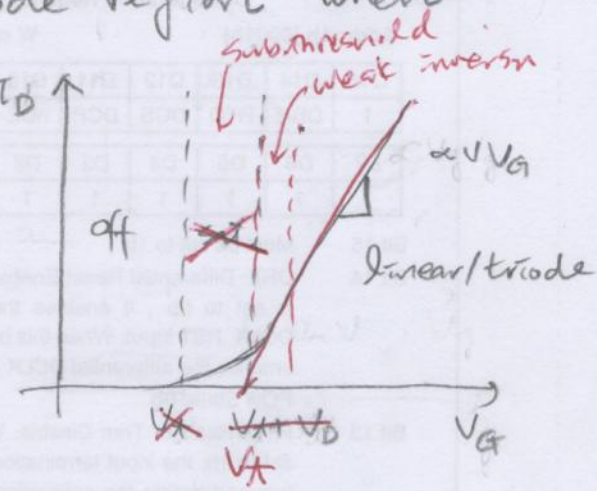
EE 140 Homework #2 Fall 2009

#1

[prob.1] (The exact numbers can be different for probl & prob2)

(a) The devices are in the triode region when

$V_G \geq V_t + V_D = V_t + 50mV$.



$$I_D = K \cdot \frac{W}{L} \cdot \left[(V_G - V_t) \cdot V_D - \frac{V_D^2}{2} \right] \quad (\frac{V_D^2}{2} \approx 0)$$

$$\simeq K \cdot \frac{W}{L} \cdot V_D (V_G - V_t)$$

W: 2.5μm, L: 0.3μm, slope, m.

$$m_{NMOS} = K_n \cdot \frac{2.5\mu m}{0.3\mu m} \cdot 50mV = \frac{8\mu - 2\mu}{0.8V - 0.6V}$$

$$\therefore K_n = \frac{8\mu - 2\mu}{0.8 - 0.6} \cdot \frac{0.3}{2.5} \cdot \frac{1}{50m} \simeq \underline{72.0 \mu A/V^2}.$$

$$m_{PMOS} = K_p \cdot \frac{2.5\mu m}{0.3\mu m} \cdot 50mV = \frac{4\mu - 2\mu}{1.0V - 0.8V}$$

$$\therefore K_p = \frac{4\mu - 2\mu}{1.0 - 0.8} \cdot \frac{0.3}{2.5} \cdot \frac{1}{50m} \simeq \underline{24.0 \mu A/V^2}$$

(b) From Fig. 5,

$$|V_t| = |V_{t0}| + \gamma \left(\sqrt{|2\phi_f| + |V_{SB}|} - \sqrt{|2\phi_f|} \right)$$

$\lvert V_{SB}\rvert$	$\lvert V_{t,n}\rvert$	$\lvert V_{t,p}\rvert$
0	0.45V	0.50V
1	0.65V	0.70V
2	0.75V	0.80V
3	0.80V	0.90V

V_{t0}

γ, ϕ_f can be found by MATLAB's "fminsearch" (see appendix for codes)

$\gamma_N = 0.21$, $\phi_{f,N} = 4.3 \times 10^{-4} \simeq 0$

$\gamma_P = 0.28$, $\phi_{f,P} = 6.4 \times 10^{-5} \simeq 0$.

#2

Unrealistic values of ϕ_f can be from either low doping concentration, inaccurate V_t, or measurement errors.

(c)

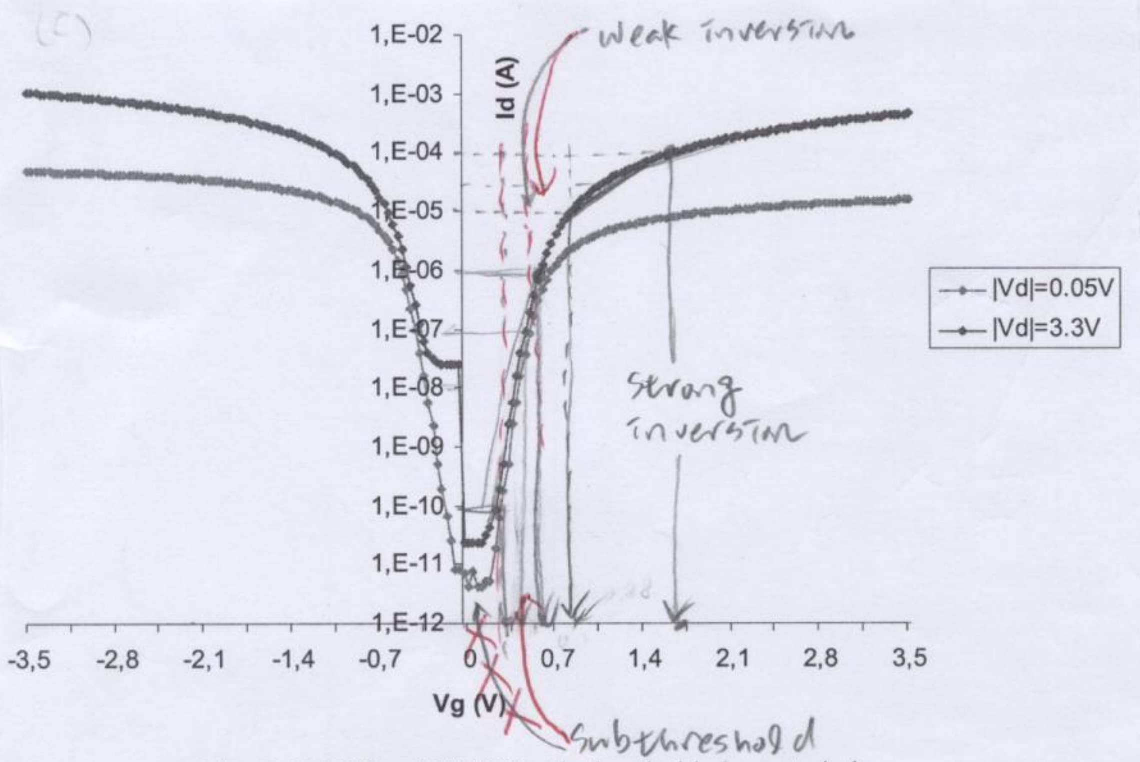


Fig. 6. PMOS and NMOS sub-threshold characteristics

(i) strong inversion

$$g_m = \frac{10^{-4}A - 10^{-5}A}{1.75V - 0.88V} \approx 1.15\times10^{-4}\,S$$

$$\frac{g_m}{I_D} = \frac{1.15\times10^{-4}\,S}{5\times10^{-5}\,A} = 2.3\,V^{-1}$$

(ii) weak inversion

$$g_m = \frac{10^{-6}A - 10^{-10}A}{0.7V - 0.35V} = 2.86\times10^{-6}\,S$$

$$\frac{g_m}{I_D} = \frac{2.86\times10^{-6}\,S}{5\times10^{-7}\,A} = 5.72\,V^{-1}$$

(iii) subthreshold

$$\cancel{g_m \approx 0}$$

$$\cancel{\frac{g_m}{I_D} \approx 0}$$

~~Probably because of the measurement accuracy limitation.~~

We can expect better efficiency from subthreshold and weak inversion than from strong inversion.

#3.

(d)

$\lvert V_g \rvert$	$I_{D,NMOS}$	$I_{D,PMOS}$
1	0.05mA	0.02mA
2	0.40mA	0.15mA
3	0.80mA	0.35mA
4	1.20mA	0.55mA

The devices looks like linear devices because of the velocity saturation.

(e) At $\lvert V_g \rvert = 3V$, for example,

$$r_o = \frac{4.0V - 1.6V}{0.8mA - 0.75mA} \simeq \underline{\underline{48K\Omega}}$$

$$r_o = \frac{1+\lambda V_{DS}}{\lambda I_D} \Rightarrow \lambda = \frac{1}{r_o I_D - V_{DS}} \simeq \underline{\underline{0.03}}$$

(f) $\text{Variation} \triangleq \dfrac{\text{max} - \text{min}}{\text{mode}}$

$$\text{NMOS : \% variation} = \frac{0.5875V - 0.5125V}{0.5375V} \times 100 \simeq \underline{\underline{14.0\%}}$$

$$\text{PMOS : } \quad " \quad = \frac{0.5875V - 0.4875V}{0.5375V} \times 100 \simeq \underline{\underline{18.6\%}}$$

In Appendix C, we can see gradual changes of V_t across a wafer, while we can also observe some random effects that contribute to the V_t variations.

[prob. 2]

(a) (i) NMOS

$$g_m \simeq \frac{0.85mA - 0.4mA}{0.8V - 0.6V} = 2.25mS$$

$$r_o \simeq \frac{1.0V - 0.5V}{0.75mA - 0.60mA} = 3.33k\Omega$$

$$\Rightarrow g_m r_o \simeq \underline{7.5}$$

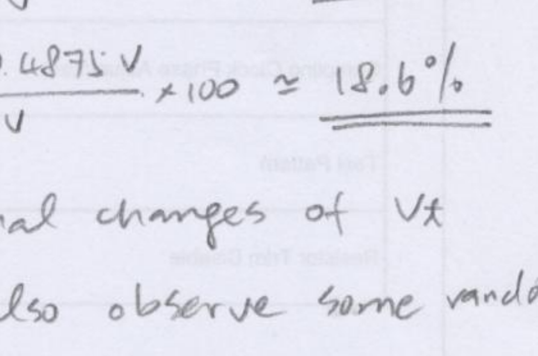


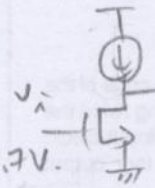
Figure 5: Transistor I-V curves

#4

(ii) PMOS

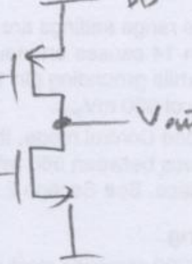
$$g_m \simeq \frac{0.75mA - 0.35mA}{0.8V - 0.6V} = 2.0mS \Rightarrow g_m r_o = \underline{\underline{10.0}}$$

$$r_o \simeq \frac{1.0V - 0.5V}{0.65mA - 0.55mA} = 5.0K\Omega$$

(b) 

$$A_v = g_m r_o = \underline{\underline{7.5}}$$

(c) $|V_{GS,PMOS}| = 0.6V$



$$r_{op,V_{GS}=0.6V} = \frac{1.0V - 0.5V}{0.50mA - 0.35mA} = \underline{3.33K\Omega}.$$

$$g_{mp,V_{GS}=0.6V} \simeq \frac{0.55mA - 0.2mA}{0.7V - 0.5V} = \underline{1.75mS}.$$

Assume that $V_{GS,N} = 0.7V$

$$A_v = g_{m,n}(r_{op} \parallel r_{on})$$

$$= 2.25mS \cdot (3.33K\Omega \parallel 5.0K\Omega)$$

$$= \underline{\underline{4.50}}$$

Therefore, we cannot get high gains from individual transistors in advanced technology.

[prob. 3] $g_m = K \cdot \left(\frac{W}{L}\right) \cdot V_{DSAT} \quad \cdots (1)$

$$= \frac{2I_D}{V_{DSAT}} \quad \cdots (2)$$

$$= \sqrt{2K \cdot \left(\frac{W}{L}\right) I_D} \quad \cdots (3) \quad (\because \lambda = 0)$$

(a) From (2), $V_{DSAT} = \frac{2I_D}{g_m} = \frac{2 \times 1mA}{1m\mho} = \underline{2.0V}$

From (1) $\left(\frac{W}{L}\right) = \frac{g_m}{V_{DSAT}} \cdot \frac{1}{K}$.

$$\left(\frac{W}{L}\right)_n = \frac{1m\mho}{2.0V} \times \frac{1}{200\mu A/V^2} = \underline{2.5} = \left(\frac{1.25M}{0.5M}\right)$$

$$\left(\frac{W}{L}\right)_p = \frac{1m\mho}{2.0V} \times \frac{1}{100\mu A/V^2} = \underline{5.0} = \left(\frac{2.5M}{0.5M}\right)$$

(b) From (2), $I_D = \frac{g_m \cdot V_{DSAT}}{2} = \frac{1m\mho \cdot 0.2V}{2} = \underline{100\mu A}$

From (1), $\left(\frac{W}{L}\right)_n = \frac{1m\mho}{0.2V} \cdot \frac{1}{200\mu A/V^2} = \underline{25.0} = \left(\frac{12.5M}{0.5M}\right)$

$$\left(\frac{W}{L}\right)_p = \frac{1m\mho}{0.2V} \cdot \frac{1}{100\mu A/V^2} = \underline{50.0} = \left(\frac{25M}{0.5M}\right)$$

(c) From (3), $I_D = \frac{g_m^2}{2 \cdot K \left(\frac{W}{L}\right)}$

$$I_{D,N} = \frac{(1m)^2}{2 \cdot 200\mu A/V^2 \cdot \frac{50}{0.5}} = \underline{25\mu A}.$$

$$I_{D,p} = \frac{(1m)^2}{2 \cdot 100\mu A/V^2 \cdot \frac{50}{0.5}} = \underline{50\mu A}.$$

From (2),

$$V_{DSAT,N} = \frac{2 I_{D,N}}{g_m} = \frac{2 \times 25\mu A}{1m\mho} = \underline{50mV}$$

$$V_{DSAT,P} = \;'' \; = \frac{2 \times 50\mu A}{1m\mho} = \underline{100mV}$$

#6.

(d) From ②, $I_D = \dfrac{g_m \cdot V_{DSAT}}{2} \geq \dfrac{g_m \cdot V_{DSAT,min}}{2}$.

$\therefore I_D \geq \dfrac{1mS \cdot 0.1V}{2} = \underline{50\mu A}$

$\left(\dfrac{W}{L}\right)_n = \dfrac{g_m}{K \cdot V_{DSAT.min}} = \dfrac{1mS}{200\mu A/V^2 \cdot 0.1V} = \underline{\underline{50}} = \left(\dfrac{25\mu}{0.5\mu}\right)$

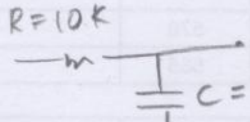
$\left(\dfrac{W}{L}\right)_p = \quad " \quad = \dfrac{1mS}{100\mu A/V^2 \cdot 0.1V} = \underline{\underline{100}} = \left(\dfrac{50\mu}{0.5\mu}\right)$

(e) From ①, $\left(\dfrac{W}{L}\right) = \dfrac{g_m}{K \cdot V_{DSAT}} \geq \dfrac{g_m}{K \cdot V_{DSAT.max}}$

$V_{DSAT,max} = V_{DD} - V_t \triangleq 2.5V.$

$\left(\dfrac{W}{L}\right)_{min} = \dfrac{1mS}{200\mu A/V^2 \cdot 2.5V} = \underline{\underline{\left(\dfrac{1.0\mu}{0.5\mu}\right)}}. \ (NMOS)$

[prob.4]



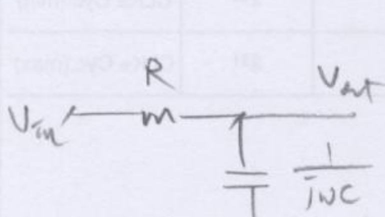
(a) $\tau = RC = 10^{-9}s$

$R = \dfrac{1}{\omega_c C} \Rightarrow \omega_c = \dfrac{1}{RC} = 10^9 \ rad/s$

$f_c = \dfrac{\omega_c}{2\pi} = 159.2MHz$

$\omega_c = \dfrac{1}{\tau}, \quad f_c = \dfrac{\omega_c}{2\pi}, \quad \tau = \dfrac{1}{\omega_c} = \dfrac{1}{2\pi f_c}$

(b)



$\dfrac{V_{out}(\omega)}{V_{in}(\omega)} = \dfrac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \dfrac{1}{1 + RCj\omega}$

$= \dfrac{1}{1 + \frac{\omega}{\omega_c} j}$

$$V_{out}(0.1\omega_c) = \frac{1}{1+0.1j} = \frac{1}{\sqrt{1+(0.1)^2}} \angle -\arctan 0.1$$

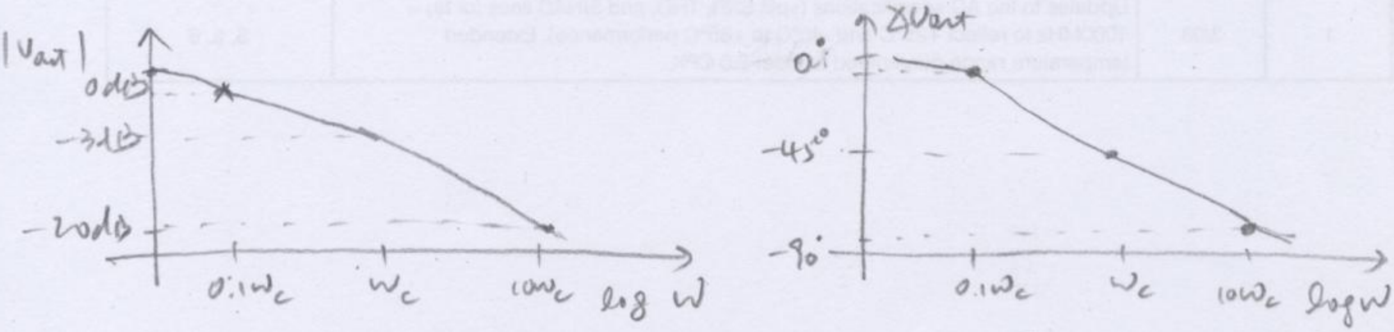
$$\simeq 0.995 \angle -5.71^\circ \quad (-0.044\,dB)$$

$$V_{out}(\omega_c) = \frac{1}{1+j} = \frac{1}{\sqrt{2}} \angle -\arctan 1$$

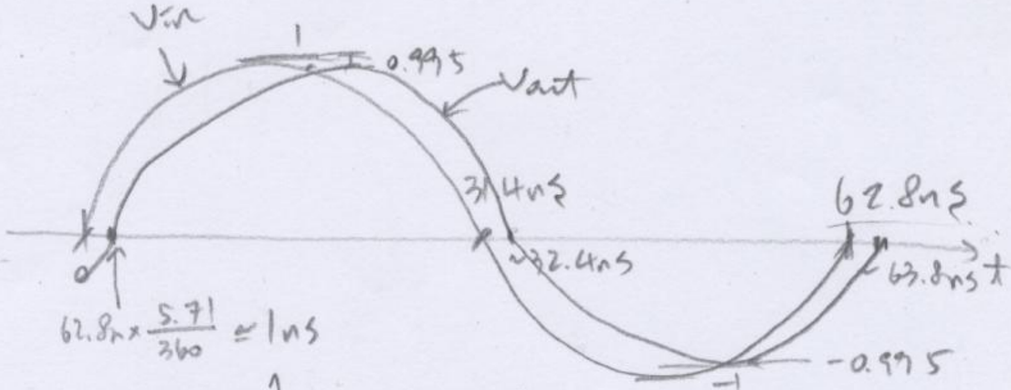
$$= 0.707 \angle -45^\circ \quad (-3\,dB)$$

$$V_{out}(10\omega_c) = \frac{1}{1+10j} = \frac{1}{\sqrt{1+10^2}} \angle -\arctan 10$$

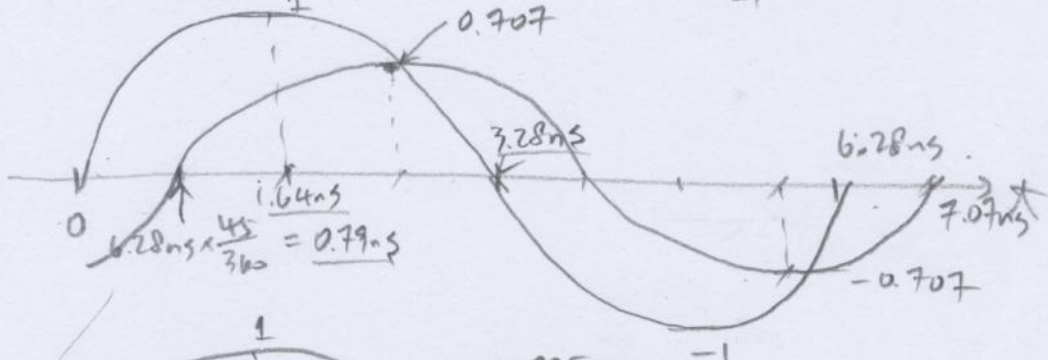
$$= 0.0995 \angle -84.3^\circ \quad (-20.0\,dB)$$



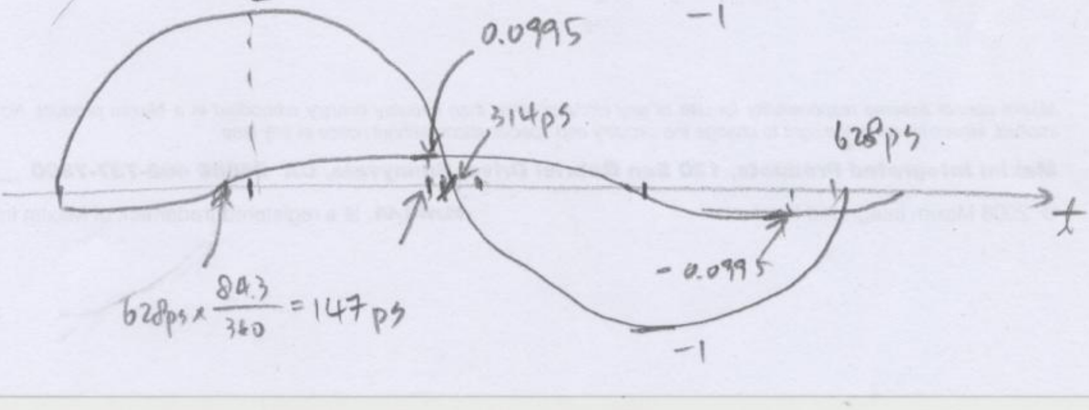
(i) $0.1\omega_c$



(ii) ω_c



(iii) $10\omega_c$

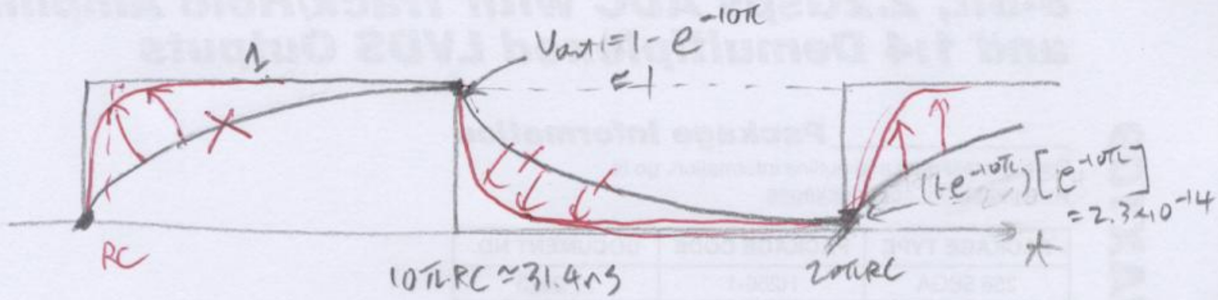


#8

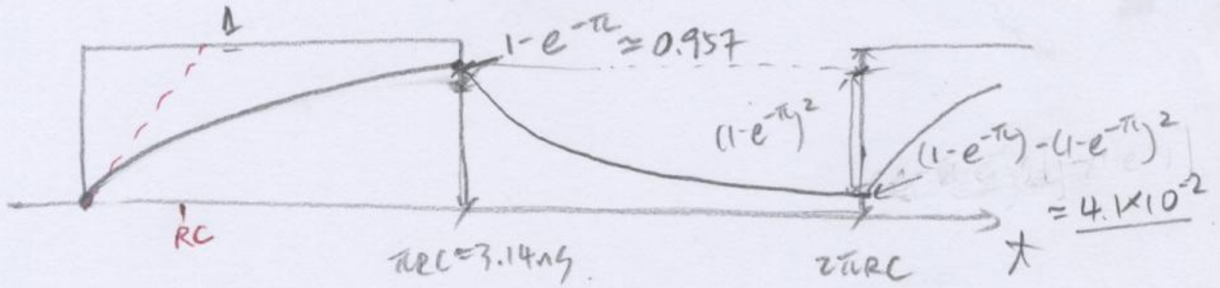
(c) $H(s) = \frac{1}{1+RCs}$

$$h(t) = 1 - e^{-\frac{1}{RC}t} = 1 - e^{-\omega_c t}$$

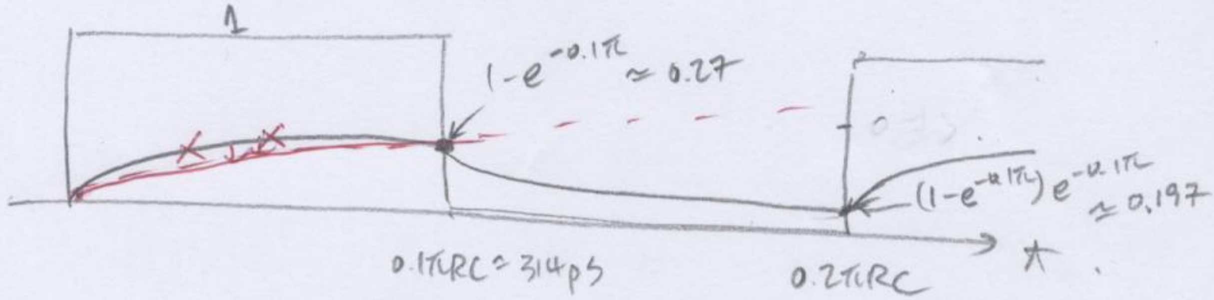
(i) $0.1\omega_c$



(ii) ω_c

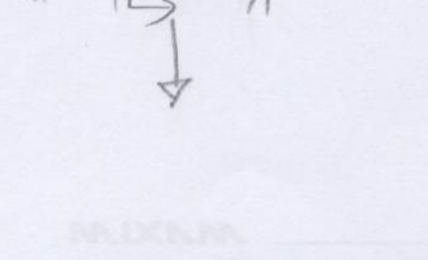


(iii) $10\omega_c$



[prob. 5]

(a)



$$\overset{1.5V}{|V_{DS}|} \geq |V_{GS}| - |V_{t,p}| = 0.5 \quad (\text{sat.})$$

$$I_{D,p} = \frac{k_p}{2}\left(\frac{W}{L}\right)(|V_{GS}| - |V_{t,p}|)^2(1 + \lambda|V_{DS}|)$$

$$= \frac{100\mu A/V^2}{2}\left(\frac{50}{1}\right)(0.5)^2(1 + 0.1\times1.5)$$

$$= \underline{\underline{0.72mA}}$$

(b) $I_{D.p} = I_{D.N}$ @ $V_{out} = 1.5V$ #9

$$\cancel{\frac{k'}{2}}\left(\frac{W}{L}\right)_p (|V_{GS.p}| - |V_{Tp}|)^2 (1 + \cancel{\lambda |V_{DS}|})$$ (∵ NMOS in sat.)

$$= \cancel{\frac{k'}{2}}\left(\frac{W}{L}\right)_n (V_{GS.n} - V_{Tn})^2 (1 + \cancel{\lambda V_{DS}})$$

$$\therefore (0.5)^2 = 2 \cdot (V_{GS.n} - 0.5)^2$$

$$V_{GS.n} = 0.5 \pm \frac{0.5}{\sqrt{2}} \cong \underline{0.85\ V}$$ (∵ $V_{GS.n} \geq V_{Th.n}$ to turn on)

(c) $$g_{m.n} = k_n' \cdot \left(\frac{W}{L}\right) \cdot V_{DSAT} (1 + \lambda V_{DS})$$

$$= 200\mu A/V^2 \left(\frac{50}{1}\right) \cdot \frac{0.5}{\sqrt{2}} (1 + 0.1 \times 1.5)$$

$$= \underline{4.07 mS}$$

$$r_{o.n} = \frac{1 + \lambda V_{DS}}{\lambda I_D} = \frac{1 + 0.1 \times 1.5}{0.1 \times 0.72 mA} = \underline{15.97 k\Omega}$$

$$g_{m.p} = 100\mu A/V^2 \left(\frac{50}{1}\right) \times 0.5 \times (1 + 0.1 \times 1.5) = \underline{2.88 mS}$$

$$r_{o.p} = \frac{1 + 0.15}{0.1 \times 0.72 mA} = \underline{15.97 k\Omega}$$

$$C_{gs.n} = \frac{2}{3} WL \cdot C_{ox} + W C_{ol}'$$

$$= \frac{2}{3} \times 50\mu \times 1\mu \times 5fF/\mu m^2 + 50\mu \times 0.5 fF/\mu m$$

$$= \underline{192.7 fF} = \underline{C_{gs.p}}$$

$$C_{gd,n} = W \cdot C_{ol}' = \underline{25 fF} = C_{gd.p}$$

$$C_{db,n} = AD \cdot \frac{C_{db0}}{\sqrt{1 + \frac{1.5}{0.5}}} = \frac{50\mu \times 1\mu}{2} \times 1 fF/\mu m^2 = \underline{\cancel{50 fF}\ 25fF} = C_{db.p}$$

Both devices are in saturation.

#10.

(d)

	hand calc.	SPICE	% error
I_D	0.72mA	0.719mA	0.14%
g_{mn}	4.07mS	4.07mS	0.10%
g_{mp}	2.88mS	2.88mS	0.50%
$r_{o,n}$	15.97KΩ	15.99KΩ	0.16%
r_{op}	15.97KΩ	16.00KΩ	0.19%
C_{gs}	192.7fF	191.82fF	0.46%
C_{db}	~~50.0fF~~ 25.0fF	~~49.99fF~~ ~~25.01fF~~ 24.99fF	~~0.02%~~ 0.04%
C_{gd}	25.0fF	25.0fF	0.00%

For all the parameters, the % error should be less than 1%.

[Prob1(b)] MATLAB Code

```
options = optimset('TolFun', 1e-12);
[x, fval] = fminsearch(@(x) myfun(x), [100, 100 ], options)

function f = myfun(x)

gamma = x(1);
phi_f = x(2);

% NMOS
% measurement = [
% 0.65    0.75    0.8 ];
%approx = 0.45+ gamma *(sqrt(2*phi_f + [ 1 2 3 ]) - sqrt(2*phi_f));

% PMOS
measurement = [
0.7    0.8    0.9 ];
approx = 0.5+ gamma *(sqrt(2*phi_f + [ 1 2 3 ]) - sqrt(2*phi_f));

f = norm((approx-measurement) ./ approx)^2;
```

[Prob5 (c)]

```
* HW#2 prob5. PMOS-load NMOS common source amplifier
.param vtn=0.5
.param vtp=-0.5
.model nmos1 nmos vto='vtn'
+ tox=6.9nm kp=200u lambda=0.1 gamma=0.5 phi=0.6
+ capop=0 cgso=0.5n cgdo=0.5n pb=0.5 cj=1e-3
.model pmos1 pmos vto='vtp' tox=6.9nm kp=100u lambda=0.1 gamma=0.5 phi=0.6
+ capop=0 cgso=0.5n cgdo=0.5n pb=0.5 cj=1e-3

m1 out bias dd dd pmos1 w=50u l=1u AD=5e-11 AS=5e-11
m2 out in 0 0 nmos1 w=50u l=1u AD=5e-11 AS=5e-11

Vdd dd 0 3v
Vg in 0 dc 0.8536v
Vbias bias 0 2v

.options post
.op

.end
```

```
**** mosfets
 subckt
 element  0:m1      0:m2
 model    0:pmos1   0:nmos1
 region     Saturati  Saturati
  id     -718.8447u 718.8447u
  ibs       0.        0.
  ibd      15.0152f -14.9848f
  vgs      -1.0000   853.6000m
  vds      -1.5015    1.4985
  vbs       0.        0.
  vth    -500.0000m  500.0000m
  vdsat  -500.0000m  353.6000m
  vod    -500.0000m  353.6000m
  beta      5.7508m   11.4985m
  gam eff  500.0000m  500.0000m
  gm        2.8754m   4.0659m
  gds      62.5000u  62.5165u
  gmb     928.0246u   1.3123m
  cdtot    49.9905f  50.0095f     = Cgd + Cdb .
  cgtot   216.8194f 216.8194f
  cstot   241.8194f 241.8194f
  cbtot    74.9905f  75.0095f
  cgs     191.8194f 191.8194f
  cgd      25.0000f  25.0000f
```