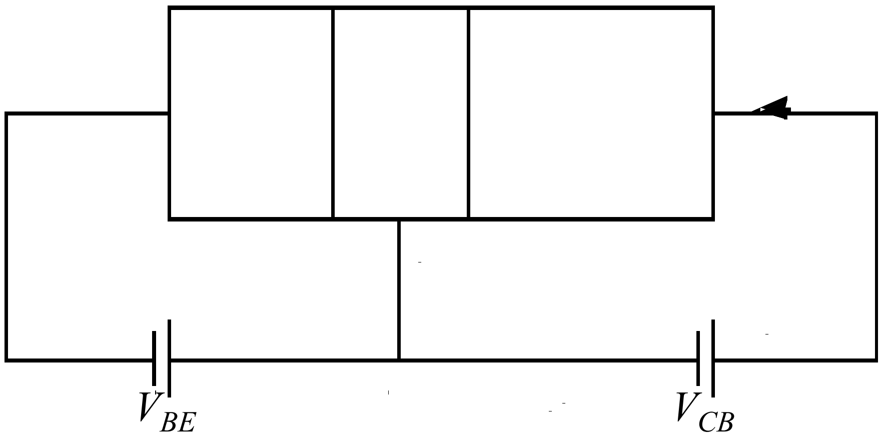
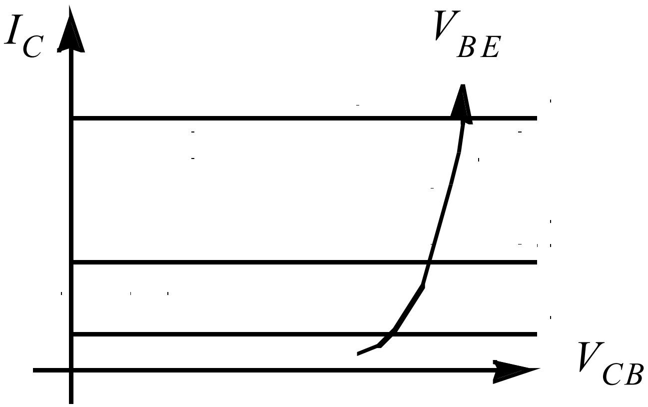
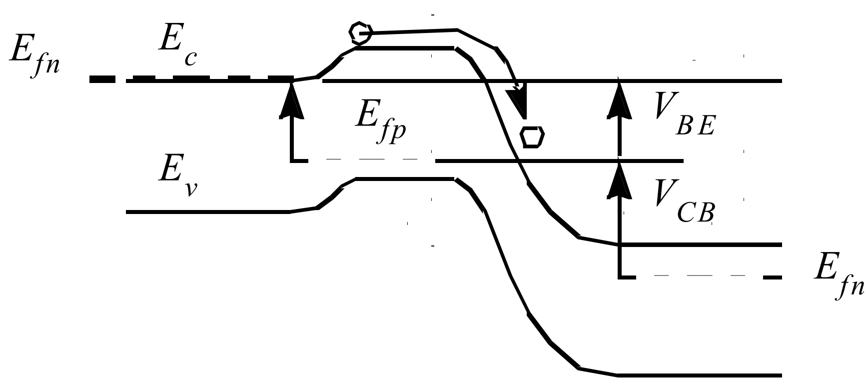
Chapter 8   Bipolar Junction Transistors

Question

8.1 Introduction to the BJT

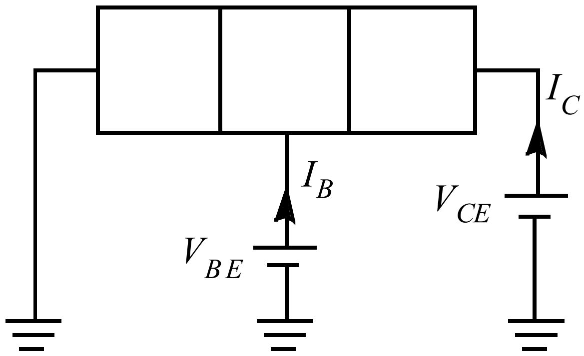


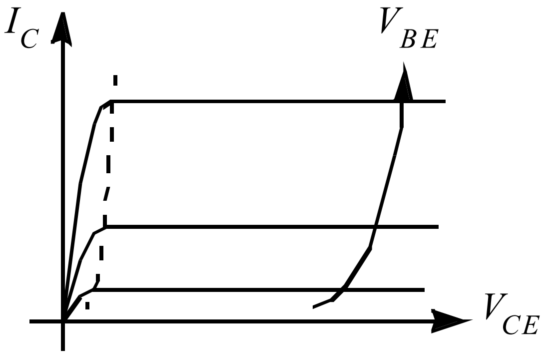


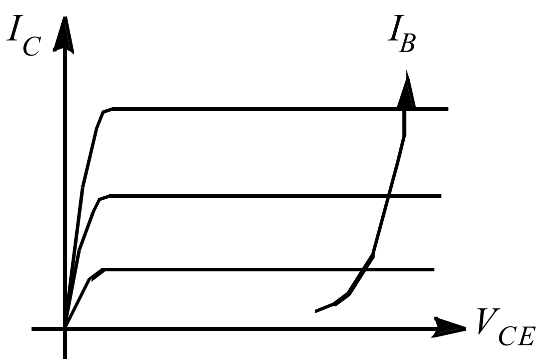


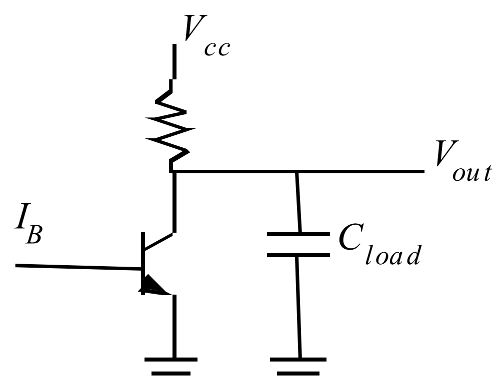
Common-Emitter Configuration

Question: I_c V_{BE} V_{CB}



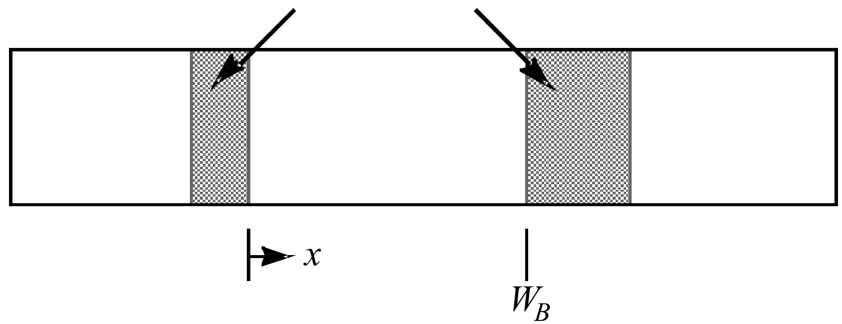






Question I_B V_{BE}

8.2 Collector Current



$$\frac{d\ \ n'}{dx} = \frac{n'}{L_B}$$

τ_B

D_B

$$L_B \equiv \sqrt{\tau_B D_B}$$

$$n' \qquad = n_B \quad e^{qV_{BE} \quad kT} -$$

$$n' \ W_B \ = n_B \quad e^{qV_{BC} \quad kT} - \quad \approx -n_B \ \approx$$

8.2 Collector Current

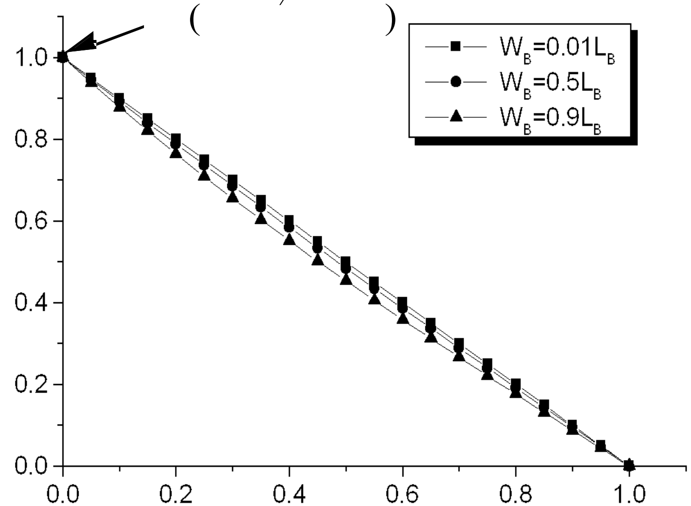
$$n'\ x\ = n_B\ \ e^{qV_{BE}\ \ kT} - \ \ \frac{\left(\frac{W_B - x}{L_B}\right)}{\left(W_B\ \ L_B\right)}$$

$$\boxed{\begin{aligned} n'\ x\ &= n' \qquad - x\ \ W_B \\ &= \frac{n_{iB}}{N_B}\ \ e^{qV_{BE}\ \ kT} - \qquad - x\ \ W_B \end{aligned}}$$

$'$

$'(\ )$

$(\qquad /\qquad)$



$x\ W_B$

$$I_C = \left| A_E q D_B \frac{dn}{dx} \right|$$

$$= A_E q \frac{D_B}{W_B} \frac{n_{iB}}{N_B}\ \ e^{qV_{BE}\ \ kT} -$$

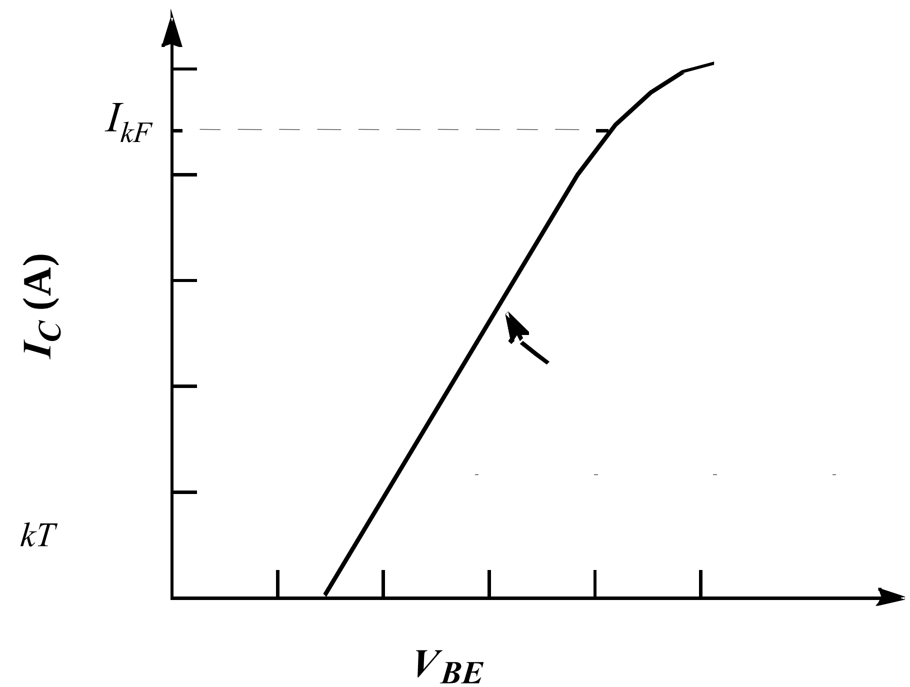
$$I_C = I_S\ \ e^{qV_{BE}\ \ kT} -$$

$$I_C = A_E \frac{qn_i}{G_B}\ \ e^{qV_{BE}\ \ kT} -$$

$$G_B \equiv \int^{W_B} \frac{n_i}{n_{iB}} \frac{p}{D_B} dx$$

G_B ***base Gummel number***

Gummel Plot



p N_B ,

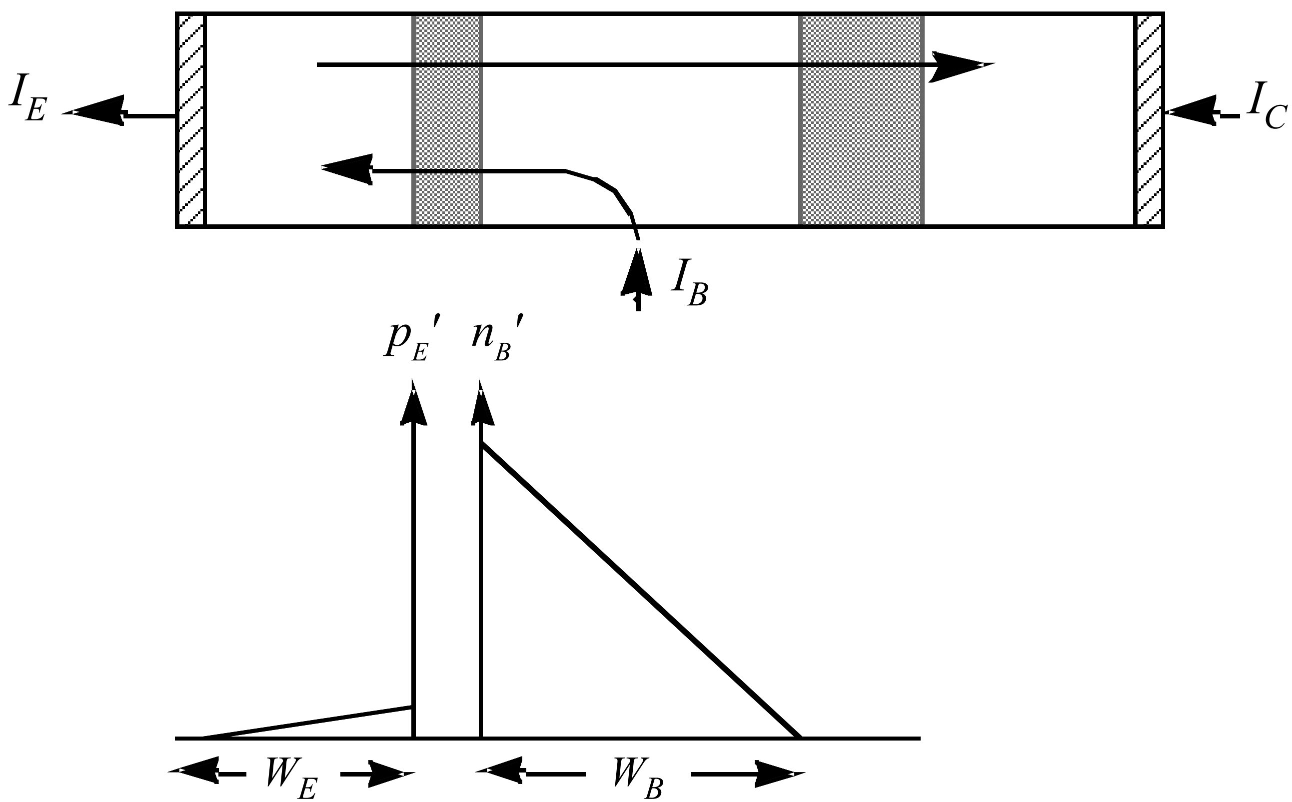
$I_C \propto e^{qV_{BE}} \quad kT$

V_{BE}

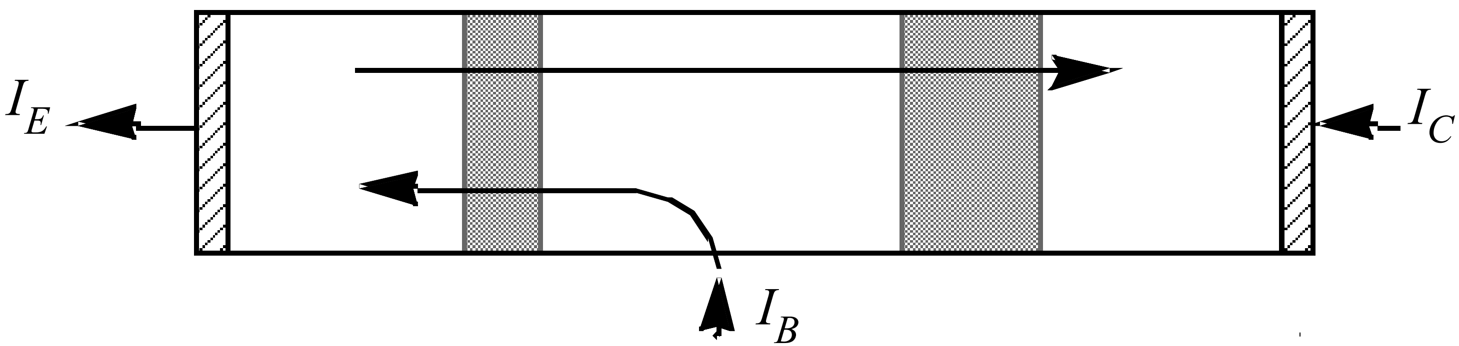
I_C

8.3 Base Current

I_B



8.3 Base Current



$$I_B = A_E \frac{qn_i}{G_E} \; e^{qV_{BE} \;\; kT} -$$

$$G_E \equiv \int^{W_E} \frac{n_i}{n_{iE}} \frac{n}{D_E} dx$$

$$I_B = A_E q \frac{D_E n_{iE}}{W_E N_E} \; e^{qV_{BE} \;\; kT} -$$

Question I_B

8.4 Current Gain

β_F

$$\boxed{\beta_F \equiv \frac{I_C}{I_B}}$$

$$I_C = \alpha_F I_E$$

$$\alpha_F \equiv \frac{I_C}{I_E} = \frac{I_C}{I_B + I_C} = \frac{I_C \quad I_B}{+ I_C \quad I_B} = \frac{\beta_F}{+ \beta_F}$$

$$\beta_F = \frac{\alpha_F}{-\alpha_F}$$

$$\boxed{\beta_F = \frac{G_E}{G_B}} = \frac{D_B W_E N_E n_{iB}}{D_E W_B N_B n_{iE}}$$

β_F

EXAMPLE: Current Gain

A BJT has $I_C = 1\ mA$ *and* $I_B = 10\ \mu A$. *What are* I_E, β_F *and* α_F?

Solution

$$I_E = I_C + I_B = \qquad + \qquad =$$

$$\beta_F = I_C \quad I_B = \qquad\qquad =$$

$$\alpha_F = I_C \quad I_E = \qquad\qquad =$$

We can confirm

$$\alpha_F = \frac{\beta_F}{+\beta_F} \qquad\qquad \beta_F = \frac{\alpha_F}{-\alpha_F}$$

8.4.1 Emitter Bandgap Narrowing

$$\beta \propto \frac{N_E}{N_B}\frac{n_{iB}}{n_{iE}}$$

β_F N_E

N_E $n_{iE} > n_i$

$$n_i = N_C N_V e^{-E_g\ kT}$$

n_i E_g

$$n_{iE} = n_i e^{\Delta E_{gE}\ kT}$$

ΔE_{gE} N_E

β_F

8.4.2 Narrow-Bandgap (SiGe) Base

$$\beta \propto \frac{N_E}{N_B} \frac{n_{iB}}{n_{iE}}$$

β_F n_{iB}

η η

η E_{gB} n_{iE}

EXAMPLE: Emitter Bandgap Narrowing and SiGe Base

Assume $D_B = 3D_E$, $W_E = 3W_B$, $N_B = 10^{18}$ cm^{-3}, and $n_{iB}^2 = n_i^2$. What is β_F for (a) $N_E = 10^{19}$ cm^{-3}, (b) $N_E = 10^{20}$ cm^{-3}, and (c) $N_E = 10^{20}$ cm^{-3} and a SiGe base with $\Delta E_{gB} = 60$ meV ?

(a) At $N_E = 10^{19}$ cm^{-3}, $\Delta E_{gE} \approx$ 50 meV,

$$n_{iE} = n_i\, e^{\Delta E_{gE} \quad kT} = n_i\, e \qquad\qquad = n_i\, e \quad = \quad n_i$$

$$\beta_F = \frac{D_B W_E}{D_E W_B} \cdot \frac{N_E n_i}{N_B n_{iE}} = \frac{\cdot \quad \cdot n_i}{\cdot \quad n_i} =$$

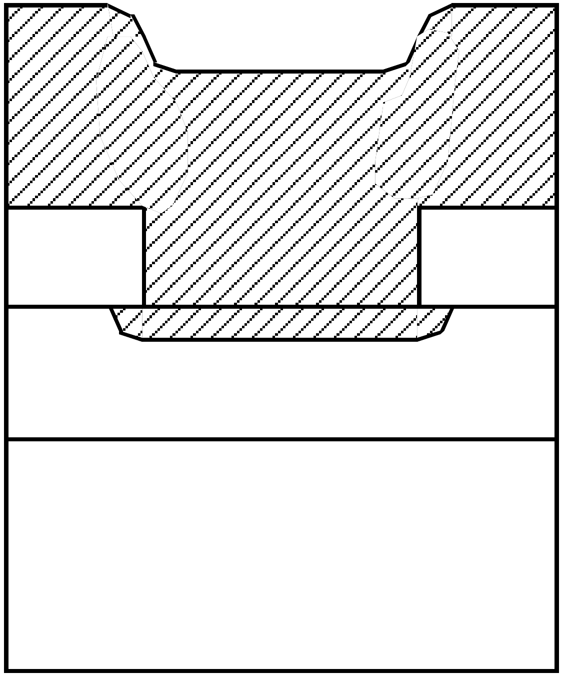
(b) At $N_E = 10^{20}$ cm^{-3}, $\Delta E_{gE} \approx$ 95 meV

$$n_{iE} = \quad n_i \qquad\qquad \beta_F =$$

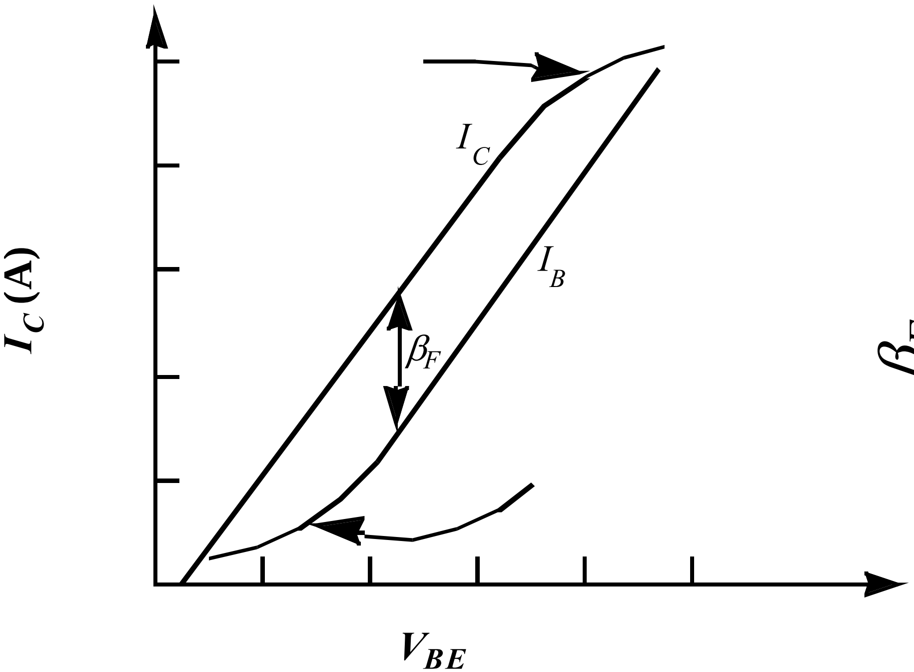
$$\text{(c)} \quad n_{iB} = n_i\, e^{\Delta E_{gB} \quad kT} = n_i\, e \qquad\qquad = \quad n_i \qquad\qquad \beta_F =$$

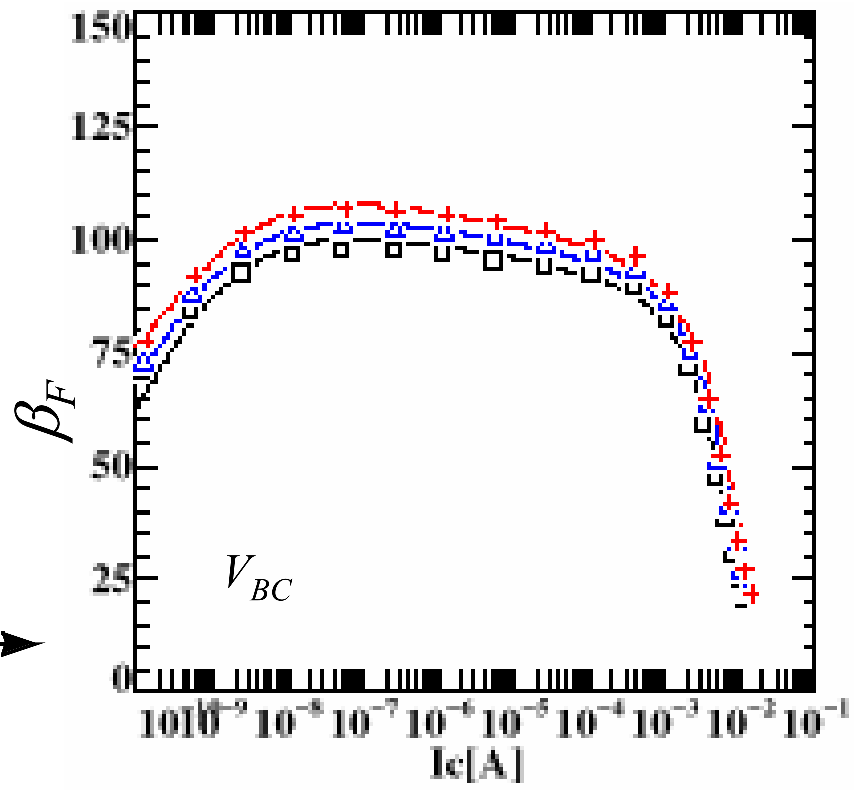
8.4.3 Poly-Silicon Emitter

β_F W_E



8.4.4 Gummel Plot and β_F Fall-off at High and Low I_c

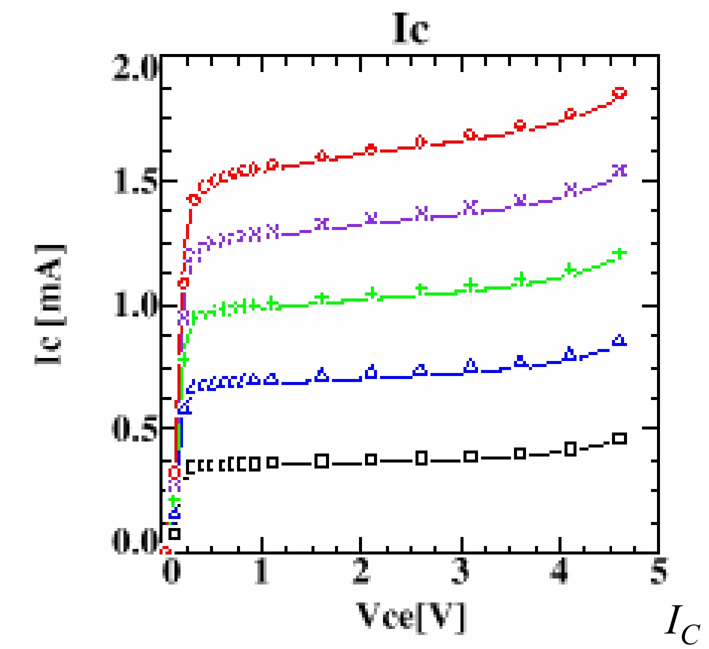




I_C I_C

β_F V_{BC}

8.5 Base-Width Modulation by Collector Voltage

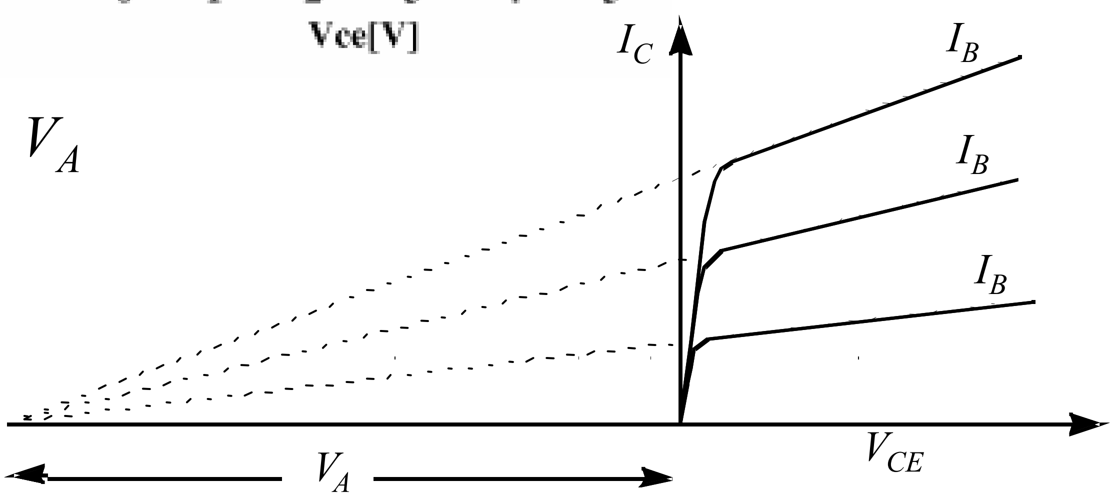


$$r \equiv \left(\frac{\partial I_C}{\partial V_{CE}} \right)^{-} = \frac{V_A}{I_C}$$

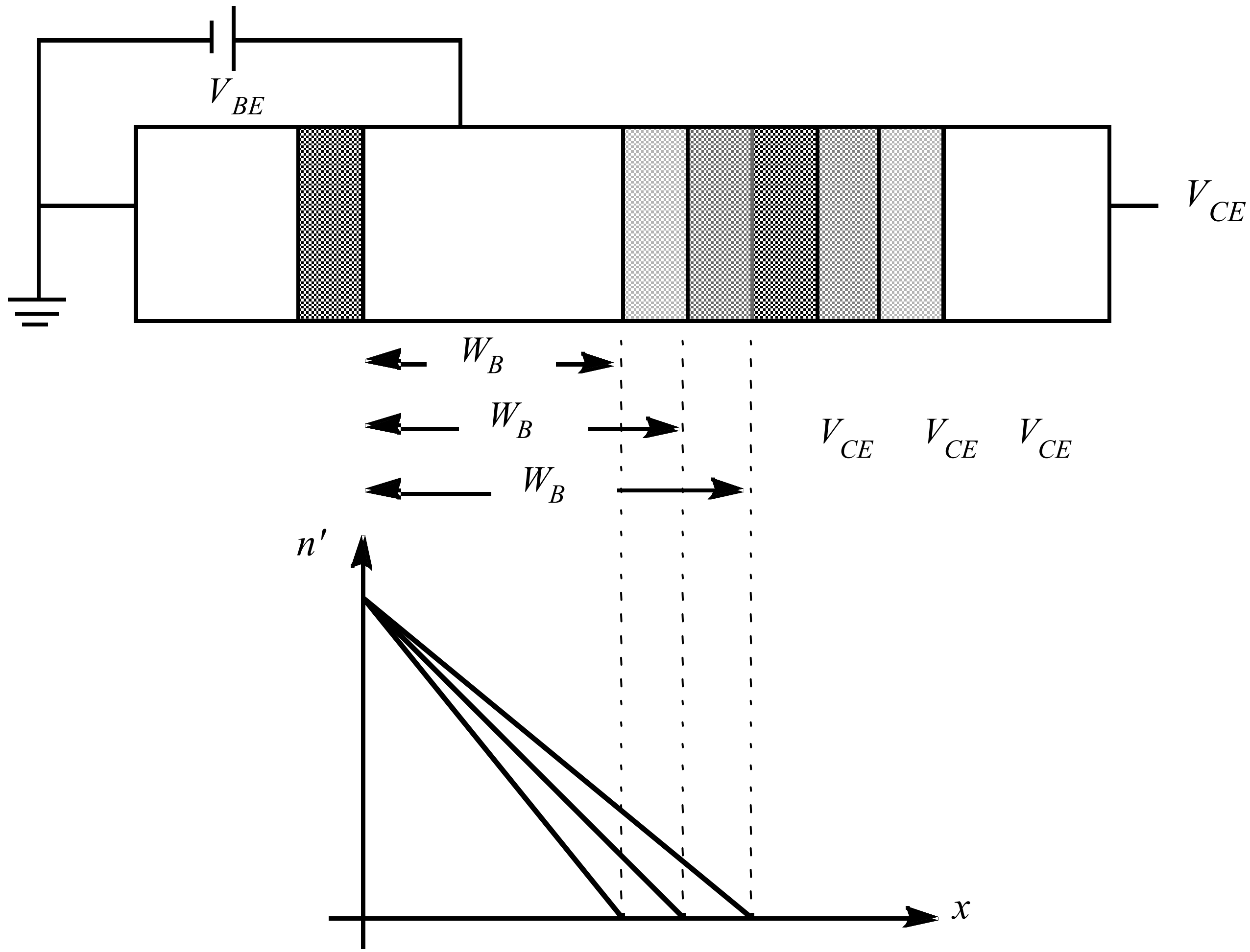
V_A

V_A

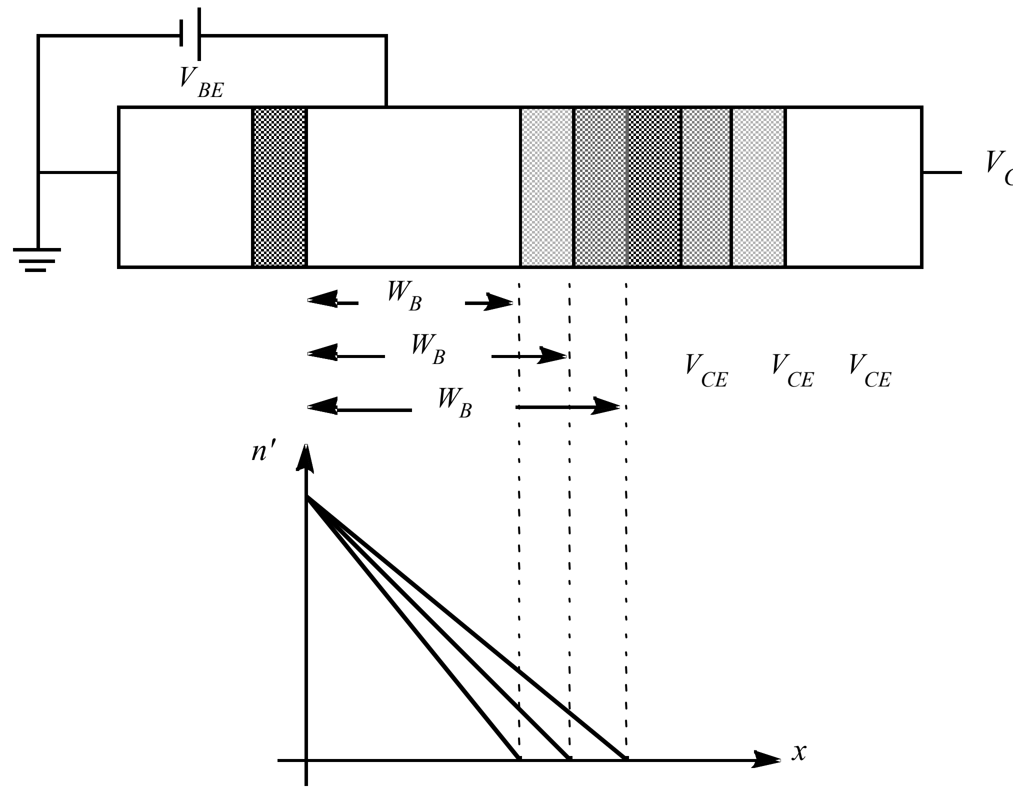
r_o



8.5 Base-Width Modulation by Collector Voltage



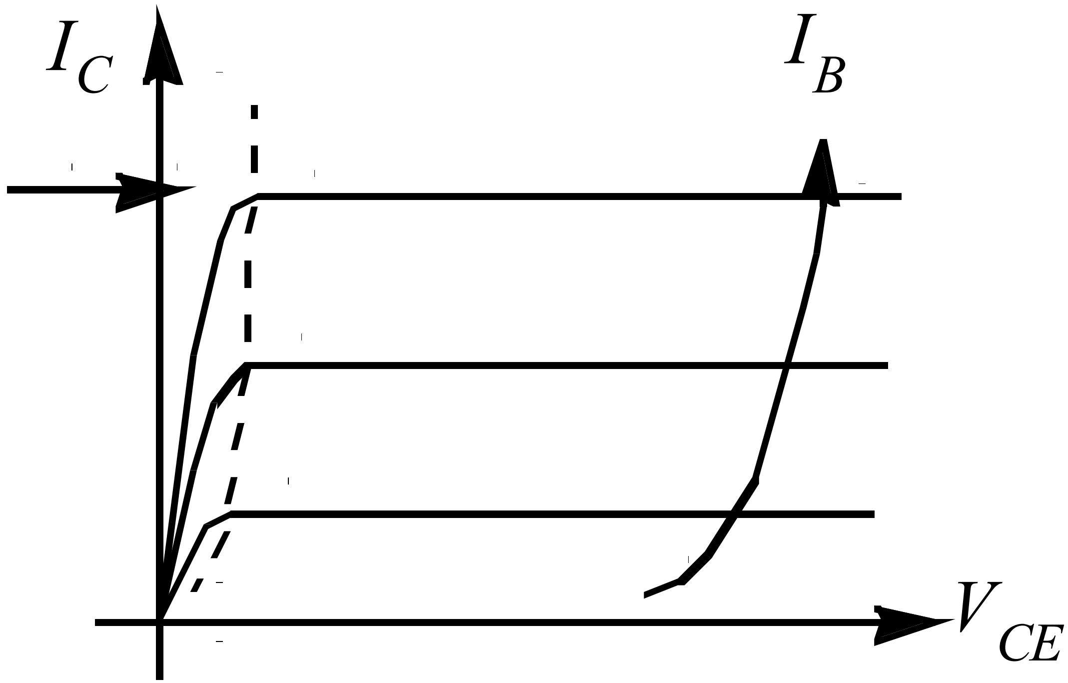
8.5 Base-Width Modulation by Collector Voltage



N_B

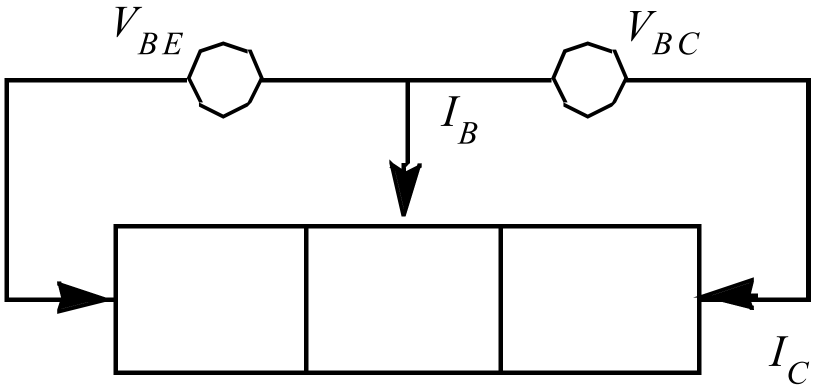
N_C

8.6 Ebers-Moll Model



8.6 Ebers-Moll Model

I_C V_{BE} V_{BC}



V_{BE}

$$I_C = I_S \; e^{qV_{BE} \;\; kT} -$$

$$I_B = \frac{I_S}{\beta_F} \; e^{qV_{BE} \;\; kT} -$$

V_{BC}

$$I_E = I_S \; e^{qV_{BC} \;\; kT} -$$

$$I_B = \frac{I_S}{\beta_R} \; e^{qV_{BC} \;\; kT} -$$

β_R

β_F

$$I_C = -I_E - I_B = -I_S \quad + \frac{\quad}{\beta_R} \quad e^{qV_{BC} \;\; kT} -$$

8.6 Ebers-Moll Model

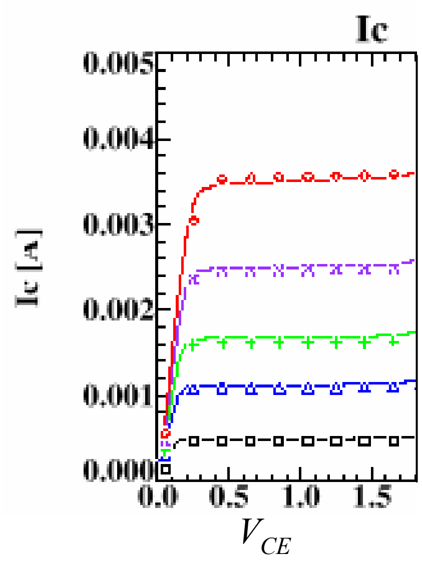
V_{BE} $\quad$ V_{BC}

$$I_C = I_S \; e^{qV_{BE} \;\; kT} - \quad - I_S \quad + \frac{\quad}{\beta_R} \; e^{qV_{BC} \;\; kT} -$$

$$I_B = \frac{I_S}{\beta_F} \; e^{qV_{BE} \;\; kT} - \quad + \frac{I_S}{\beta_F} \; e^{qV_{BC} \;\; kT} -$$

V_{BC}

I_B $\quad$ I_C



8.7 Transit Time and Charge Storage

Q_F

$$\boxed{\tau_F \equiv \frac{Q_F}{I_C}}$$

τ_F

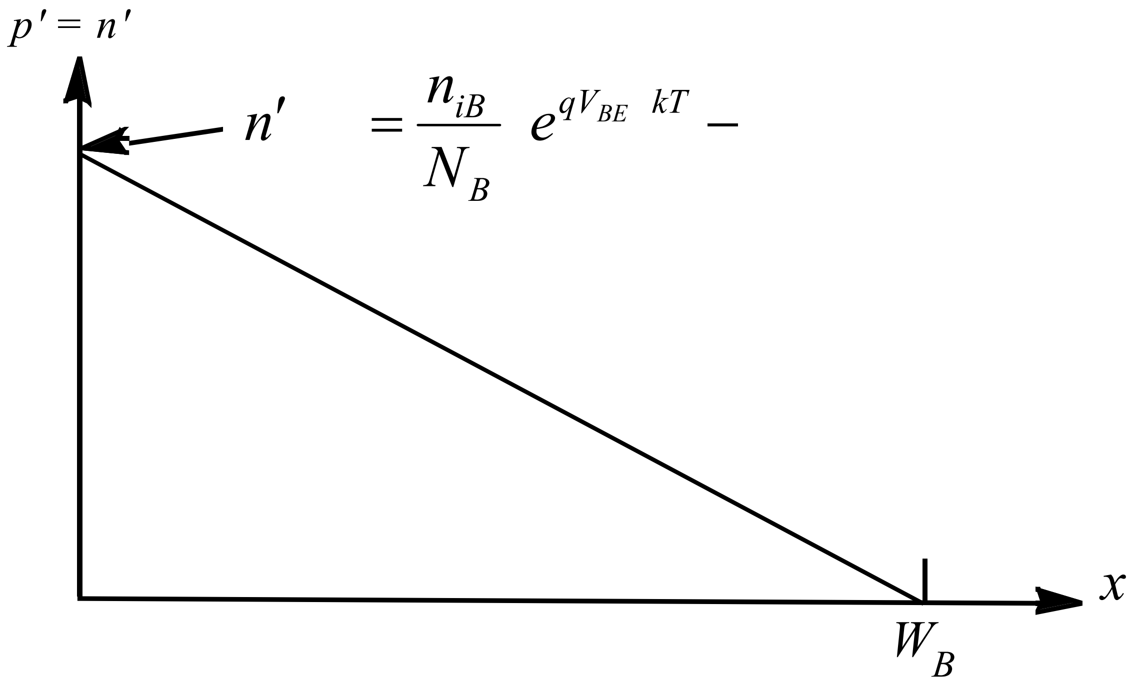
τ_F determines the high-frequency limit of BJT operation.

8.7.1 Base Charge Storage and Base Transit Time

$$Q_{FB} = qA_E n' \quad W_B$$

$$\frac{Q_{FB}}{I_C} \equiv \tau_{FB} = \frac{W_B}{D_B}$$

$$n' \quad = \frac{n_{iB}}{N_B} \ e^{qV_{BE} \ \ kT} -$$



EXAMPLE: Base Transit Time

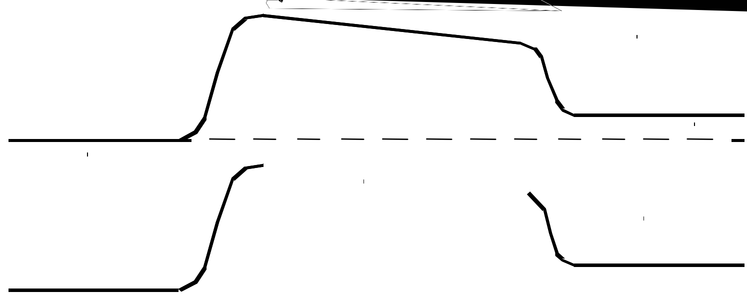
What is τ_{FB} *if* $W_B = 70$ *nm and* $D_B = 10$ *cm*^{2}*/s?*

Answer:

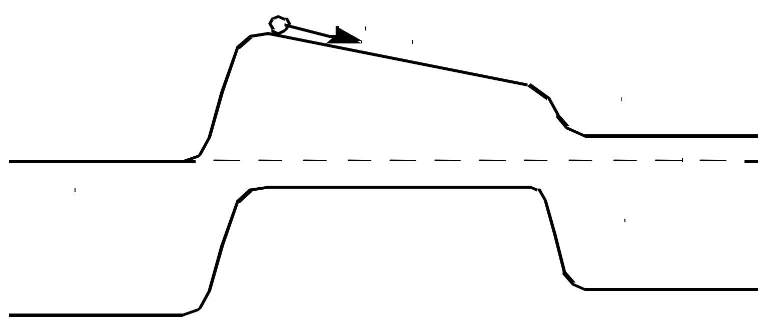
$$\tau_{FB} = \frac{W_B}{D_B} = \frac{\times \quad ^{-}}{\times} = \quad \times \quad ^{-} \quad =$$

2.5 ps is a very short time. Since light speed is $3{\times}10^8$ *m/s, light travels only 1.5 mm in 5 ps.*

E_{gB} N_B



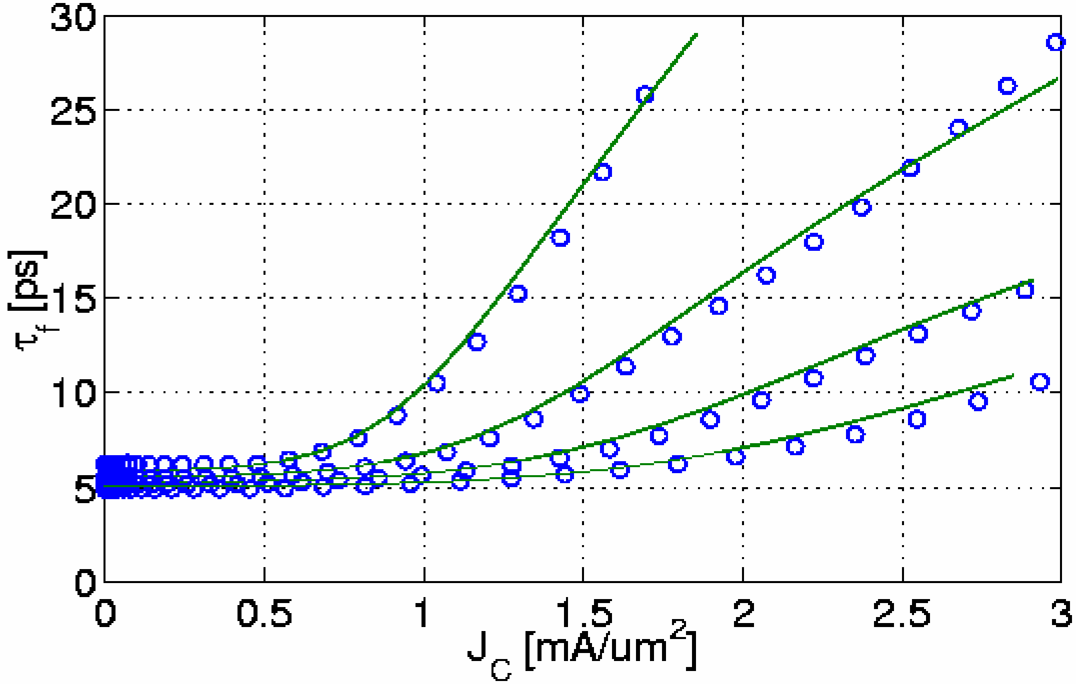
N_B E_{gB}



8.7.3 Emitter-to-Collector Transit Time and Kirk Effect

n N_C

τ_F



V_{CE}

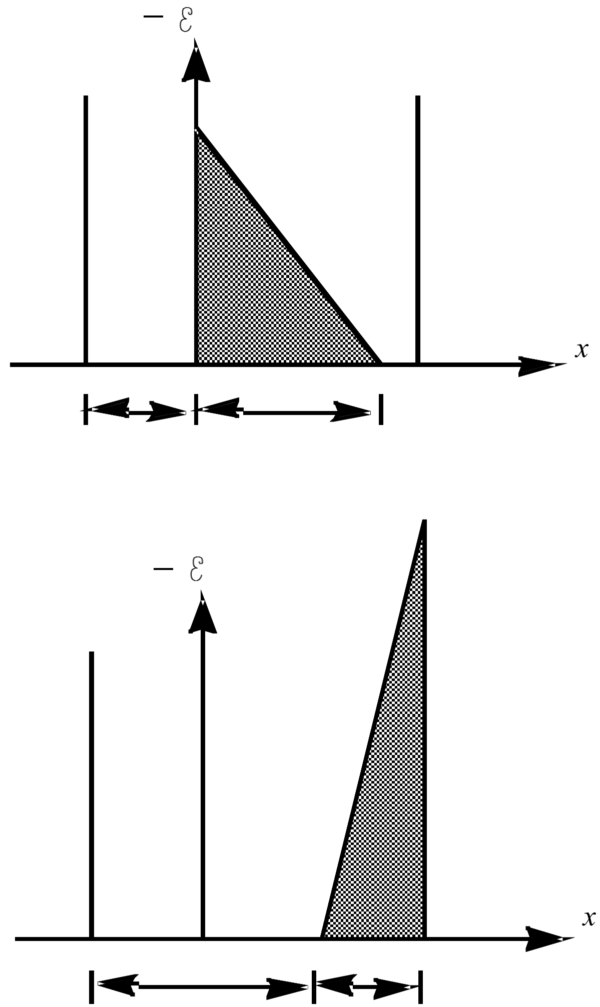
Base Widening at Large I_c

$$I_C = A_E q n v_{sat}$$

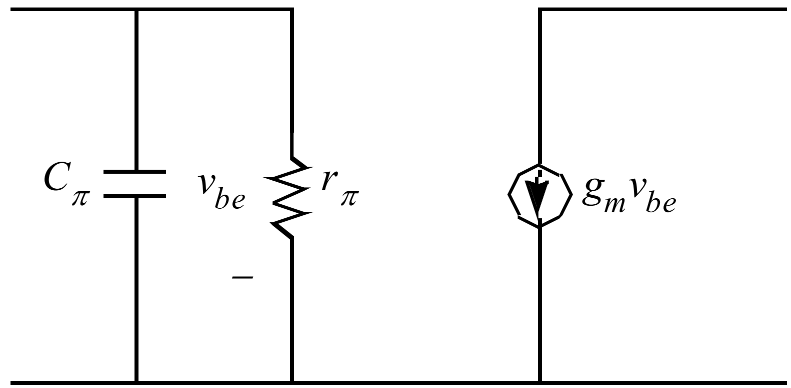
$$\rho = qN_C - qn$$

$$= qN_C - \frac{I_C}{A_E v_{sat}}$$

$$\frac{d\mathcal{E}}{dx} = \rho \quad \varepsilon_s$$



8.8 Small-Signal Model



$$I_C = I_S e^{qV_{BE} \quad kT}$$

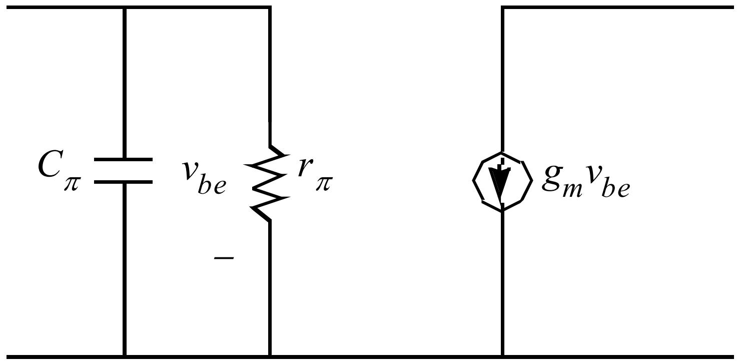
$$g_m \equiv \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}} \; I_S e^{qV_{BE} \quad kT}$$

$$= \frac{q}{kT} I_S e^{qV_{BE} \quad kT} = I_C \quad kT \quad q$$

$$\boxed{g_m = I_C \quad kT \quad q}$$

g_m=I_C/

8.8 Small-Signal Model



$$\frac{}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{}{\beta_F}\frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta_F} \qquad r_\pi = \beta_F \quad g_m$$

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \frac{d}{dV_{BE}}\tau_F I_C = \tau_F g_m$$

diffusion capacitance

$$C_{dBE}$$

$$C_\pi = \tau_F g_m + C_{dBE}$$

EXAMPLE: Small-Signal Model Parameters

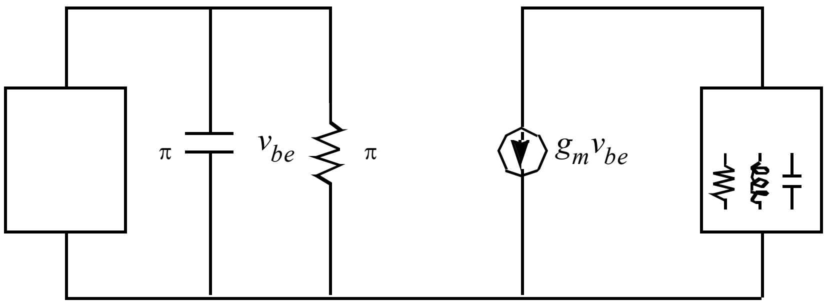
A BJT is biased at I_C = 1 mA and V_{CE} = 3 V. β_F=90, τ_F=5 ps, and T = 300 K. Find (a) g_m, (b) r_π, (c) C_π.

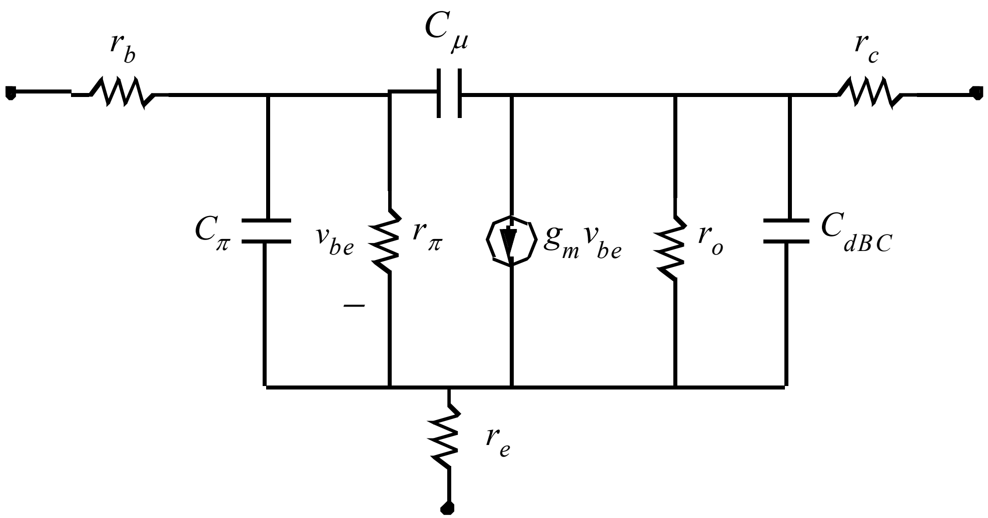
Solution:

(a) $g_m = I_C \quad kT \quad q = \underline{\hspace{2cm}} = \quad \underline{\hspace{1cm}} =$ *(milli siemens)*

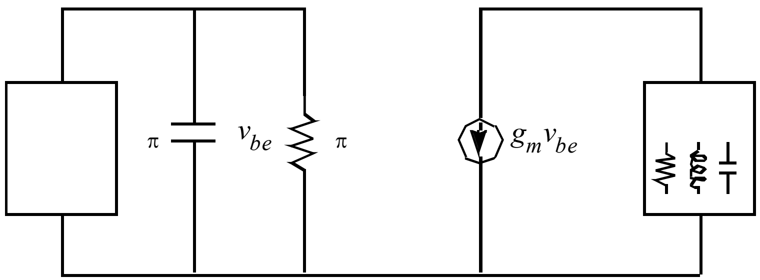
(b) $r_\pi = \beta_F \quad g_m = \underline{\hspace{2cm}} = \qquad \Omega$

(c) $C_\pi = \tau_F g_m = \ \times \ ^{-} \ \times \qquad \approx \ \times \ ^{-} \ =$ *(femto farad)*


π
v_{be}
π
$g_m v_{be}$


r_b
C_μ
r_c
C_π
v_{be}
r_π
$g_m v_{be}$
r_o
C_{dBC}
–
r_e

8.9 Cutoff Frequency



$\beta =$

$$f_T = \frac{\quad}{\pi \;\; \tau_F + C_{dBE} kT \;\; qI_C}$$

i_b f

$$\beta \equiv i_c \quad i_b$$

$$v_{be} = \frac{i_b}{\qquad} = \frac{i_b}{r_\pi + j\omega C_\pi}$$

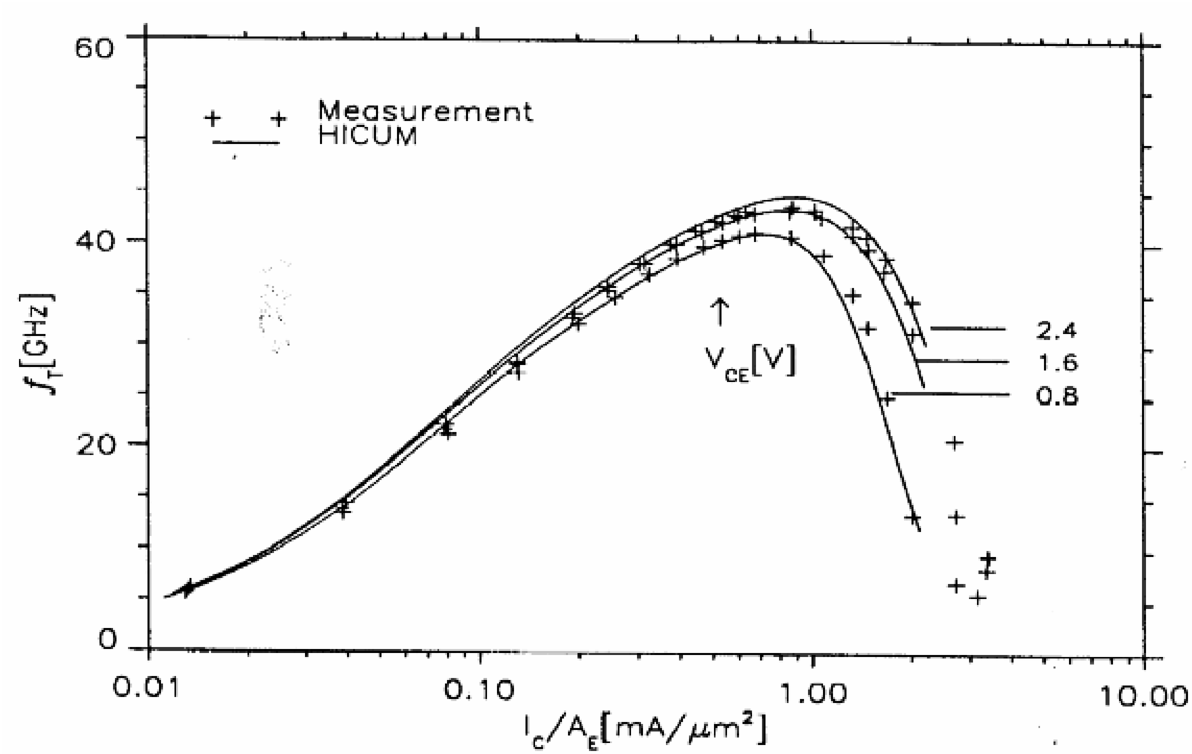
$$C_\pi = \tau_F g_m + C_{dBE}$$

$$i_c = g_m v_{be}$$

$$\beta \;\; \omega \;\; = \left|\frac{i_c}{i_b}\right| = \frac{g_m}{\left| \quad r_\pi + j\omega C_\pi \right|} = \frac{\quad}{\left| \quad \beta_F + j\omega\tau_F + j\omega C_{dBE} kT \;\; qI_C \right|}$$

8.9 Cutoff Frequency

$$f_T \propto \qquad \tau_F \quad C_{dBE} kT\, qI_C$$

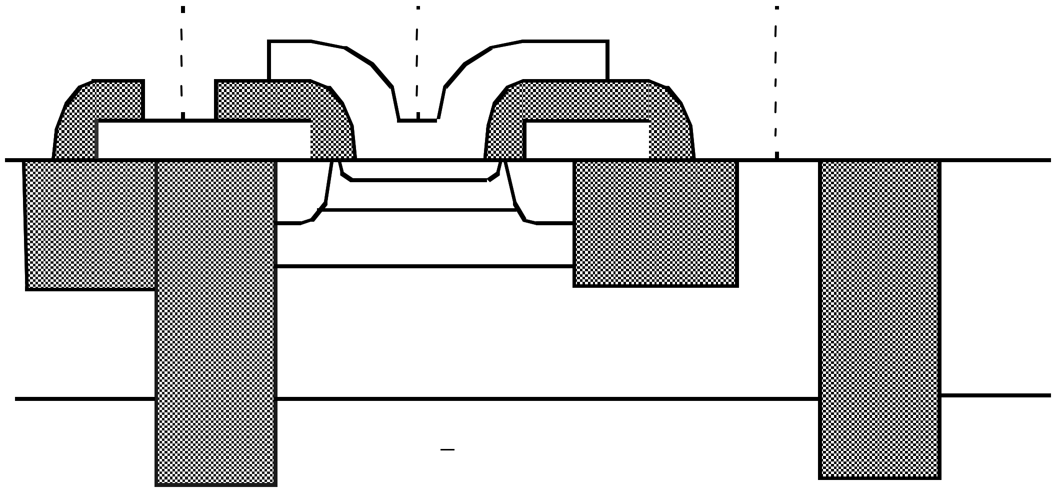


f_T

f_T I_C

f_T I_C

BJT Structure for Minimum Parasitics and High Speed



isolation

8.10 Charge Control Model

$$I_B = I_C \quad \beta_F = \frac{Q_F}{\tau_F \beta_F}$$

I_B

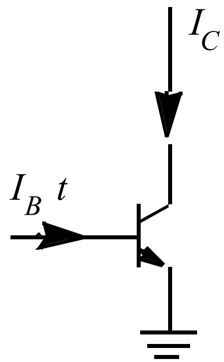
I_B $\quad Q_F \quad \tau_F \beta_F$

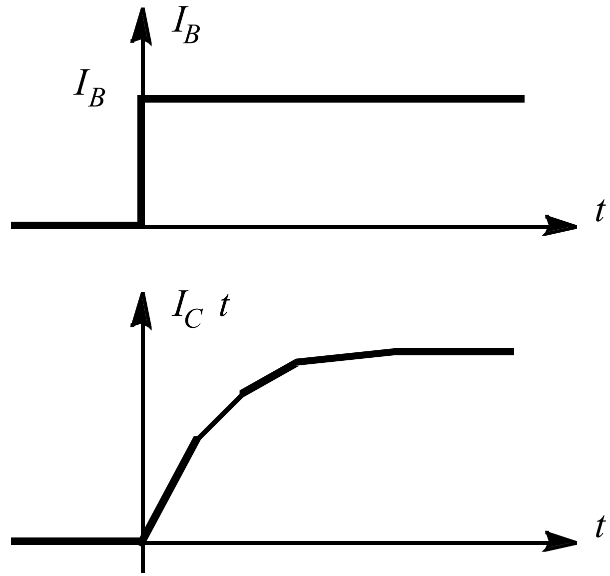
$$\boxed{\frac{dQ_F}{dt} = I_B \ t \ - \frac{Q_F}{\tau_F \beta_F}}$$

$Q_F \ t \quad I_B \ t \ .$

$I_C \ t \quad I_C \ t \quad Q_F \ t \quad \tau_F \ .$

EXAMPLE : Find $I_C(t)$ for a Step $I_B(t)$



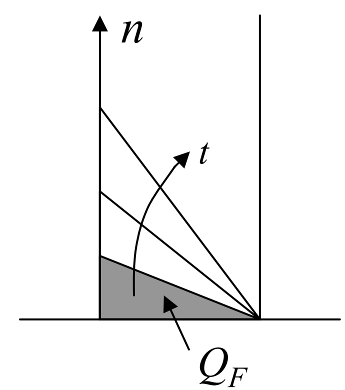


$$\frac{dQ_F}{dt} = I_B\ \ t\ \ - \frac{Q_F}{\tau_F \beta_F}$$

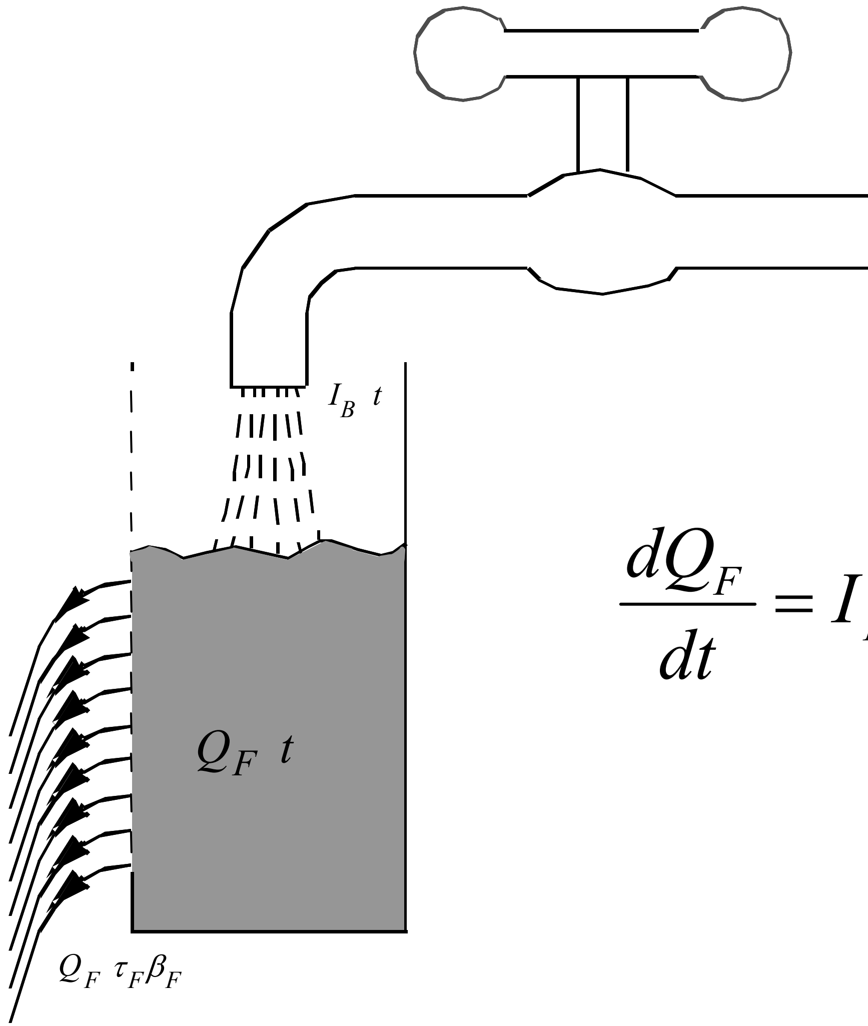
$$Q_F = \tau_F \beta_F I_B \quad\ \ - e^{-t\ \ \tau_F \beta_F}$$

$$I_C\ \ t\ \ = Q_F\ \ t\quad \tau_F = \beta_F I_B \quad\ \ - e^{-t\ \ \tau_F \beta_F}$$

$$I_B\ \ \infty \qquad Q_F \qquad\qquad Q_F\ \ \infty$$

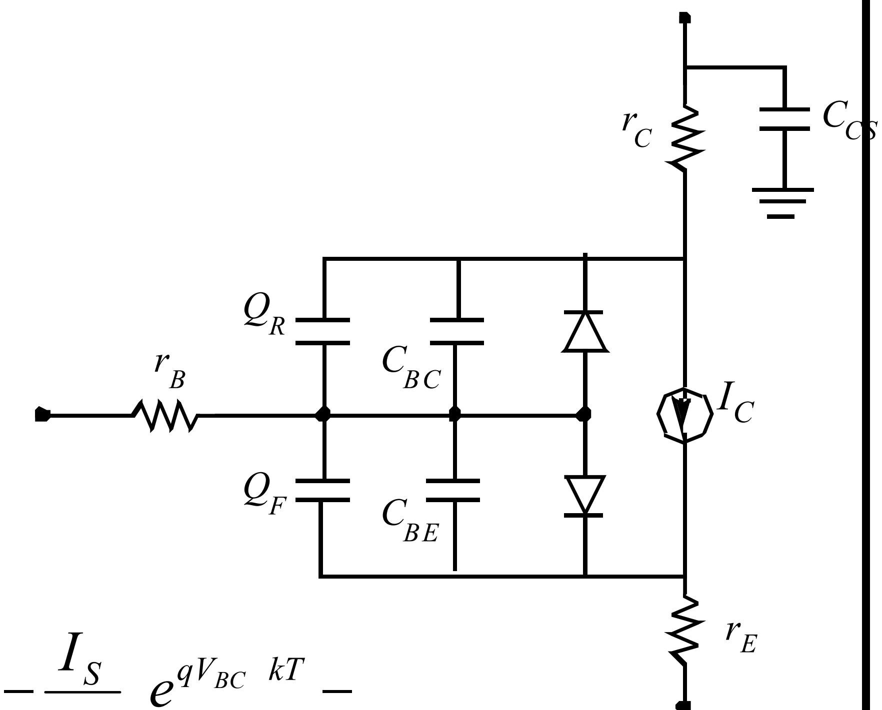


Visualization of $Q_F(t)$



$$\frac{dQ_F}{dt} = I_B\ t\ - \frac{Q_F}{\tau_F\beta_F}$$

8.11 Model for Large-Signal Circuit Simulation



$$I_C = I_S{}' \; e^{qV_{BE} \;\; kT} - e^{qV_{BC} \;\; kT} \left(\; + \frac{V_{CB}}{V_A} \right) - \frac{I_S}{\beta_F} \; e^{qV_{BC} \;\; kT} -$$

8.11 Model for Large-Signal Circuit Simulation

I_C

8.12 Chapter Summary

V_{BE}

I_C

$$I_C = A_E \frac{qn_i}{G_B} \; e^{qV_{BE} \;\; kT} \; -$$

$$G_B \equiv \int^{W_B} \frac{n_i}{n_{iB}} \frac{p}{D_B} dx$$

G_B

I_C

8.12 Chapter Summary

I_B I_C

β_F

α_F

$$\beta_F = \frac{I_C}{I_B} \approx \frac{G_E}{G_B} \qquad \alpha_F = \frac{I_C}{I_E} = \frac{\beta_F}{+\beta_F}$$

β_F I_C

I_C

V_{CB}

I_C V_{CE}

8.12 Chapter Summary

V_{BE}

Q_F I_C

$$Q_F \equiv I_C \tau_F$$

τ_F

$$\tau_F = \tau_{FB} = \frac{W_B}{D_B}$$

$Q_F\ t$ $I_B\ t$

$I_C\ t$

$$\frac{dQ_F}{dt} = I_B\ t\ - \frac{Q_F}{\tau_F \beta_F}$$

$$I_C\ t\ = Q_F\ t\ \quad \tau_F$$

8.12 Chapter Summary

$$g_m \equiv \frac{dI_C}{dV_{BE}} = I_C \quad \frac{kT}{q}$$

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \tau_F g_m$$

$$r_\pi = \frac{dV_{BE}}{dI_B} = \beta_F \quad g_m$$