HOMEWORK SET NO. 1
Due: Thursday, 26th January, 2006

1. (a) How many silicon atoms are there in each unit cell?
   (b) How many silicon atoms are there in one cubic centimeter?
   (c) Knowing that the length of a side of the unit cell (the silicon lattice constant) is 5.43 Å, the atomic weight of Si is 28.1 and Avogadro’s number is $6.02 \times 10^{23}$ atoms/mole, find the density of silicon in g/cm$^3$.

2. GaAs is doped with silicon atoms.
   (a) If silicon replaces the gallium atoms, is the semiconductor n or p type?
   (b) If silicon replaces the arsenic atoms, is the semiconductor n or p type?
   (c) Draw neatly and label the energy band diagram for GaAs when doped with with Si on (i) Ga sites and (ii) As sites.

3. (a) Under equilibrium conditions and $T > 0$ K, what is the probability of an electron state being occupied if it is located at the Fermi level?
   (b) If $E_F$ is positioned at $E_c$, determine (numerical answer required) the probability of finding electrons in states at $E_c + kT$.
   (c) The probability a state is filled at $E_c + kT$ is equal to the probability a state is empty at $E_c + kT$. Where is the Fermi level located?

4. The Maxwell-Boltzmann distribution function
   \[ f(E) = e^{-(E-E_F)/kT} \]
   is often used as an approximation to the Fermi-Dirac function. Using this approximation and the densities of states in the conduction band
   \[ D_c(E) = A(E-E_c)^{1/2}, \]
   find:
   (a) the energy at which one finds the most electrons (1/cm$^3$ eV).
   (b) the conduction band electron concentration (explain any approximation made).
   (c) the ratio of the electron concentration at the energy of (a) to the electron concentration at $E = E_c + 40kT$ (about 1eV above $E_c$ at 300 K). Does this result justify one of the approximations in part (b)?
   (d) the average kinetic energy, $E - E_c$ of the electrons.
These relationships may be useful

\[ \int_{0}^{\infty} x^{n-1} e^{-x} \, dx = \Gamma(n) \text{ (gamma function)} \]

\[ \Gamma(2) = \Gamma(1) = 1, \quad \Gamma(3) = 2, \quad \Gamma(4) = 6 \]

\[ \Gamma(1/2) = \sqrt{\pi}, \quad \Gamma(3/2) = \frac{1}{2}\sqrt{\pi}, \quad \Gamma(5/2) = \frac{3}{4}\sqrt{\pi} \]

5. The carrier distributions (or number of carriers as a function of energy) in the conduction and valence bands were noted to peak at an energy very close to the band edges. (Refer to there carrier distributions in Fig. 1-19). Taking the semiconductor to be non-degenerate show that the energy at which the carrier distributions peaks is \( E_c + \frac{kT}{2} \) and \( E_v - \frac{kT}{2} \) for the conduction and valence bands respectively. [5]