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EECS 130
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Fall 2005

## Midterm II

Name: $\qquad$

Closed book. Two sheet of notes is allowed.
There are 8 pages of this exam including this page.

| Problem 1 |  | 39 |
| :--- | :--- | :--- |
| Problem 2 |  | 31 |
| Problem 3 |  | 30 |
| Total |  | 100 |

## Problem 1:

Consider an N-channel MOSFET. You may assume that this MOSFET has no oxide charge. Fill in the blank cells in the table, using the following symbols: $\uparrow$ for increase, $\downarrow$ for decrease, and $\rightarrow$ for no change. If the cell has already been provided with an X it means that you are not responsible for filling that cell out. When moving along a row consider only the change brought on due to the parameter specified in the first cell of that row. (Each cell is worth 3 points)

|  | $\mathrm{V}_{\mathrm{t}}$ | $\mathbf{V F B}_{\text {FB }}$ | $\mu_{\text {s }}$ | $\mathrm{I}_{\text {ds }}$ | gmsat |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{T}_{\text {ox }} \uparrow$ | increase | No change | Assume $V_{t}$ is unchanged <br> increase | Assume $\mu_{\text {s }}$ remains unchanged <br> decrease | decrease |
| $\mathrm{N}_{\text {sub }} \uparrow$ |  |  |  |  |  |
|  | increase | increase | decrease | decrease | decrease |
| Temperature $\uparrow$ | $X$ | decrease | Assume phonon scattering dominates <br> decrease | Assume $V_{t}$ is constant and phonon scattering dominates <br> decrease | $X$ |

## Problem 2:

Assume an N -channel MOSFET with an $\mathrm{N}^{+}$poly gate and a substrate with an idealized retrograde substrate doping profile as shown in the figure below.

a. Draw the energy band diagram of the MOSFET along the $x$ direction from the gate through the oxide and the substrate, when the gate is biased at threshold voltage. (Hint: Since the P region is very lightly doped you may assume that the field in this region is constant or $\mathrm{d} \varepsilon / \mathrm{dx}=0$ ). Assume that the Fermi level in the $\mathrm{P}^{+}$region coincides with Ev and the Fermi level in the $\mathrm{N}^{+}$gate coincides with Ec. Remember to label Ec, Ev and Ef.

b. Find an expression for $V_{t}$ of this ideal retrograde device in terms of $V_{o x}$. Assume $V_{o x}$ is known. (Hint: use the diagram from part (a) and remember that $\mathrm{V}_{\mathrm{t}}$ is the difference between the Fermi levels in the gate and in the substrate. At threshold, at the $\mathrm{Si}-\mathrm{SiO}_{2}$ interface, Ec of Si coincides with the Fermi level). (10)

From the figure above it can be seen that $\mathrm{Vt}=$ Vox.
Note - You need to apply a positive gate bias to reach inversion. Thus the Fermi level of the poly gate is below the Fermi level of the substrate and the difference is Vt.
c. Now write an expression for $\mathrm{V}_{\mathrm{t}}$ in terms of $\mathrm{X}_{\mathrm{rg}}, \mathrm{T}_{\mathrm{ox}}, \varepsilon_{\mathrm{ox}}, \varepsilon_{\mathrm{si}}$ and any other common parameters you see fit, but not in terms of $\mathrm{V}_{\mathrm{ox}}$. (11)

$$
\begin{aligned}
& \varepsilon_{\mathrm{ox}} \varepsilon_{\mathrm{ox}}=\varepsilon_{\mathrm{si}} \varepsilon_{\mathrm{si}} \\
& \varepsilon_{\mathrm{ox}} \mathrm{~V}_{\mathrm{ox}} / \mathrm{T}_{\mathrm{ox}}=\varepsilon s i *\left(\mathrm{E}_{\mathrm{g}}\right) / \mathrm{q}^{*} \mathrm{X}_{\mathrm{rg}} \\
& \mathrm{Vt}=\varepsilon_{\mathrm{si}} *\left(\mathrm{E}_{\mathrm{g}}\right) / \mathrm{q} * \mathrm{X}_{\mathrm{rg}} * \mathrm{C}_{\mathrm{ox}}
\end{aligned}
$$

## Problem 3:

Any errors made in the earlier parts of the problem will not be penalized as the error propagates through the problem.

Consider a $\mathrm{P}^{+} \mathrm{N}$ diode where the neutral region on the N side has a graded doping profile, which creates an electric field because $\mathrm{E}_{\mathrm{c}}-\mathrm{E}_{\mathrm{f}}$ decreases with increasing doping concentration. Assume that the neutral region on the N side extends from $\mathrm{x}=0$, to $\mathrm{x}=\infty$. You are given that the E-field $\varepsilon$ is a known constant.

a. Write a general expression for $\mathrm{J}_{\mathrm{p}}(\mathrm{x})$ in the neutral N region, including the drift and diffusion current. You may leave the answer in terms of $\mathrm{D}_{\mathrm{p}}, \mu_{\mathrm{p}}, \mathrm{p}(\mathrm{x})$ and $\boldsymbol{\varepsilon}$. (6)

The total current is the sum of hole drift and diffusion currents.

## $\mathrm{J}_{\mathrm{p}}(\mathbf{x})=\mathbf{q} \mu \varepsilon p(\mathrm{x})-\mu \mathrm{kTdp}(\mathrm{x}) / \mathrm{dx}$

b. It can be shown that the presence of an electric field gives rise to a new continuity equation $\mathrm{qD}_{\mathrm{p}} \mathrm{d}^{2} \mathrm{p} / \mathrm{dx}^{2}+\mathrm{q}_{\mathrm{p}} \varepsilon \mathrm{dp} / \mathrm{dx}=\mathrm{qp} / \tau_{\mathrm{p}}$. Assume that the electric field, $\varepsilon$. is a constant from 0 to infinity. The general solution to this new differential equation is $p(x)=A e^{(-x / h)}$. Find $h$ in terms of $\mu_{\mathrm{p}}, \mathrm{D}_{\mathrm{p}}, \varepsilon$ and $\tau_{\mathrm{p} .}$ (6)

We need to plug in $p(x)=A e^{(-x / h)}$ into the differential equation and solve for $h$ :
$p(x)=A e^{-x / h}$ and $\frac{d p}{d x}=\frac{-A e^{-x / h}}{h}$ and $\frac{d^{2} p}{d x^{2}}=\frac{A e^{-x / h}}{h^{2}}$
Then substituting this into the differential equation we obtain:
$q \mu \varepsilon A\left(e^{-x / h}\right)\left(\frac{-1}{h}\right)+\frac{q \mu k T}{q}\left(\frac{A e^{-x / h}}{h^{2}}\right)=q \frac{A e^{-x / h}}{\tau}$
$\mu \varepsilon\left(\frac{-1}{h}\right)+D_{p}\left(\frac{1}{h^{2}}\right)=\frac{1}{\tau}$
$-\mu \varepsilon h+D_{p}=\frac{1}{\tau} h^{2}$
Simplifying further we obtain the second order differential equation:
$h^{2}+\mu \varepsilon h \tau-D_{p} \tau$
$h=\frac{-\mu \varepsilon \tau+\sqrt{(\mu \varepsilon \tau)^{2}+4 D_{p} \tau}}{2}$
The negative sign is not included because $h<0$ is not a physical solution.

## From here on, for all other subsections, you may leave expressions in terms of $h$.

c. You may assume that the current in this diode is equal to hole current at $\mathrm{x}=0$ on the N side. Now that you know the general form for $\mathrm{p}(\mathrm{x})$, find J as a function of V (the applied bias). The boundary condition at $x=0$ is still $p^{\prime}(0)=n_{i}^{2} / N_{d}(0)\left(e^{q V / k T}-1\right)$. You may also assume, $\mathrm{p}(\mathrm{x})=\mathrm{p}^{\prime}(\mathrm{x})$ (i.e.) the equilibrium hole concentration $\left(\mathrm{p}_{0}\right)$ of the N side can be neglected for mathematical simplicity. (6)

We know that two boundary conditions (1) $p^{\prime}(0)=n_{i}^{2} / N_{d}(0)\left(e^{q V / k T}-1\right)$ and (2) $p^{\prime}(\infty)=0$ are satisfied with $p(x)=\frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) e^{-x / h}$

Therefore the sum of the total current as a function of $V$ is still expressed as drift and diffusion.
$J_{p}(x)=q \mu \varepsilon \frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) e^{-x / h}-\frac{q D_{p}}{h} \frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) e^{-x / h}$
Then the current total current is $J_{p}(0)$
$J_{\text {total }}=J_{p}(0)=q \mu \varepsilon \frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right)-\frac{q D_{p}}{h} \frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right)$
d. Find the total stored charge Q in the neutral N region. (6)

The total charge $Q$ is the injected excess hole charge.
$Q=\int_{0}^{\infty} p^{\prime}(x) d x=\int_{0}^{\infty} \frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) e^{-x / h} d x$
$Q=\frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) h$
e. Find the storage time $\tau_{\mathrm{s} \text {, for }}$ the stored charge in part (d). (Hint: $\tau_{\mathrm{s}}=\mathrm{Q} / \mathrm{J}_{\mathrm{p}}$ ). (6)

$$
\begin{aligned}
& \tau_{s}=\frac{Q}{J_{p}}=\frac{\frac{n_{i}{ }^{2}}{N_{d}(0)}\left(e^{q V / k T}-1\right) h}{J_{p}} \\
& \tau_{s}=\frac{h}{q \mu \varepsilon-q \frac{D_{p}}{h}}
\end{aligned}
$$

