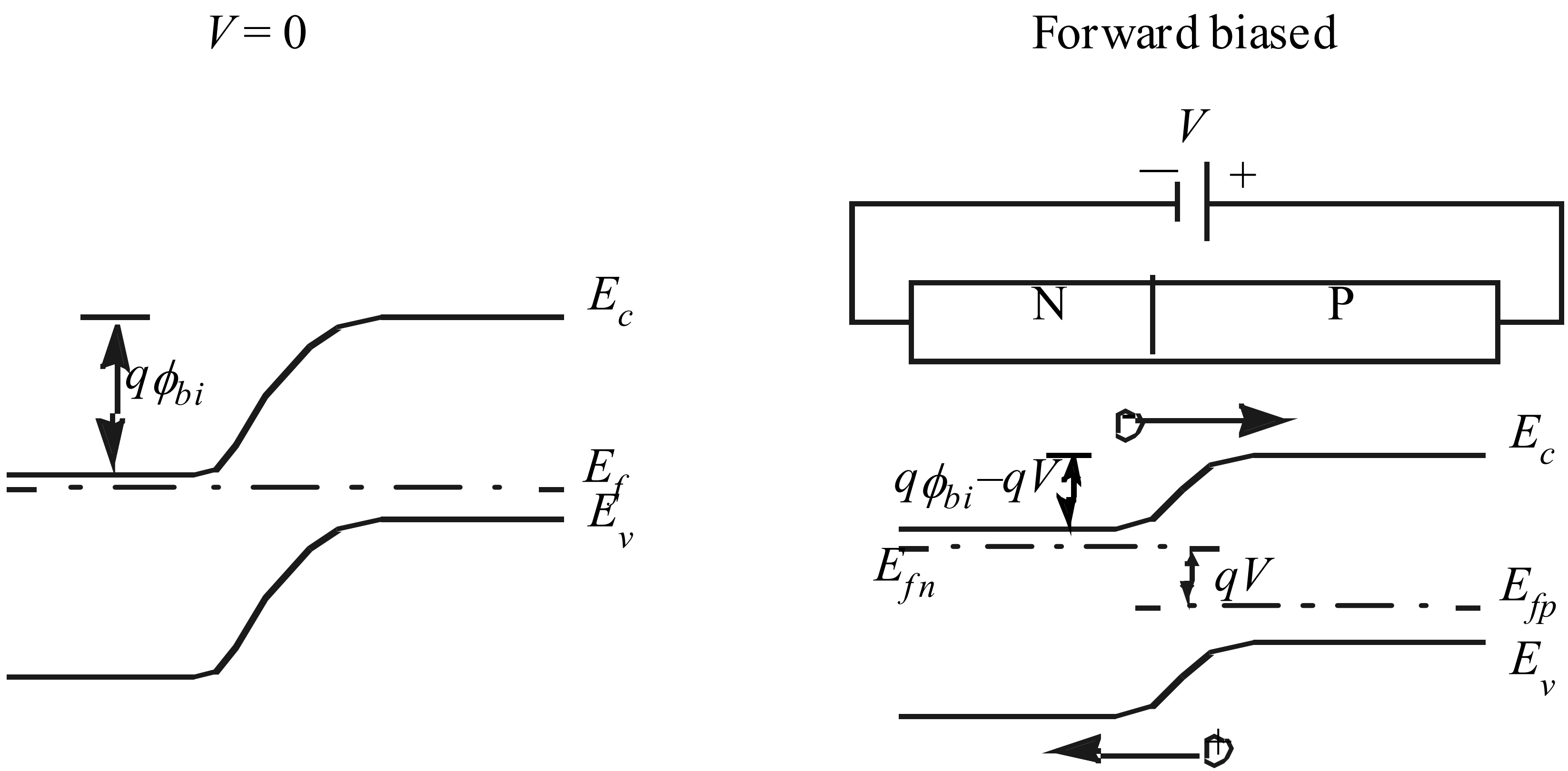
↓

avalanche breakdown

$$V_B = \frac{\varepsilon_s \mathcal{E}_{crit}^{\;2}}{2qN}$$

$$\boxed{V_B \propto \frac{1}{N}} = \frac{1}{N_a} + \frac{1}{N_d}$$

4.6 Carrier Injection Under Forward Bias– Quasi-equilibrium Boundary Condition

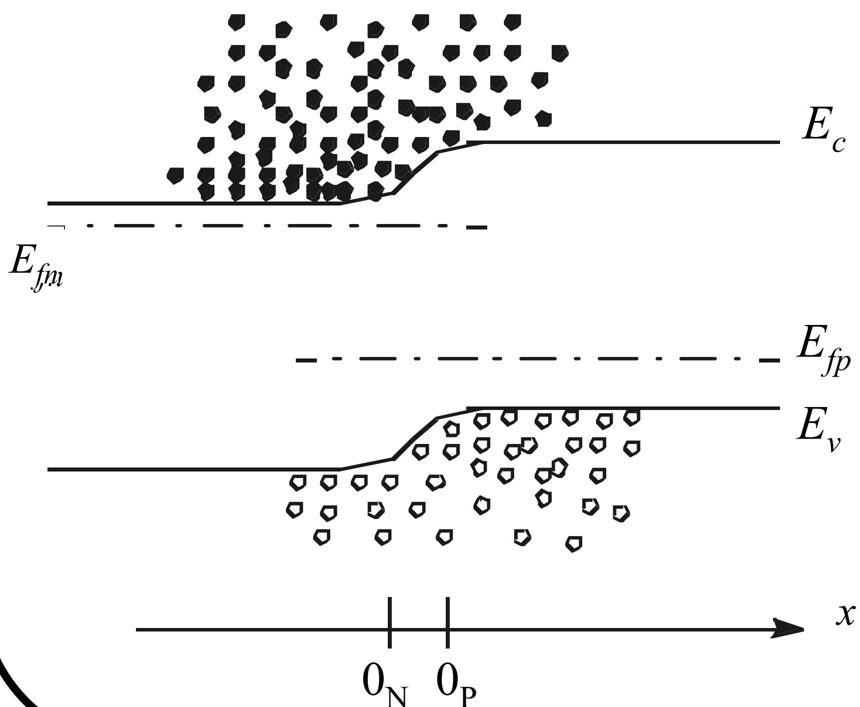


minority carrier injection

4.6 Carrier Injection Under Forward Bias– Quasi-equilibrium Boundary Condition

$$n(0_P) = N_c e^{-(E_c - E_{fn})/kT} = N_c e^{-(E_c - E_{fp})/kT} e^{(E_{fn} - E_{fp})/kT}$$

$$= n_{P0} e^{(E_{fn} - E_{fp})/kT} = n_{P0} e^{qV/kT}$$



- The minority carrier densities are raised by $e^{qV/kT}$
- Which side gets more carrier injection?

4.6 Carrier Injection Under Forward Bias– Quasi-equilibrium Boundary Condition

$$n(0) = n_{P0} e^{qV/kT} = \frac{n_i^2}{N_a} e^{qV/kT}$$

$$p(0) = p_{N0} e^{qV/kT} = \frac{n_i^2}{N_d} e^{qV/kT}$$

$$n'(0) \equiv n(0) - n_{P0} = n_{P0}(e^{qV/kT} - 1)$$

$$p'(0) \equiv p(0) - p_{N0} = p_{N0}(e^{qV/kT} - 1)$$

EXAMPLE: Carrier Injection

A PN junction has N_a=10^{19}cm^{-3} and N_d=10^{16}cm^{-3}. The applied voltage is 0.6 V.

***Question**: What are the minority carrier concentrations at the depletion-region edges?*

***Solution**:*

$$n(0) = n_{P0}e^{qV/kT} = 10 \times e^{0.6/0.026} = 10^{11}\ \text{cm}^{-3}$$

$$p(0) = p_{N0}e^{qV/kT} = 10^{4} \times e^{0.6/0.026} = 10^{14}\ \text{cm}^{-3}$$

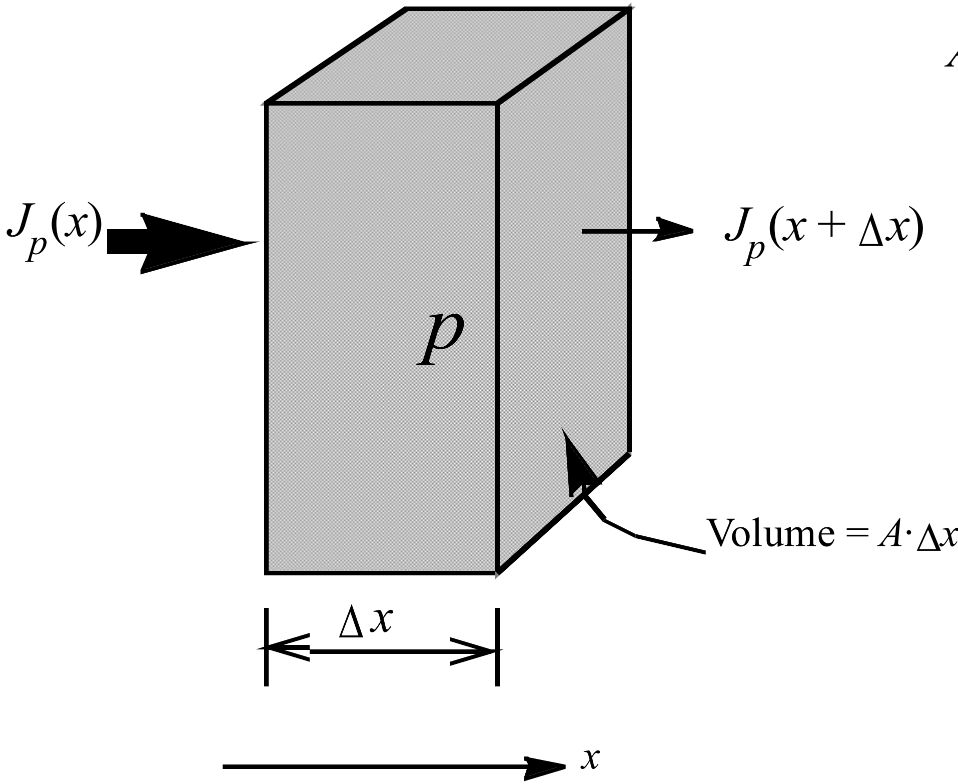
***Question**: What are the excess minority carrier concentrations?*

***Solution**:*

$$n'(0) = n(0) - n_{P0} = 10^{11} - 10 = 10^{11}\ \text{cm}^{-3}$$

$$p'(0) = p(0) - p_{N0} = 10^{14} - 10^{4} = 10^{14}\ \text{cm}^{-3}$$

4.7 Current Continuity Equation



$$A \cdot \frac{J_p(x)}{q} = A \cdot \frac{J_p(x+\Delta x)}{q} + A \cdot \Delta x \cdot \frac{p'}{\tau}$$

$$-\frac{J_p(x+\Delta x) - J_p(x)}{\Delta x} = q\frac{p'}{\tau}$$

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

4.7 Current Continuity Equation

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

Minority drift current is negligible;

$\therefore\ J_p = -qD_p dp/dx$

$$qD_p\frac{d^2p}{dx^2} = q\frac{p'}{\tau_p}$$

$$\boxed{\frac{d^2p'}{dx^2} = \frac{p'}{D_p\tau_p} = \frac{p'}{L_p{}^2}}$$

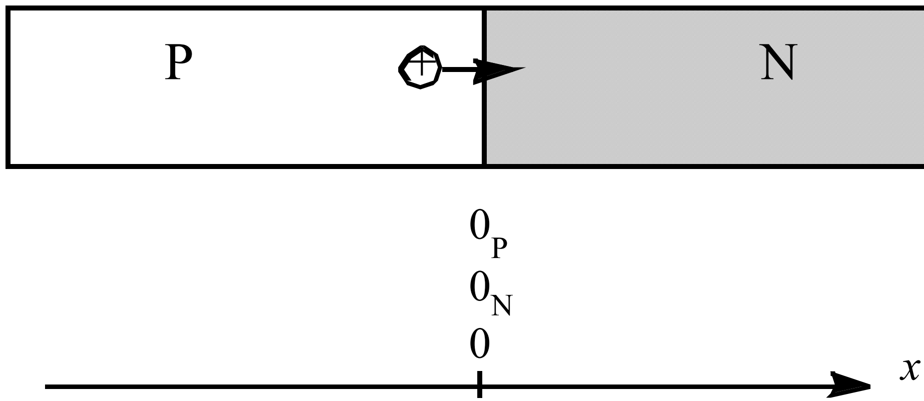
$$\boxed{\frac{d^2n'}{dx^2} = \frac{n'}{L_n{}^2}}$$

L_p and L_n are the diffusion lengths

$$\boxed{L_p \equiv \sqrt{D_p\tau_p}}$$

$$\boxed{L_n \equiv \sqrt{D_n\tau_n}}$$

4.8 Excess Carrier Distribution in Biased PN Junction



$$\frac{d^2 p'}{dx^2} = \frac{p'}{{L_p}^2}$$

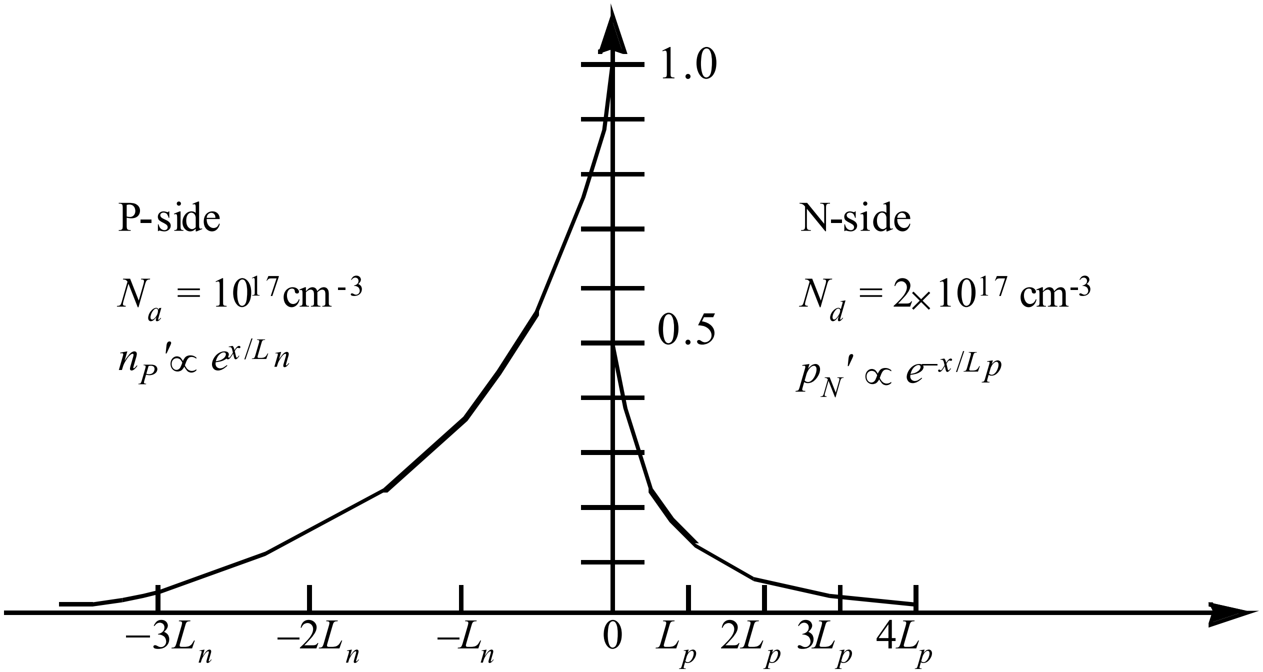
$$p'(\infty) = 0$$

$$p'(0) = p_{N0}(e^{qV/kT} - 1)$$

$$p'(x) = Ae^{x/L_p} + Be^{-x/L_p}$$

$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-x/L_p}, \;\; x > 0$$

4.8 Excess Carrier Distribution in Biased PN Junction



$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-x/L_p},\ x > 0$$

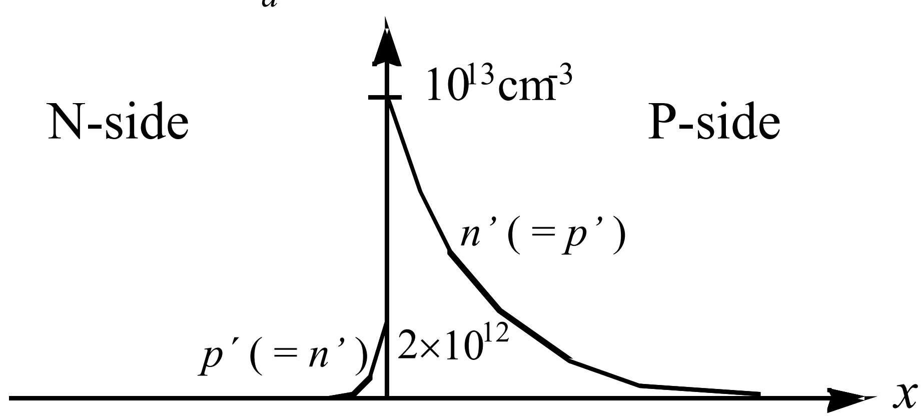
$$n'(x) = n_{P0}(e^{qV/kT} - 1)e^{x/L_n},\ x < 0$$

EXAMPLE: Carrier Distribution in Forward-biased PN Diode

N-type	P-type
N-type $N_d = 5\times10^{17}$ cm^{-3} $D_p = 12$ cm^2/s $\tau_p = 1\ \mu$s	P-type $N_a = 10^{17}$ cm^{-3} $D_n = 36.4$ cm^2/s $\tau_n = 2\ \mu$s

- *Sketch n'(x) on the P-side.*

$$n'(0) = n_{P0}(e^{qV/kT} - 1) = \frac{n_i^2}{N_a}(e^{qV/kT} - 1) = \frac{10^{20}}{10^{17}} e^{0.6/0.026} = 10^{13}\ \text{cm}^{-3}$$



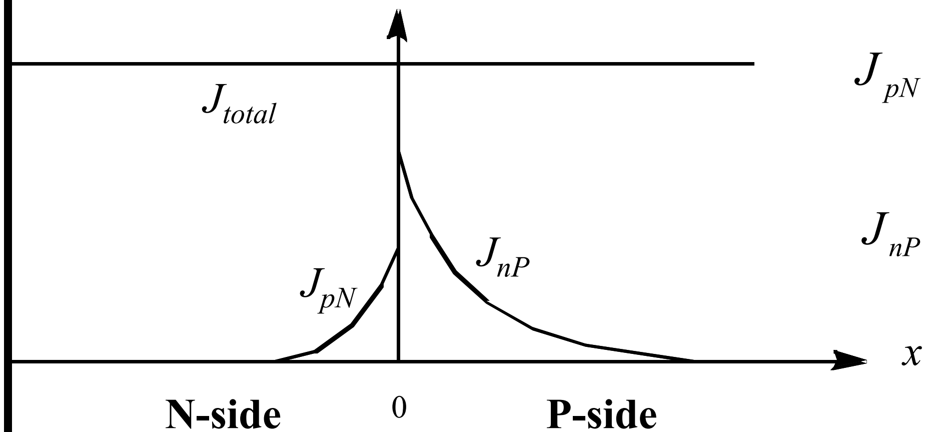
EXAMPLE: Carrier Distribution in Forward-biased PN Diode

- *How does L_n compare with a typical device size?*

$$L_n = \sqrt{D_n \tau_n} = \sqrt{36 \times 2 \times 10^{-6}} = 85\ \mu\text{m}$$

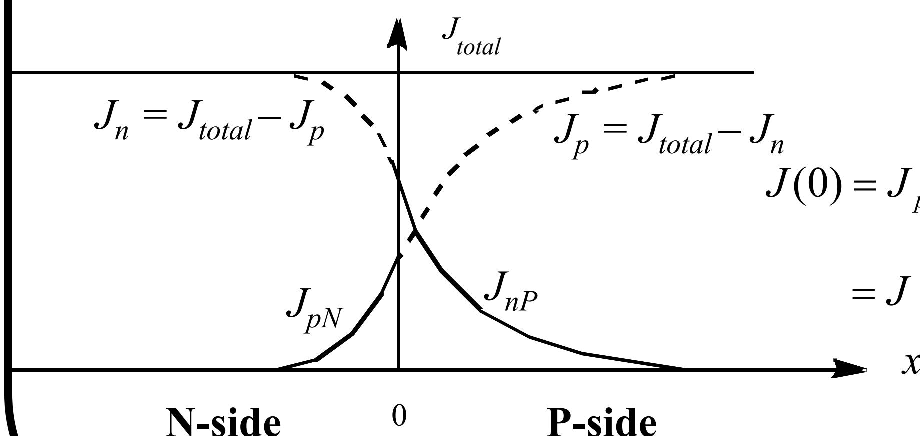
- *What is p'(x) on the P- side?*

4.9 PN Diode IV Characteristics



$$J_{pN} = -qD_p \frac{dp'(x)}{dx} = -q\frac{D_p}{L_p} p_{N0}(e^{qV/kT} - 1)e^{x/L_p}$$

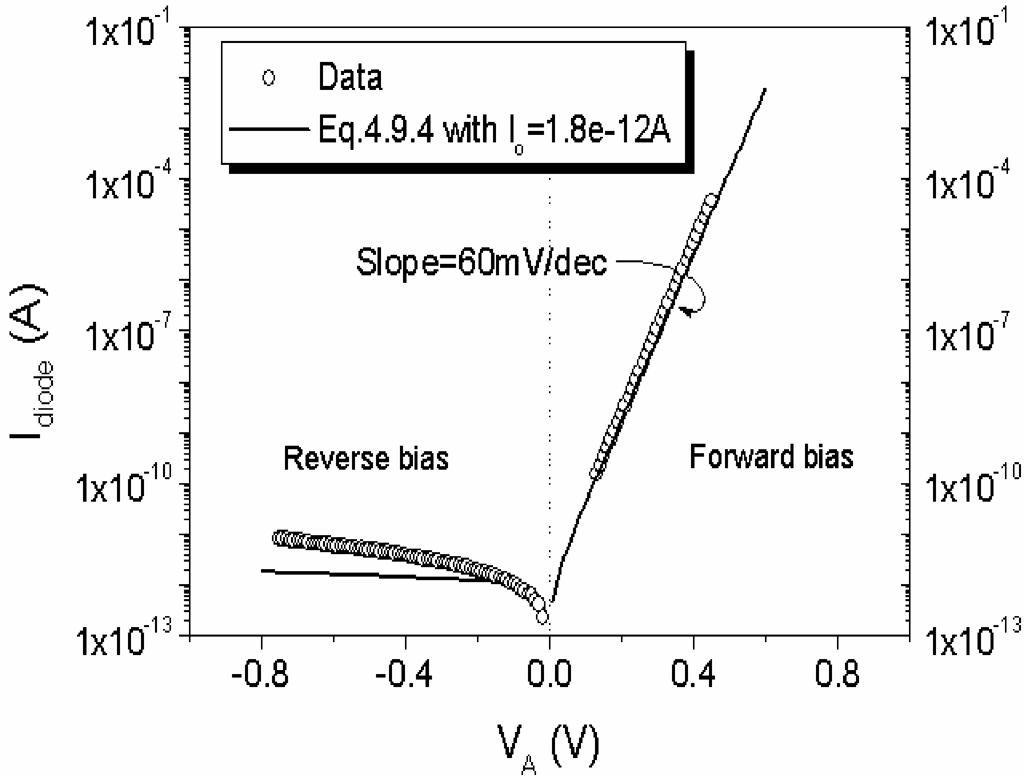
$$J_{nP} = qD_n \frac{dn'(x)}{dx} = -q\frac{D_n}{L_n} n_{P0}(e^{qV/kT} - 1)e^{-x/L_n}$$



$$J(0) = J_{pN}(0) + J_{nP}(0) = \left(q\frac{D_p}{L_p} p_{N0} + q\frac{D_n}{L_n} n_{P0}\right)(e^{qV/kT} - 1)$$

$$= J \text{ at all } x$$

4.9 PN Diode IV Characteristics

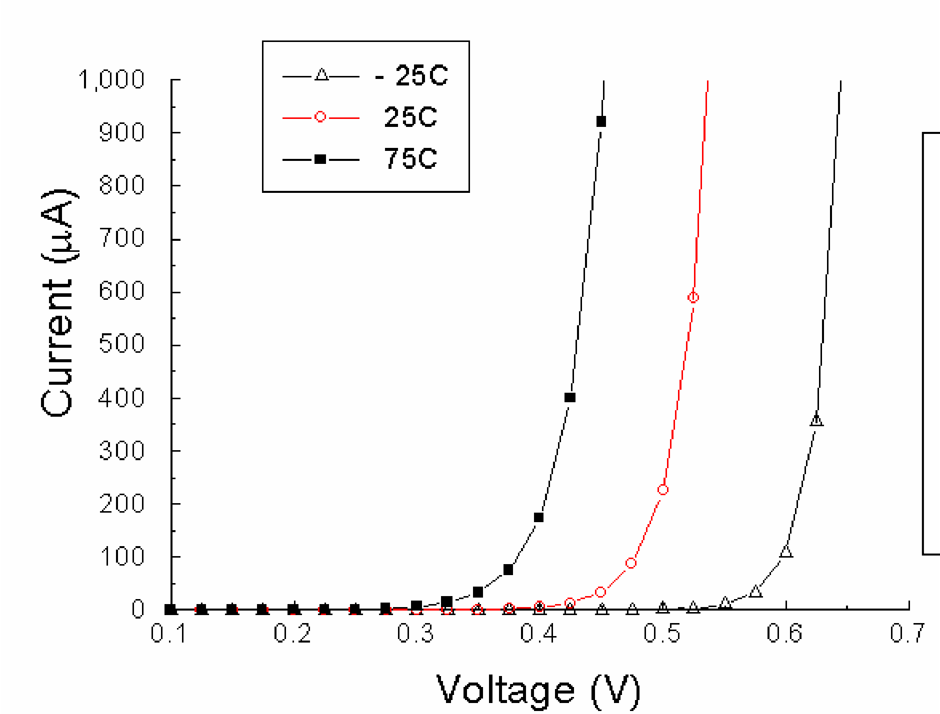


$$I = I_0(e^{qV/kT} - 1)$$

$$I_0 = Aqn_i^{\,2}\left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right)$$

$$I_r = I_0 + A\frac{qn_i W_{dep}}{\tau_{dep}}$$

The PN Junction as a Temperature Sensor

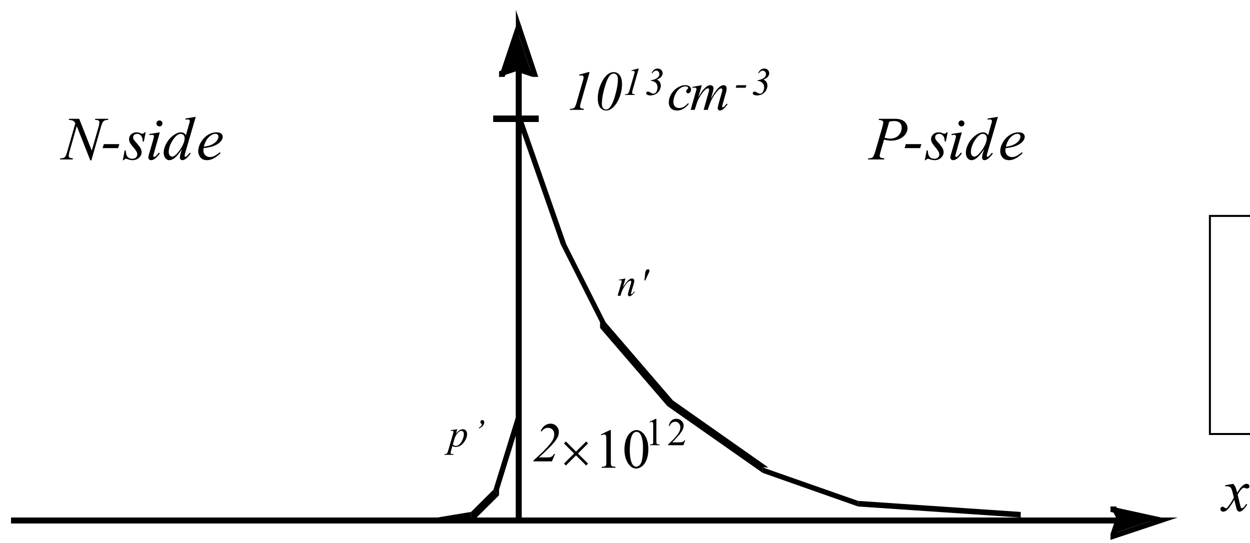


$$I = I_0(e^{qV/kT} - 1)$$

$$I_0 = Aqn_i^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)$$

What causes the IV curves to shift to lower V at higher T ?

4.10 Charge Storage

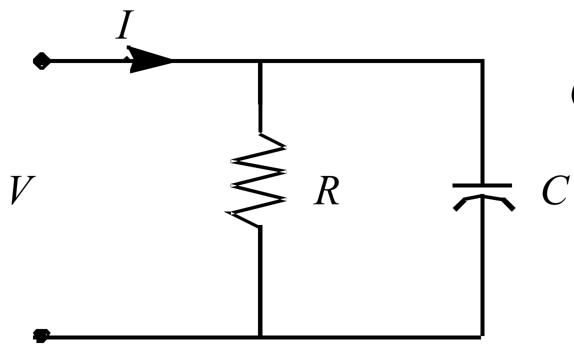


$Q \propto I$

$$I = Q/\tau_s$$
$$Q = I\tau_s$$

What is the relationship between τ_s (charge-storage time) and τ (carrier lifetime)?

4.11 Small-signal Model of the Diode



$$G \equiv \frac{1}{R} = \frac{dI}{dV} = \frac{d}{dV} I_0 (e^{qV/kT} - 1) \approx \frac{d}{dV} I_0 e^{qV/kT}$$

$$= \frac{q}{kT} I_0 (e^{qV/kT} - 1) = I_{DC} / \frac{kT}{q}$$

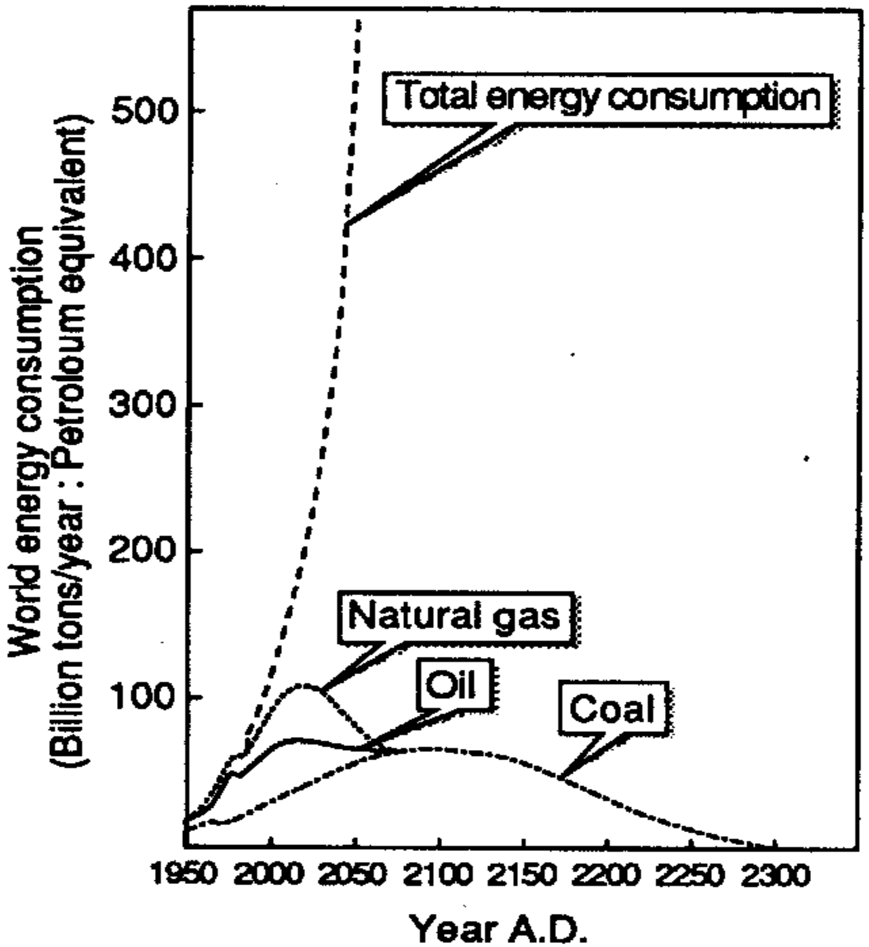
What is G at 300K and $I_{DC} = 1$ mA?

Diffusion Capacitance:

$$C = \frac{dQ}{dV} = \tau_s \frac{dI}{dV} = \tau_s G = \tau_s I_{DC} / \frac{kT}{q}$$

Which is larger, diffusion or depletion capacitance?

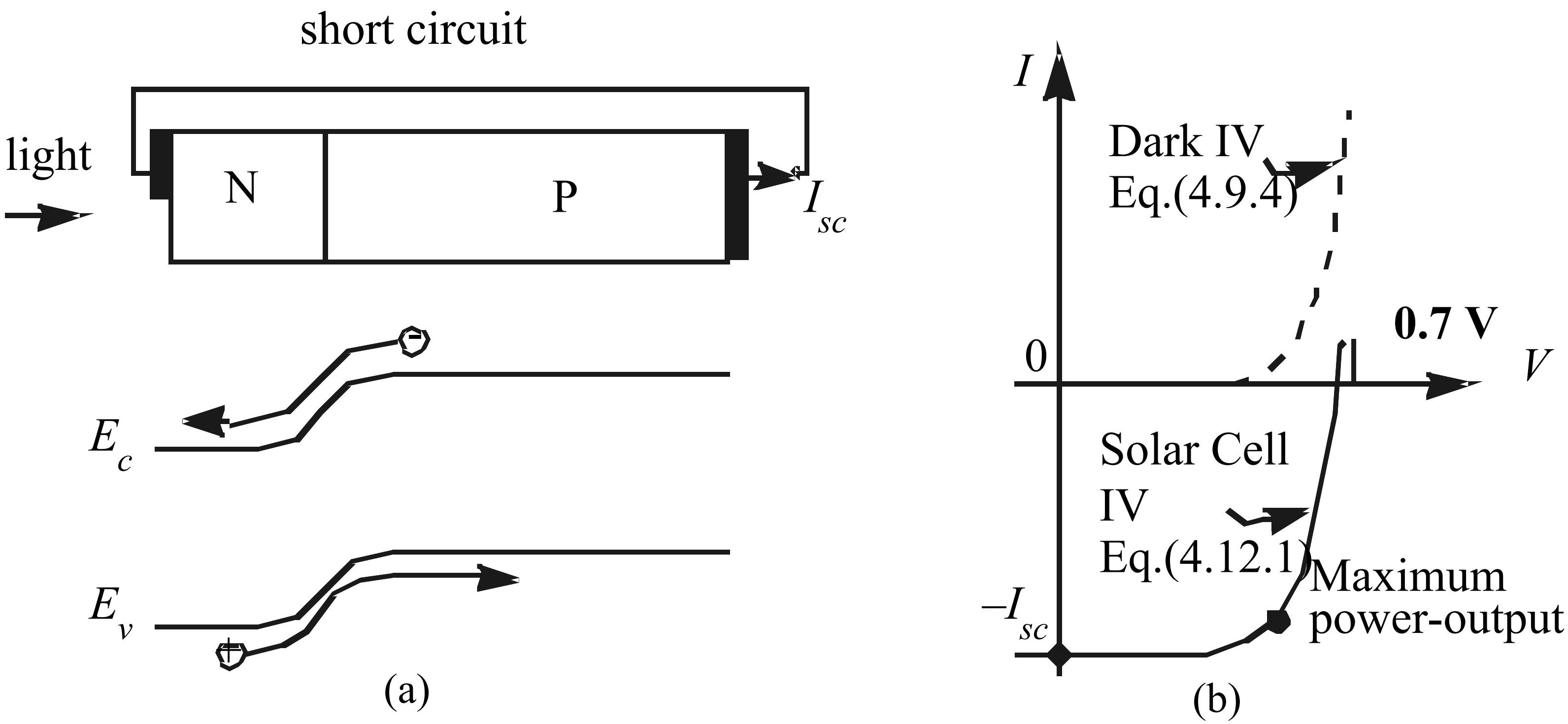
4.12 Other PN Junction Devices–From Solar Cells to Laser Diodes



Solar Cells

Also known as ***photovoltaic cells,*** solar cells can convert sunlight to electricity with 15-30% energy efficiency

4.12.1 Solar Cells



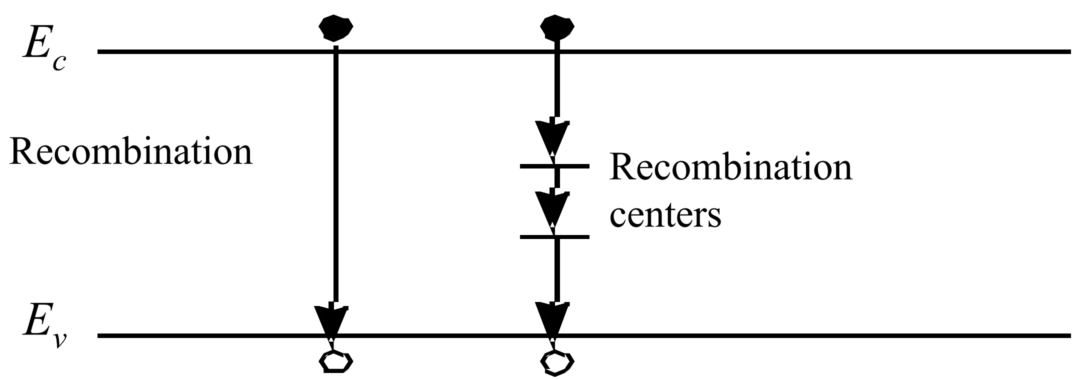
$$I = I_0(e^{qV/kT} - 1) - I_{sc}$$

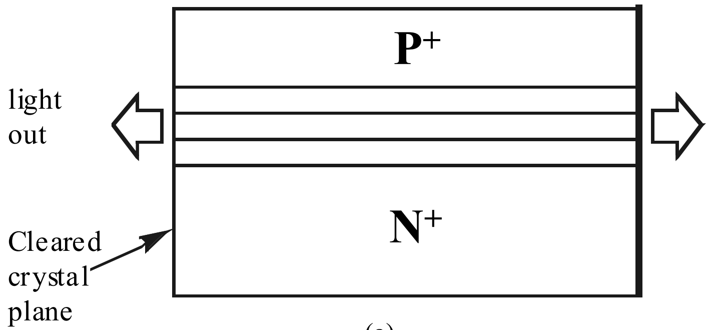
4.12.2 Photodiodes and Avalanche Photodiodes

4.12.3 Light Emitting Diodes (LEDs)

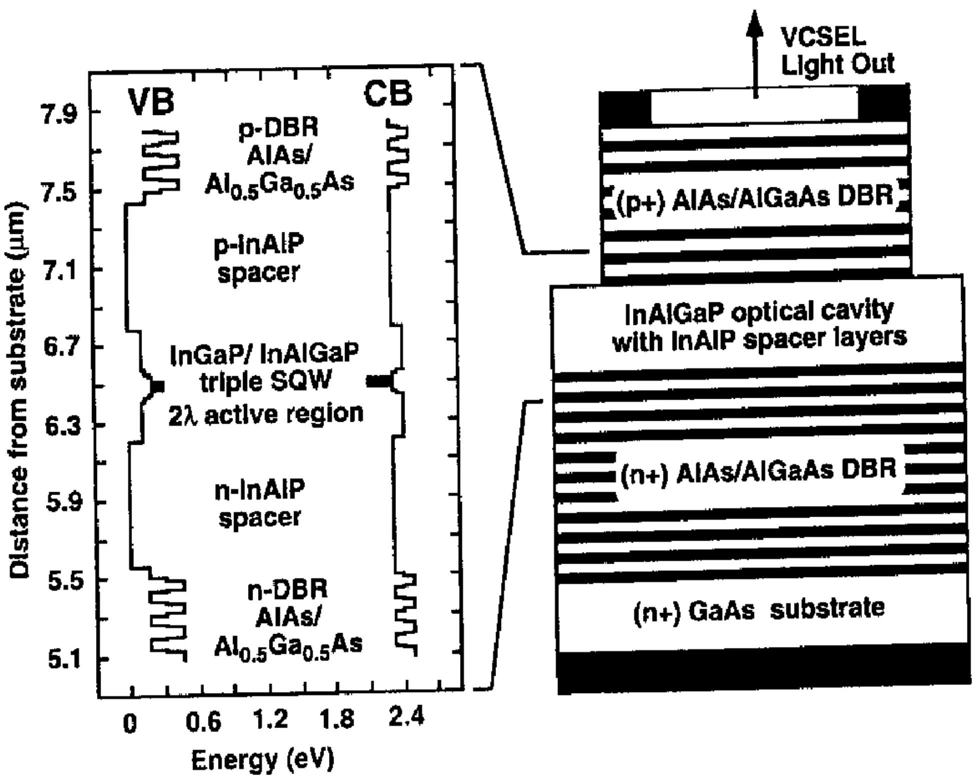
- LEDs are made of compound semiconductors such as InP and GaN.

- Terms: ***direct band-gap, radiative recombination***





(a)



(b)

4.12.4 Diode Lasers

- A diode laser must be strongly forward-biased to make $E_{fn} - E_{fp} > E_g$.

- Terms: ***population inversion, heterojunctions, cleaved mirror, distributed Bragg reflector (DBR)***

4.13 Chapter Summary

$$\boxed{\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^{\,2}}}$$

The potential barrier increases by 1 V if a 1 V reverse bias is applied

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s \cdot potential\ barrier}{qN}}}$$

depletion width

$$\boxed{C_{dep} = A\frac{\varepsilon_s}{W_{dep}}}$$

junction capacitance

4.13 Chapter Summary

- Under forward bias, there is minority carrier injection.

- The quasi-equilibrium boundary condition of minority carrier densities is:

$$n(0) = n_{P0} e^{qV/kT}$$
$$p(0) = p_{N0} e^{qV/kT}$$

- Most of the minority carriers are injected into the lighter doped side.

4.13 Chapter Summary

- Steady-state continuity equation:

$$\frac{d^2 p'}{dx^2} = \frac{p'}{D_p \tau_p} = \frac{p'}{{L_p}^2}$$

$$L_p \equiv \sqrt{D_p \tau_p}$$

- Minority carriers diffuse outward $\propto e^{-|x|/L_p}$ or $e^{-|x|/L_n}$
- L_p and L_n are the diffusion lengths

$$I = I_0 (e^{qV/kT} - 1)$$

$$I_0 = Aq{n_i}^2 \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right)$$

4.13 Chapter Summary

Charge storage: $$Q = I\tau_s$$

Diffusion capacitance: $$C = \tau_s G$$

Diode conductance: $$G = I_{DC} / \frac{kT}{q}$$