Lecture 15: High Gain Op Amps

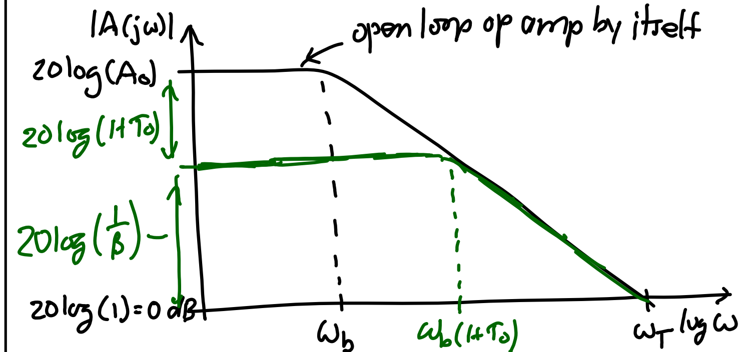
- **Announcements**:
- I am still on travel today
  - This is a video recorded lecture
  - Please watch the online lecture videos before next week
- Pre-Lecture materials online (for Vos)
- HW#6 online … now due this Friday, 10/16
- HW#1A online 240A folks
- Midterm is getting closer: Oct. 29, in the evening
- **Lecture Topics**:
  - Finish offset Voltage (bipolar)
  - Finite Gain BW
  - Effect of FB (a first pass)
  - High Gain Op Amps
- ------------------------------------
- **Last Time**:
- SCP input offset voltage
- Go through offset voltage handout (skim in lecture, then you should go through it more slowly later)

over

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$.

In reality, the gain is given by: $A(s) = \dfrac{A_o}{1 + s/\omega_b}$



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ $(= 0\,dB)$

At ω_T:

$$|A(j\omega_T)| = 1 = \frac{A_o}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}}$$

$$[\omega_T \gg \omega_b] \Rightarrow \frac{A_o}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = \underbrace{A_o\omega_b}_{\text{Gain-Bandwidth Product}}}$$

For $\omega \gg \omega_b$:

$$A(s) \approx \frac{A_o}{\frac{s}{\omega_b}} = \frac{A_o\omega_b}{s} = \frac{\omega_T}{s} = \frac{f_T}{f}$$

[Integrator w/ time Constant $\tau = \frac{1}{\omega_T}$]

The unity gain bandwidth f_T is usually specified on op amp data sheets. Knowing f_T, one can easily determine the op amp gain at a given frequency f.

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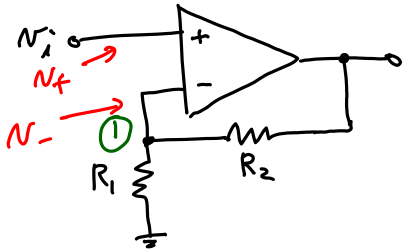
Frequency Response of Closed Loop Amplifiers

Example. Non-Inverting Amplifier



$v_o = A(s)(v_+ - v_-)$

$v_- = v_+ - \dfrac{v_o}{A(s)}$

$v_- = v_i - \dfrac{v_o}{A(s)}$

Find an expression for the gain as a function of frequency.

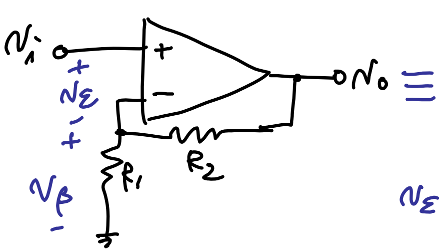
① Brute force derivation:

KCL①: $\dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} \rightarrow \dfrac{v_o}{R_2} = v_-\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow \frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)} = *$$

$$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}}\right] \Rightarrow \boxed{\frac{v_o}{v_i}(s) = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + \frac{s}{A_o\omega_b\left(\frac{R_1}{R_1+R_2}\right)}}}$$

② More insightful way to do this:



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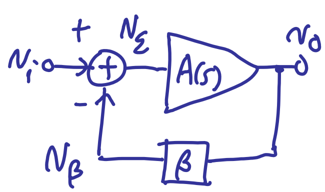
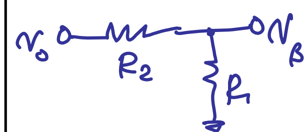


$v_\varepsilon = v_i - v_\beta \rightarrow \dfrac{v_o}{v_i} = \dfrac{A(s)}{1 + \beta A(s)}$

What is β?



$$\boxed{\beta = \frac{v_\beta}{v_o} = \frac{R_1}{R_1 + R_2}}$$

feedback factor

Recall f/ previous FB analysis:

$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1 + \beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}}\right] \rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{A_o}{1 + (s/\omega_b)}}{1 + \beta\left(\frac{A_o}{1 + (s/\omega_b)}\right)}$$

$$\frac{v_o}{v_i}(s) = \underbrace{\frac{A_o}{1 + \beta A_o}}_{\text{closed loop dc gain term}} \; \underbrace{\frac{1}{1 + \frac{s}{\omega_b(1 + \beta A_o)}}}_{\text{frequency shaping term}}$$

$T_o = \beta A_o \triangleq$ "loop gain" @ $\omega = 0$ (i.e., @ dc)

If $A_o \rightarrow \infty$,
or if $\beta A_o \gg 1$ $\Big\} \Rightarrow$ dc gain $= \dfrac{1}{\beta}$

Plug in β: [for $\beta A_o \gg 1$]

$$\frac{v_o}{v_i}(s) \cong \frac{1}{\beta}\,\frac{1}{1 + \frac{s}{\omega_b \beta A_o}} = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + \frac{s}{\omega_b A_o\left(\frac{R_1}{R_1+R_2}\right)}} \; *$$

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Observations:

① Closed loop DC gain $= \dfrac{A_o}{1+\beta A_o} = \dfrac{A_o}{1+T_o} \approx \dfrac{A_o}{T_o}$ $[T_o \gg 1]$

i.e., the closed loop gain is reduced from the open loop gain by $1+T_o$ → show this on graph

② Alternatively, closed loop DC gain $\approx \dfrac{A_o}{\beta A_o} = \dfrac{1}{\beta}$ $[T_o \gg 1]$

③ ω_{-3dB} has increased from $\omega_b \rightarrow \omega_b(1+A_o\beta) = \omega_b(1+T_o)$

To draw the Bode plot, just find the dc gain, draw a horizontal line across, then follow the open loop response after running into it!

④ Gain-BW Product $= \dfrac{A_o}{1+\beta A_o}\,\omega_b(1+\beta A_o) = A_o\omega_b = \omega_T$

∴ the Gain-BW product remains the same for the open & closed loop FB cases!

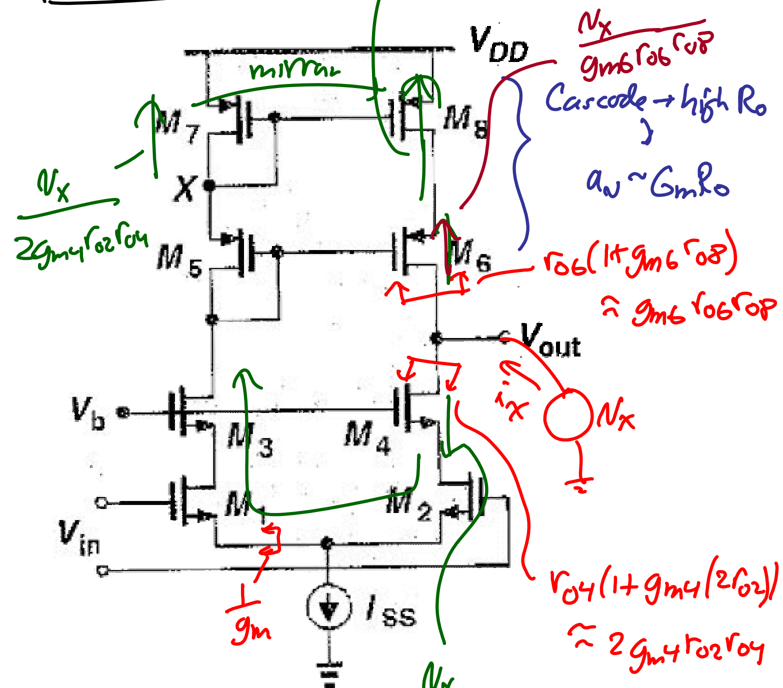
High Gain Op Amps

How can we increase gain?

① Cascode

② Cascode Amplifier

Telescopic Op Amp w/ Single-Ended Output

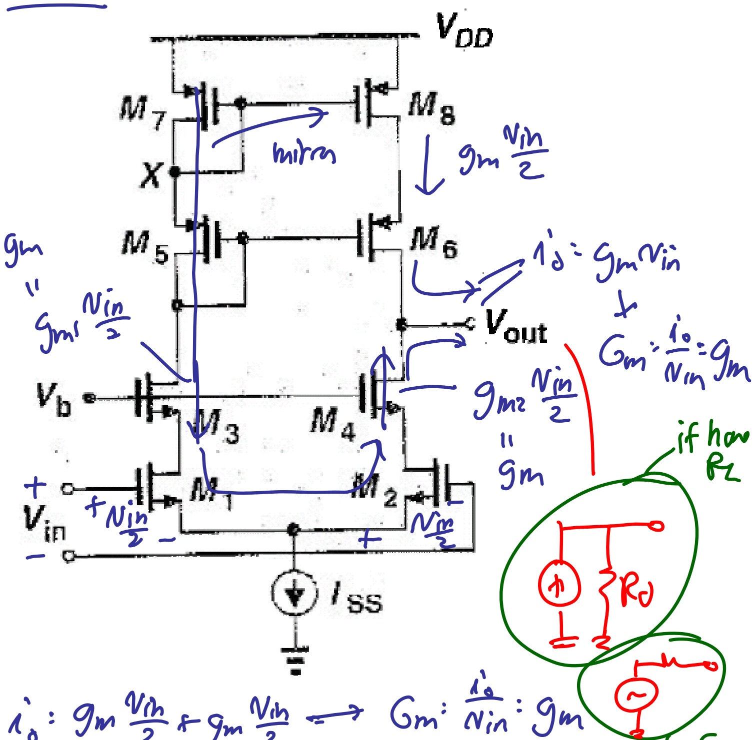


Get R_o:

$$i_x = \frac{v_x}{g_{m6}r_{o6}r_{op}} + \frac{v_x}{2g_{m4}r_{o2}r_{o4}} + \frac{v_x}{2g_{m4}r_{o2}r_{o4}}$$

$$R_o = \frac{v_x}{i_x} = (g_{m6} r_{o6} r_{o8}) \| (g_{m4} r_{o2} r_{o4})$$
$$= (g_{mp} r_{op}^2) \| (g_{mN} r_{oN}^2)$$

Get G_m:

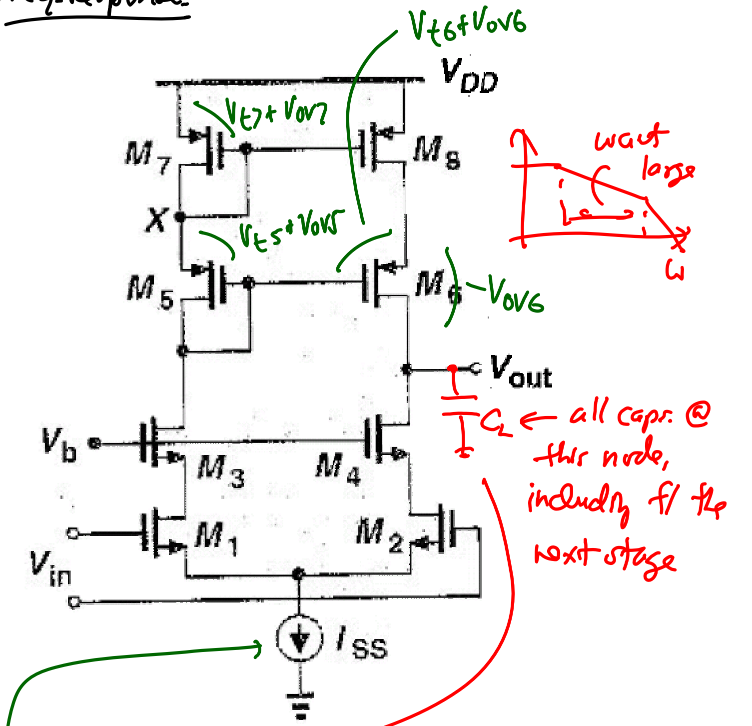


$$i_o = g_m \frac{v_{in}}{2} + g_m \frac{v_{in}}{2} \longrightarrow G_m = \frac{i_o}{v_{in}} = g_m$$

$$\therefore \text{Gain} = A_v = g_{mN} \left[\underbrace{(g_{mN} r_{oN}^2)}_{\text{huge } R!} \| \underbrace{(g_{mp} r_{op}^2)}_{\text{huge } R!} \right]$$

Gain will be BIG!

Freq. Response:



This contributes the dominant pole since R_o = huge:

$$\omega_H = \frac{1}{R_o C_L}$$

Problems/Issues:

① Limited output swing:

Problem: Not so large!

$$V_{omax} = V_{DD} - |V_{t7}| - |V_{ov7}| - |V_{t5}| - |V_{ov5}| + |V_{t6}| + \cancel{|V_{ov6}|} - \cancel{|V_{ov6}|}$$

$$V_{omin} = V_{ovSS} + V_{ov2} + V_{ov4}$$

Current source V_{ov}

$$V_{oswing} = V_{omax} - V_{omin}$$

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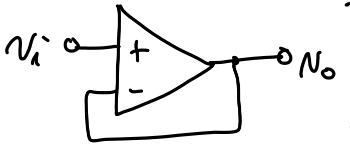
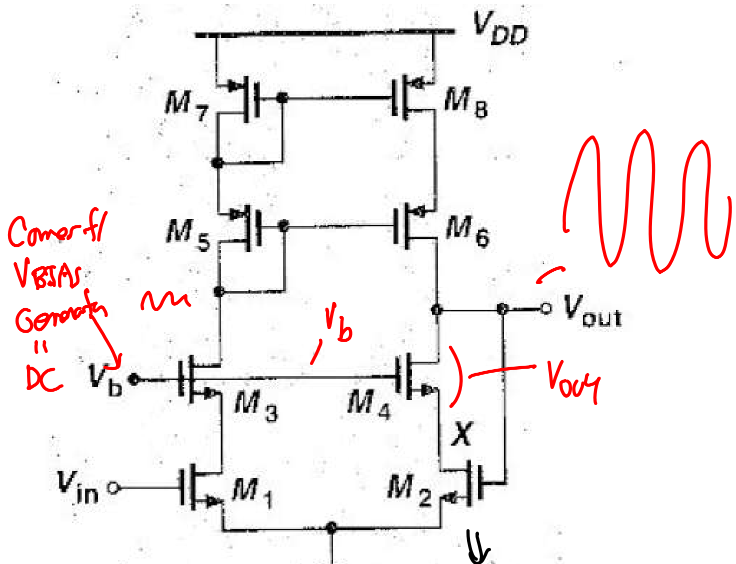
Problem ②: Difficult to tie the input to output!



unity gain buffer (very useful)



Must keep M_2 & M_4 saturated!

M_4: