1. a) \( R_{out} = R_{o1} + R_{o2} + g_m R_{o1} R_{o2} \)

The "turn on" voltage is when the two right transistors are in saturation region.

It's set by \( V_G \)

\[ V_{G4} = V_{G3} + V_T + V_{SAT} \]
\[ = 2 (V_T + V_{SAT}) \]

\[ V_{S2} = V_{G2} - V_T - V_{SAT} \]
\[ = V_{G4} - V_T - V_{SAT} \]
\[ = V_T + V_{SAT} \]

\[ V_{out} = V_{D2} = V_{S2} + V_{SAT} \quad (E0S) \]
\[ = V_T + 2 V_{SAT} = V_{min} \ (turn \ on \ voltage) \]

To find \( V_{SAT} \):

\[ M3 & 4: \quad \frac{1}{2} K' \frac{W}{L} \cdot V_{SAT}^2 \cdot (1 + \lambda (V_{SAT} + V_T)) = 1 \ \text{m A} \]

\[ V_{SAT} = 0.304 \ 23 \ V \approx 0.304 \ V \]

\[ V_{min} = 1.108 \ V \]

\[ R_{o1} = \frac{1 + \lambda V_{SAT}}{\lambda I} = \frac{1 + \lambda (V_T + V_{SAT})}{\lambda I} \approx 10.8 \ \text{k} \Omega \]

\[ R_{o2} = \frac{1 + \lambda V_{SAT}}{\lambda I} = \frac{1 + \lambda V_{SAT} + V_T}{\lambda I} \approx 10.8 \ \text{k} \Omega \]

\[ g_m = \frac{2 I}{V_{SAT} (1 + \lambda V_T)} \approx 6.1 \ \text{mS} \]

\[ g_{m1} = \frac{2 I}{V_{SAT} (1 + \lambda V_T)} \approx 6.1 \ \text{mS} \]
i. \[ R_{out} = \frac{g_m}{I_{out}} + R_o + R_{in} \]
\[ = 732 \text{ k}\Omega \]

Note: if you guys use \( R_o = \frac{1}{n} \), \( g_m = \frac{2I}{V_{osat}} \),
you will end up with \( R_{out} \approx 678 \text{ k}\Omega \)

b) see SPICE deck and plots.

2. a) VBN sets the source voltage of M2 and the drain voltage of M1, so VBN determines M1 in saturation region or not.
\[ V_{D1} = V_{Psat} \] (EOS of M1)
\[ V_{BN,min} = V_{D1} + V_T + V_{DSAT} \]
\[ = 2V_{Psat} + V_T \]
\[ = 1.10 \text{ V} \]

Turn on voltage makes sure M1 & M2 are in saturation region.
\[ V_{o,bin} = V_{Psat} + V_{Psat} \]
\[ = 2V_{Psat} \]
\[ = 0.608 \text{ V} \]

\[ R_{out} = R_o + R_{in} + \frac{g_m}{I_{out}} \] the same as prob.1

b) see SPICE deck and plots.
\[ R_o = \frac{1}{(I(out))} - I(\text{out}) \] use expression builder

c) Cascade mirror w/ VBN is best.
Simulation should give very close results.
3. \[ V_{os} = 2 V_{os1} \]

\[ I = \frac{1}{2} k'(\frac{V_o}{2V_{os1}})^2 \left( \frac{R}{\lambda (V_{os1}-V_o)} \right) \]

\[ V_{os} \Rightarrow \frac{V_o}{V_{os1}} = 24.35 \]

To verify, put this into SPICE and it works fine.

4. \( Assume \; g_m R_o \gg 1 \)

a) \[ \frac{V_o}{V_i} = \frac{g_m}{(g_m R_o + (R_{L} || R_L))} \frac{g_m}{g_m} \] (Cascode)

\[ \frac{V_o}{V_i} = -g_m \cdot R_{out} \]

\[ = -g_m \left( R_o || \frac{R_L}{R_{L} || V_o} \right) \frac{1}{g_m} \] (CS)

\[ \frac{V_o}{V_{BN}} = CS \; W/ \; source \; degeneration \]
\[
\frac{V_o}{V_{in}} = R_{out} \cdot G_m \cdot \left[ \frac{1 + g_m R_o}{1 + g_m R_o} \right] \frac{R_o}{R_2} \cdot \frac{-g_m}{1 + g_m R_o} \\
\frac{V_c}{V_{in}} = \frac{g_m}{1 + \frac{R_c}{R_o}} \cdot \left( \frac{R_{11} \cdot \frac{R_o}{R_{11} \cdot R_{12}}} {1 + \frac{R_c}{R_o}} \right) \cdot \frac{C_D}{1 + \frac{R_c}{R_o}}
\]

b) \( R_L = R_o \), \( g_m = \frac{2 I}{V_{os}} = 6.3 \, \text{mS}, \ R_o = 10k \Omega \)

\[
v_o \approx -g_m \left( R_{11} \cdot \frac{R_o}{R_{11} \cdot R_{12}} \right) \approx -g_m R_o = -6.3
\]

\[
\frac{v_c}{v_{in}} = -g_m \left( R_{11} \cdot \frac{R_o}{R_{11} \cdot R_{12}} \right) \approx -g_m \cdot \frac{2}{g_m} = -2
\]

\[
v_o \approx R_o \cdot \frac{-g_{m2}}{1 + g_{m2} R_o} \approx -1
\]

\[
v_c \approx \frac{g_m}{1 + \frac{R_c}{R_o}} \cdot \frac{R_{11} \cdot \frac{R_o}{R_{11} \cdot R_{12}}}{R_o} \approx 1
\]

Note: Assume \( g_{m2} \gg 1 \) approximately.

c) \( R_L = g_m R_o \)

\[
v_o \approx -g_m \left( g_{m2} \cdot \frac{R_o}{R_{in}} \right) = -\frac{1}{2} g_m R_o = -1084.5
\]

\[
v_c \approx -g_m \cdot \left( R_{11} \cdot \frac{R_o}{R_{in}} \right) = -\frac{1}{2} g_m R_o = -31.5
\]
\[
\frac{V_0}{V_{aw}} = \frac{9m}{2} - \frac{9m}{1 + 9m} = -31.5
\]

\[
\frac{V_c}{V_{aw}} = \frac{9m}{1 + 9m} \cdot \left( \frac{1}{10} \right) = \frac{1}{10} \cdot \frac{9}{2} = \frac{1}{2}
\]

5. open end design problem, no solution