EE 140 Homework #3 Fall 2009

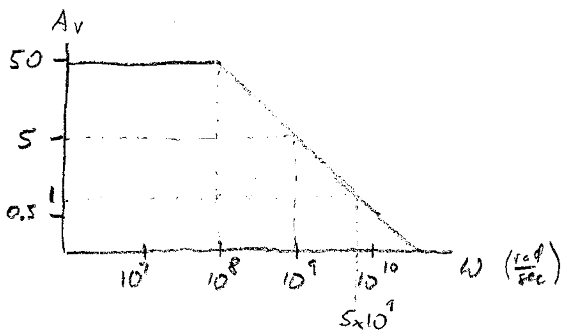
1 A) $A_v = 100$ $f_p = 5\ MHz$ $C_L = 1pF$

$f_u = A_v f_p = 500\ MHz$

$R_o = \frac{1}{2\pi f_p C_L} = (10\pi \cdot 10^6 \cdot 10^{-12})^{-1} = \frac{1}{\pi} \cdot 10^5\ \Omega$

$g_m = \frac{A_v}{R_o} = \frac{100}{1/\pi \cdot 10^5} = \pi\ mS$

1 B)



A_v decreases by 10x / decade after ω_p, so ω_p must be exactly one decade before $10^9 \frac{rad}{sec}$ where $A_v = 5$, given that $A_{v,DC} = 50$.

$\omega_p = 10^8 \frac{rad}{sec}$

$\omega_u = A_{v,DC} \cdot \omega_p = 5 \cdot 10^9 \frac{rad}{sec}$

1c) $R_o = 10^7$ $G_m = 10nS$ $\omega_u = 10^9 \frac{rad}{sec}$

$A_v = G_m R_o = 10^5$

$\omega_p = \frac{\omega_u}{A_v} = 10^4 \frac{rad}{sec}$

$C_L = \frac{1}{\omega_p R_o} = 10^{-11}\ F = 10pF$

2)

	g_m [S]	R_o [Ω]	C_L [F]	A_v	ω_p [$\frac{rad}{sec}$]	ω_u [$\frac{rad}{sec}$]
a)	1m	50k	2p	50	10 M	500 M
b)	5m	20k	5p	100	10 M	1 G
c)	25m	4k	5p	100	50 M	5 G
d)	2m	15k	13.33p	30	5 M	150 M
e)	50m	100	2.5p	5	4 G	20 G

Useful relations: $A_v = g_m R_o$, $\omega_p = \frac{1}{R_o C_L}$, $\omega_u = A_v \omega_p$

3) First, a few things to note:

(1) If I can achieve the required g_m, R_o, and C_L, then I can also achieve the required A_v, ω_p, and ω_u.

(2) L presents a tradeoff between R_o (via λ) and ω_p (via C_L)

(3) W presents a tradeoff between g_m and ω_p (via C_L)

(4) V_{Dsat} presents a tradeoff between g_m and r_o (via I_D)

(5) This problem has no unique solution, so we need to make an intelligent choice about which parameter to choose first.

a) Minimum V_{Dsat} gives the greatest g_m efficiency $\left(\frac{g_m}{I_D}\right)$

b) Minimum length gives the minimum capacitance / area

$\Rightarrow$ A good place to start is to minimize either V_{Dsat} or L. Because I claim the math is easier, I'll start with minimum L.

(6) Assume $|V_{DS}| = |V_{DG}| = V_{out} = \frac{V_{DD}}{2} = 1.5\,V$. This means $W_p = 2W_n$

A) $L = 0.5\,\mu m$ $\qquad \lambda = 0.1 \cdot \frac{1\,\mu m}{L} \Rightarrow \lambda = 0.2$

$$R_o = \frac{1}{2}\,\frac{1+\lambda V_{DS}}{\lambda I_D} \Rightarrow I_D = \frac{1+\lambda V_{DS}}{2\lambda R_o}$$

so for $R_o = 50\,k\Omega$, $\quad I_D = 65\,\mu A$.

$$g_m = \frac{2 I_D}{V_{Dsat}} \Rightarrow V_{Dsat} = \frac{2 I_D}{g_m}$$

so for $g_m = 1\,mS$, $\quad V_{Dsat} = 130\,mV$

$$W = \frac{I_D}{I_D/W} = \frac{I_D}{\frac{1}{2L} k_n' V_{Dsat}^2 (1+\lambda V_{DS})} = \frac{65 \cdot 10^{-6}}{\frac{1}{2 \cdot 0.5} 200 \cdot 10^{-6} \cdot 0.13^2 \cdot (1+0.2 \cdot 1.5)}$$

$$= 14.79\,\mu m$$

$$C_L = C_{gd\,(nmos)} + C_{gd\,(pmos)} + C_{db\,(nmos)} + C_{db\,(pmos)}$$

$$= W_p C_{OL}' + (W_n + W_p)\frac{k_{eq} C_{dso}}{\sqrt{1+\frac{|V_{db}|}{\phi}}} \qquad C_{OL}' = 0.5\,fF/\mu m$$

$$\frac{k_{eq} C_{dso}}{\sqrt{1+\frac{|V_{db}|}{\phi}}} = 0.5\,fF/\mu m = C_*$$

$$= 2W_n C_{OL}' + 3W_n C_* = 2.5\,W_n\,[fF]$$

$$= 37\,fF$$

B) $L = 0.5\,\mu m \Rightarrow \lambda = 0.2$

$I_D = \frac{1+\lambda V_{DS}}{2\lambda R_o}$, so for $R_o = 20\,k\Omega$ $I_D = 162.5\,\mu A$.

$V_{Dsat} = \frac{2I_D}{g_m}$, so for $g_m = 5\,mS$ $V_{Dsat} = 65\,mV$.

This is less than $100\,mV$, the minimum allowable V_{Dsat}, so we need a new approach.

$V_{Dsat} = V_{Dsat,min} = 100\,mV$

$I_D = \frac{1}{2} g_m V_{Dsat}$, where g_m is taken from the spec.

$R_o = \frac{1}{\lambda} \frac{1+\lambda V_{DS}}{2I_D} \Rightarrow \lambda = \frac{1}{2R_o I_D - V_{DS}}$, where R_o is from the spec.

$\lambda = 0.1 \cdot \frac{1\mu m}{L} \Rightarrow L = \frac{0.1}{\lambda}$

$W = \frac{I_D}{I_D/W} = \frac{I_D}{\frac{1}{2L} k_n' V_{Dsat}^2 (1+\lambda V_{DS})}$

Plugging in using the values for B ($V_{Dsat} = 100\,mV$, $g_m = 5\,mS$, $R_o = 20\,k\Omega$),

we find $I_D = 250\,\mu A$, $L = 0.85\,\mu m$ and $W = 180.625\,\mu m$

Using $C_L = 2W_n C'_{OL} + 3W_n C^*$ from part A, $C_L = 452\,fF$

C) Using the approach from A, we would again get $V_{Dsat} = 65\,mV < V_{Dsat,min}$.

Instead, fix V_{Dsat} as in part B.

Plugging in to the same expressions in part B using the values from part C ($V_{Dsat} = 100\,mV$, $g_m = 25\,mS$, $R_o = 4\,k\Omega$), we find

$I_D = 1.25\,mA$, $L = 0.85\,\mu m$ and $W = 903.125\,\mu m$. Then $C_L = 2.26\,pF$

D) Using the approach from A, $L = 0.5\,\mu m \Rightarrow \lambda = 0.2$

$I_D = \frac{1+\lambda V_{DS}}{2\lambda R_o}$, so for $R_o = 15\,k\Omega$ $I_D = 216.67\,\mu A$

$V_{Dsat} = \frac{2I_D}{g_m}$, so for $g_m = 5\,mS$ $V_{Dsat} = 216.7\,mV$

$W = \frac{I_D}{I_D/W}$, so $W = 17.75\,\mu m$. Then $C_L = 44\,fF$

E) $L = 0.5\ \mu m \Rightarrow \lambda = 0.2$

for $R_o = 100$, $I_D = \frac{1+\lambda V_{DS}}{2\lambda R_o} = 32.5 mA$

for $g_m = 50 mS$ $V_{Dsat} = \frac{2 I_D}{g_m} = 1.3 V$

$W = \frac{I_D}{I_D / W} = 74\ \mu m$

$C_L = 185\ fF$

Caveat: Typically values of W and L in an IC process are limited to multiples of $\frac{1}{2} L_{min}$, or in this process 0.25 µm. I neglected this fact in solving this problem, but since we have plenty of margin in our bandwidth, we could certainly resize the devices slightly and still meet the spec.

Summary of results:

	g_m [S]	R_o [Ω]	C_L [F]	I_d [A]	W [µm]	L [µm]	V_{Dsat} [V]
a)	1m	50k	37f	65µ	14.79	0.5	130m
b)	5m	20k	452f	250µ	180.625	0.85	100m
c)	25m	4k	2.26p	1.25m	903.125	0.85	100m
d)	2m	15k	44f	216.67µ	17.75	0.5	216.7m
e)	50m	100	185f	32.5m	74	0.5	1.3

Note: Had I approached A, D, and E by fixing $V_{Dsat} = V_{Dsat,min} = 100 mV$, the value of L found to meet the spec would be less than L_{min}.

4)

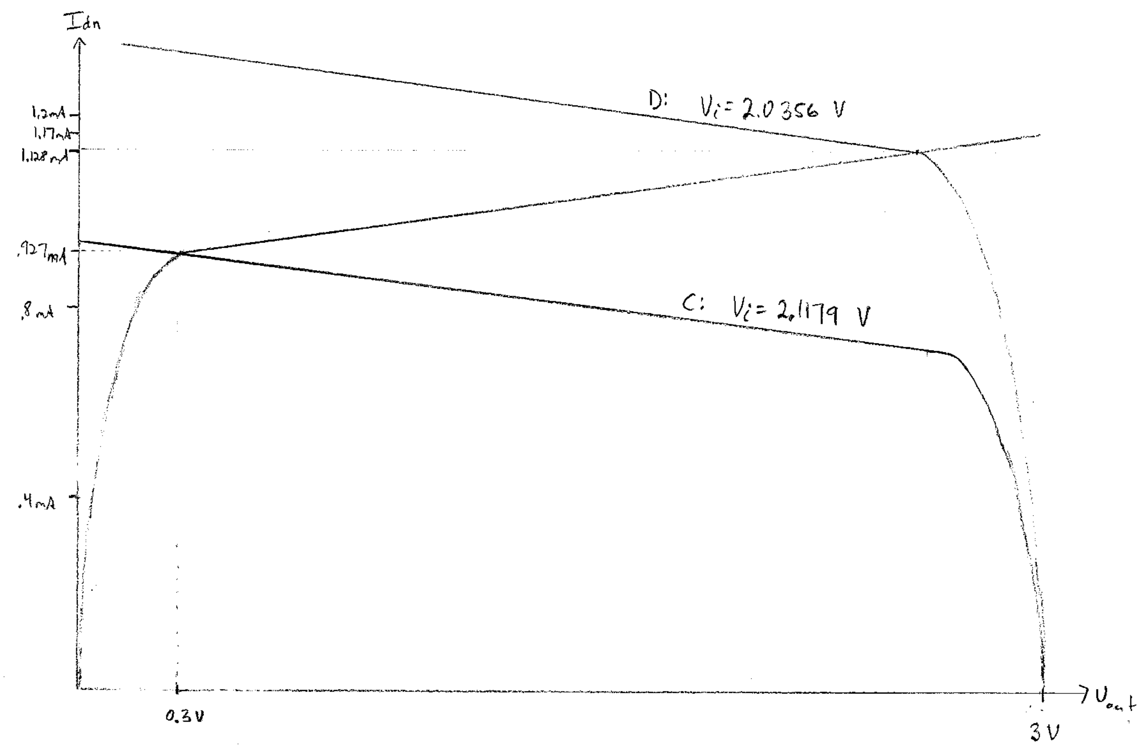
A) $V_{Dsat_n} = V_{GS} - V_T = V_{BN} - V_T = 0.8\ V - 0.5\ V = \boxed{0.3\ V}$

$I_{dn} = \frac{1}{2} k_n' \frac{W}{L} V_{Dsat_n}^2 (1 + \lambda V_{DS})$ where $V_{DS} = V_{out} = V_{Dsat_n}$

$\lambda = 0.1 \frac{\mu m}{L} = 0.1$ $W = 100\ \mu m$ $L = 1\mu m$ $k_n' = 200 \cdot 10^{-6} \frac{A}{V^2}$

$I_{dn} = \frac{1}{2} \cdot 200 \cdot 10^{-6} \cdot \frac{100}{1} \cdot 0.3^2 \cdot (1 + 0.1 \cdot 0.3) = \boxed{927\ \mu A}$

B)



Minimum I_{dn} for NMOS saturation: Found in part A to be $927\mu A$.

Maximum I_{dn} for NMOS saturation: Occurs at $V_{DS} = 3V$ (V_{DD})

Note: PMOS is out of saturation, but that isn't what the problem asked.

$I_{dn,max} = \frac{1}{2} \frac{W}{L} k_n' V_{Dsat}^2 (1 + \lambda (3)) = 9 \cdot 10^{-4} (1.3) = \boxed{1.17\ mA}$

C) $|I_{dp}| = I_{dn}$, so $|I_{dp}| = 927\ \mu A$ at this point

$|I_{dp}| = \frac{1}{2} \frac{W}{L} k_p' (|V_i - V_{DD}| - |V_{Tp}|)^2 (1 + |\lambda (V_{out} - V_{DD})|)$

$9.27 \times 10^{-4} = \frac{1}{2} \cdot \frac{100}{1} \cdot 100 \cdot 10^{-6} ((3 - V_i) - 0.5)^2 (1 + 0.1 (3 - 0.3))$

$\Rightarrow \boxed{V_i = 2.1179\ V}$

D) The PMOS leaves saturation when $|V_{out} - V_{DD}| = |V_i - V_{DD}| - |V_{Tp}|$

$$I_{Dp} = \frac{1}{2}\frac{W}{L}k_p'\,(V_{DD} - V_{out})^2\,(1 + \lambda\,(V_{DD} - V_{out}))$$

$$= I_{Dn} = \frac{1}{2}\frac{W}{L}k_n'\,(V_{BN} - V_{Tn})^2\,(1 + \lambda V_{out})$$

$$k_p'\,(V_{DD} - V_{out})^2\,(1 + \lambda\,(V_{DD} - V_{out})) = k_n'\,(V_{BN} - V_{Tn})^2\,(1 + \lambda V_{out})$$

$$(V_{DD} - V_{out})^2\,(1 + \lambda\,(V_{DD} - V_{out})) = 0.18\,(1 + \lambda V_{out})$$

$$(V_{DD} - V_{out})^2\,(1 + \lambda\,(V_{DD} - V_{out})) - 0.18\,(1 + \lambda V_{out}) = 0$$

$$(3 - V_{out})^2\,(1 + 0.1\,(3 - V_{out})) - 0.18\,(1 + 0.1 V_{out}) = 0$$

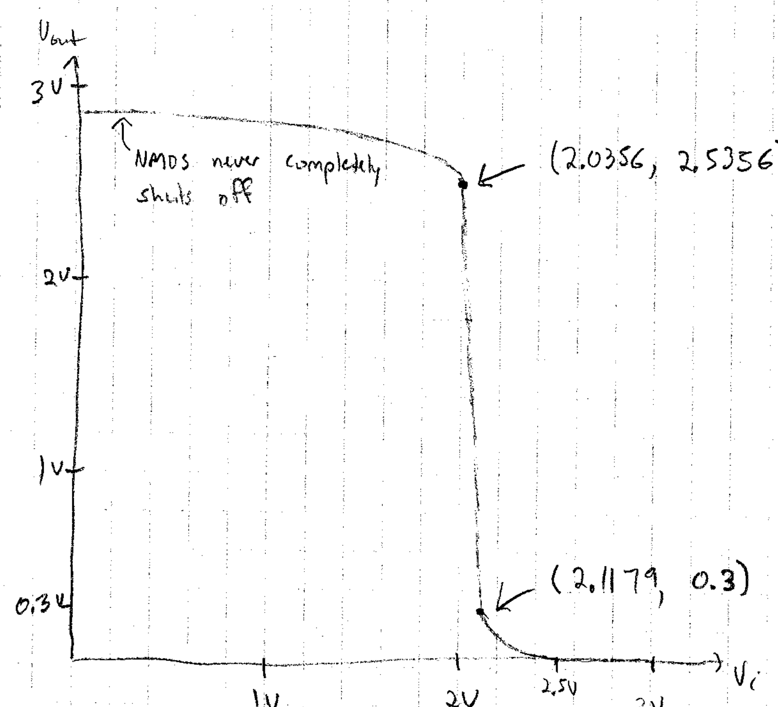
Solving in MATLAB, $\boxed{V_{out} = 2.5356\text{ V}}$

$$V_{DD} - V_{out} = V_{DD} - V_i - |V_{Tp}|$$

$$3 - 2.5356 = 3 - V_i - 0.5 \quad \Rightarrow \quad \boxed{V_i = 2.0356}$$

At this point, $I_D = \frac{1}{2}\frac{W}{L}k_n'\,(V_{BN} - V_{Tn})^2\,(1 + \lambda V_{out}) = \boxed{1.282\text{ mA}}$

E)



F) Assuming ~~approx~~ constant slope in high-gain region,

$$A_v = -\frac{\Delta V_o}{\Delta V_i} = -\frac{0.3 - 2.5356}{2.1179 - 2.0356} = -27.164$$

input range: 2.0356 V to 2.1179 V

output range: 0.3 V to 2.5356 V

G)

Left edge: (2.0356, 2.5356)

$$A_v = -g_{mp}(r_{op} \| r_{on})$$

$$= -\frac{2 \cdot 1.1282 \text{ mA}}{0.4644 \text{ V}} \cdot \left[\frac{(1 + 0.1 \cdot .4644)}{0.1 \cdot 1.1282 \times 10^{-3}} \,\Big\|\, \frac{(1 + 0.1 \cdot 2.5356)}{0.1 \cdot 1.1282 \times 10^{-3}}\right]$$

$$= -4.8587 \text{ mS} \cdot (9.275 \text{ k}\Omega \,\|\, 11.111 \text{ k}\Omega)$$

$$= -4.8587 \text{ mS} \cdot 5.055 \text{ k}\Omega$$

$$= -24.5116$$

Right edge: (2.1179, 0.3)

$$A_v = -g_{mp}(r_{op} \| r_{on}) = \frac{2 \cdot .927 \times 10^{-3}}{0.3821} \cdot \left[\frac{1 + 0.1 \cdot 2.7}{0.1 \cdot .927 \times 10^{-3}} \,\Big\|\, \frac{1 + 0.1 \cdot 0.3}{0.1 \cdot .927 \times 10^{-3}}\right]$$

$$= -4.8521 \text{ mS} \cdot (13.7 \text{ k}\Omega \,\|\, 11.111 \text{ k}\Omega)$$

$$= -29.7689$$

Middle: (2.0768, 1.4178) $\qquad I_D = \frac{1}{2}\frac{W}{L}k_n'(0.3)^2(1 + \lambda \cdot 1.4178) = 1.0278 \text{ mA}$

$$A_v = -g_{mp}(r_{op} \| r_{on}) = \frac{2 \cdot 1.0278 \times 10^{-3}}{0.4232} \cdot \left[\frac{1 + 0.1 \cdot 1.5822}{0.1 \cdot 1.0278 \times 10^{-3}} \,\Big\|\, \frac{1 + 0.1 \cdot 1.4178}{0.1 \cdot 1.0278 \times 10^{-3}}\right]$$

$$= -4.8573 \text{ mS} \cdot (11.269 \text{ k}\Omega \,\|\, 11.109 \text{ k}\Omega) = -4.8573 \text{ mS} \cdot 5.5942 \text{ k}\Omega$$

$$= -27.1727$$

Note: Actual R_o (and A_v) at edge points is less than predicted here because we are at the edge of saturation and triode. I used the saturation r_o equations above, but I just as well might have used the triode equations and gotten a much smaller R_o. The truth lies somewhere between the two.