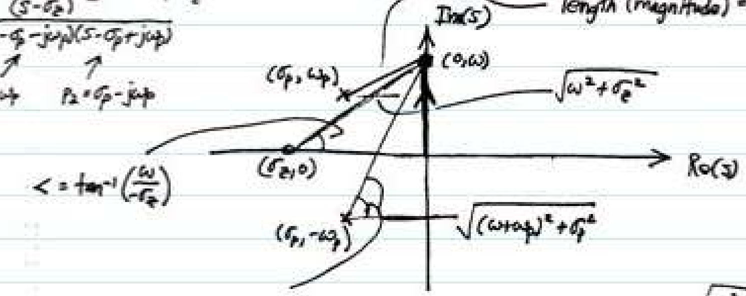
EE 140 | **Review of Pole/Zero Plots** | CTN | 94

The frequency response of a given system is completely characterized by knowledge of the poles, zeros, and dc gain factor (H_o) of the system in question. In fact, the magnitude & phase of the network can be determined graphically from a pole/zero diagram.

$$H(s) = \frac{(s-\sigma_z)}{(s-\sigma_p-j\omega_p)(s-\sigma_p+j\omega_p)} \qquad z_1 = \sigma_z$$

$p_1 = \sigma_p + j\omega_p \qquad p_2 = \sigma_p - j\omega_p$



length (magnitude) $= \sqrt{(\omega-\omega_p)^2+\sigma_p^2}$

$\sqrt{\omega^2+\sigma_z^2}$

$\sqrt{(\omega+\omega_p)^2+\sigma_p^2}$

$\angle = \tan^{-1}\left(\frac{\omega}{-\sigma_z}\right)$

$\angle = \tan^{-1}\left(\frac{\omega+\omega_p}{-\sigma_p}\right) = \tan^{-1}\left(\frac{\omega+|\omega_p|}{|\sigma_p|}\right)$

$$\therefore |H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_z^2}}{\sqrt{(\omega-\omega_p)^2+\sigma_p^2}\sqrt{(\omega+\omega_p)^2+\sigma_p^2}}$$

$$\therefore \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{|\sigma_z|}\right) - \tan^{-1}\left(\frac{\omega-|\omega_p|}{|\sigma_p|}\right) - \tan^{-1}\left(\frac{\omega+|\omega_p|}{|\sigma_p|}\right)$$

In general:

$$|H(j\omega)| = H_o \frac{\Pi\ (\text{mag. of vectors from zeros to } j\omega)}{\Pi\ (\text{mag. of vectors from poles to } j\omega)} = H_o \frac{\prod_{j=1}^{m} |j(\omega-\omega_{zj})-\sigma_{zj}|}{\prod_{i=1}^{n} |j(\omega-\omega_{pi})-\sigma_{pi}|}$$

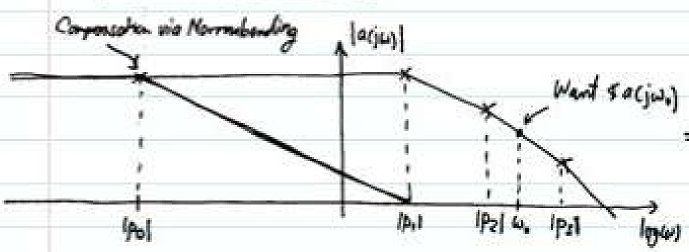
$z_j = \sigma_{zj} + j\omega_{zj}$

$p_i = \sigma_{pi} + j\omega_{pi}$

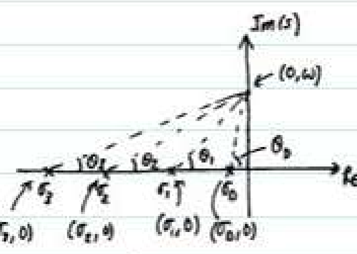
$\angle H(j\omega) = \Sigma$ angles from zeros $-\ \Sigma$ angles from poles $+ \angle H_o$

$\angle H_o$ could be $0°$ (−) or $180°$ (+)

Precise Determination of Phase



$\Longrightarrow$



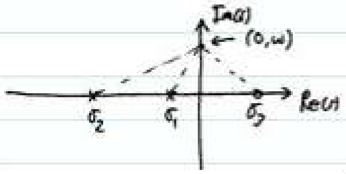
$\sigma_0 = (-),\ \sigma_1 = (-),\ \sigma_2 (-),\ \sigma_3 = (-)$

$$\measuredangle a(j\omega) = -\theta_0 - \theta_1 - \theta_2 - \theta_3$$
$$= -\tan^{-1}\left(\frac{\omega}{-\sigma_0}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) - \underbrace{\tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)}_{\tan^{-1}\left(\frac{\omega-0}{0-\sigma_3}\right)}$$

$$\therefore \measuredangle a(j\omega_0) = \underbrace{-90° - 90°}_{\text{since } \omega_0 \gg |p_0| \,\&\, |p_1|} - \tan^{-1}\left(\frac{\omega_0}{|p_2|}\right) - \tan^{-1}\left(\frac{\omega_0}{|p_3|}\right)$$

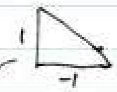
EE 140 | **Review of Pole/Zero Plots** | CTN | 95

Ex: System w/ 2 poles & 1 RHP zero



$\sigma_1 = (-)$ $\quad \sigma_2 = (-)$ $\quad$ since it's a zero! $\quad \sigma_3 = (+)$

$$\boxed{\begin{aligned}\measuredangle H(j\omega) &= -\tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) + \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)\\ &= -\tan^{-1}\left(\frac{\omega}{|\sigma_1|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_2|}\right) + \tan^{-1}\left(-\frac{\omega}{|\sigma_3|}\right)\end{aligned}}$$



$$\measuredangle H(j\omega) = -90° - 90° + (-90°) \Rightarrow \measuredangle H(j\omega) \underset{\omega\to\infty}{=} -270°$$

$$\boxed{|H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_3^2}}{\sqrt{\omega^2+\sigma_1^2}\sqrt{\omega^2+\sigma_2^2}}}$$

Note: The RHP zero contributes a (-) phase shift!

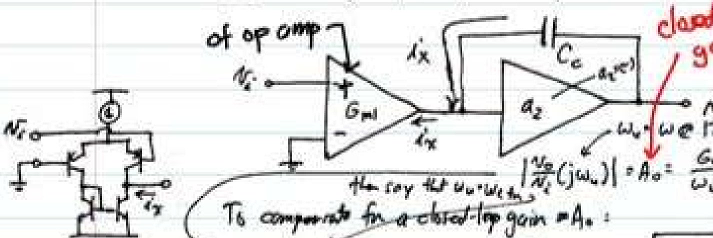
EE 140 | **Choosing C_c** | CTN | 96

Choosing C_c (assuming no RHP zeros & $p_3 \gg p_2$)

→ for cases where $\underbrace{\frac{1}{sC_c} \ll \text{(surrounding node resistance)}}_{\text{very often the case}}$, can write simple expressions for C_c in terms of ω_2

For PM=45° → A_o

① Case: Two-Stage Amplifier, Miller Compensation



$v_o = \frac{i_x}{sC_c}$, $i_x = G_{m1} v_i$ } $v_o = \frac{G_{m1} v_i}{sC_c}$

$\therefore \frac{v_o}{v_i}(s) = \frac{G_{m1}}{sC_c}$

$\omega_u = \omega @ |T(j\omega)| = 1$

$\left|\frac{v_o}{v_i}(j\omega_u)\right| = A_o = \frac{G_{m1}}{\omega_u C_c}$ → $\boxed{C_c = \frac{G_{m1}}{\omega_u A_o}}$

Then say that $\omega_u = \omega_{2}$

To compensate for a closed-loop gain $= A_o$:

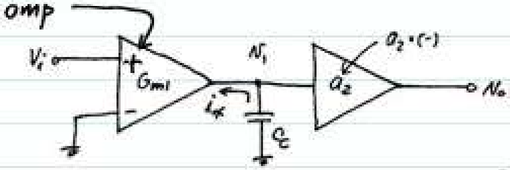
For PM=45°: $\left|\frac{v_o}{v_i}(j\omega_2)\right| = A_o = \frac{G_{m1}}{\omega_2 C_c}$ → $\boxed{C_c = \frac{G_{m1}}{\omega_2 A_o}}$ ← For 45° = PM (provided higher-order poles are far away) ($p_3 \gg p_2$)

2nd pole → for 45° PM

$\omega_u = \omega @ |T(j\omega)| = 1 = \frac{\omega_2}{1.73}$

For PM=60°: $\left|\frac{v_o}{v_i}\left(j\frac{\omega_2}{1.73}\right)\right| = A_o = \frac{G_{m1}}{\frac{\omega_2}{1.73} C_c}$ ⇒ $\boxed{C_c = \frac{1.73 G_{m1}}{\omega_2 A_o}}$ ← For PM=60°.

② Case: Two-Stage Amplifier, Shunt C_c Compensation



$v_o = a_2 v_1$, $v_1 = \frac{G_{m1} v_i}{sC_c}$ } $v_o = -\frac{G_{m1} a_2}{sC_c} v_i$

$\therefore \frac{v_o}{v_i}(s) = \underbrace{-\frac{G_{m1} a_2}{sC_c}}_{\text{ends up (+), since } a_2 = (-)}$

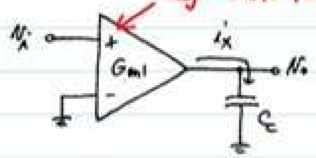
To compensate for a closed-loop gain $= A_o$: (PM=45°; $\overbrace{p_3 \text{ and above far away}}^{p_3 \gg p_2}$)

For PM=45°: $\left|\frac{v_o}{v_i}(j\omega_u)\right| = A_o = \frac{G_{m1} a_2}{\omega_u C_c}$ → $\boxed{C_c = \frac{G_{m1} a_2}{\omega_2 A_o}}$ ← PM=45° ⟸ $a_2 \times$ bigger than (consistent w/ previous analysis)

$\boxed{C_c = \frac{G_{m1} a_2}{\omega_u A_o}}$ = $\boxed{C_c = \frac{1.73 G_{m1} a_2}{\omega_2 A_o}}$ ← PM=60°

③ Case: Single-Stage Amplifier, Shunt C_c Compensation ⟹ we will see this case often in CMOS

again, (+) terminal of op amp!



$v_o = \frac{i_x}{sC_c}$, $i_x = G_{m1} v_i$ } $\frac{v_o}{v_i}(s) = \frac{G_{m1}}{sC_c}$

To compensate for a closed-loop gain $= A_o$: (PM=45° w/ $p_3 \gg p_2$)

For PM=45°: $\left|\frac{v_o}{v_i}(j\omega_u)\right| = A_o = \frac{G_{m1}}{\omega_u C_c}$ ⟹ $\boxed{C_c = \frac{G_{m1}}{\omega_2 A_o}}$ ← For PM=45°

$\boxed{C_c = \frac{G_{m1}}{\omega_u A_o}}$ = $\boxed{C_c = \frac{1.73 G_{m1}}{\omega_2 A_o}}$ ← For PM=60°

