

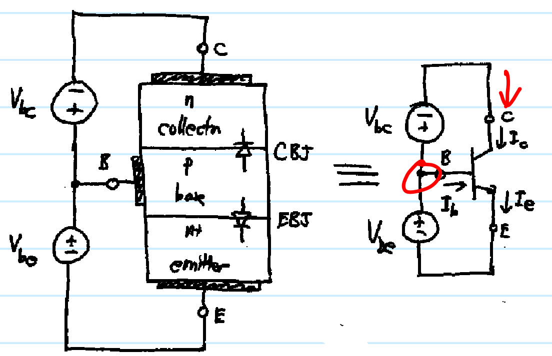
EE 140 | BJT Modeling

CTN 1

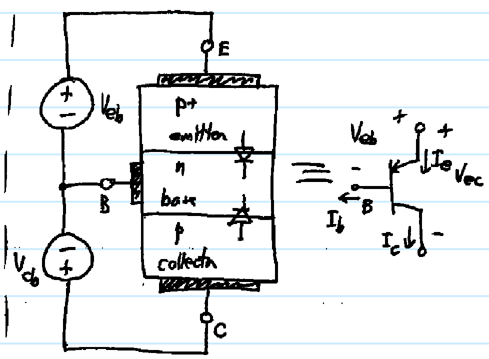
Modeling the Bipolar Junction Transistor (BJT)

⇒ physically, BJT's are just back-to-back pn junctions

npn bipolar Xsistor



pnp bipolar Xsistor

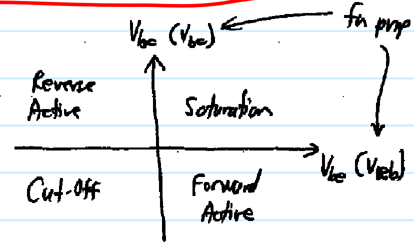


Regions of Bipolar Xsistor Operation

→ Key: R ≜ reverse-biased
F ≜ forward-biased

EBJ	CBJ	Mode
R	R	Cut-off (both diodes off)
F	R	Forward Active (widely used in analog amplifier ckts)
R	F	Reverse Active
F	F	Saturation

⇒ can also think of this in a convenient graphical sense:
→ for npn (pnp):



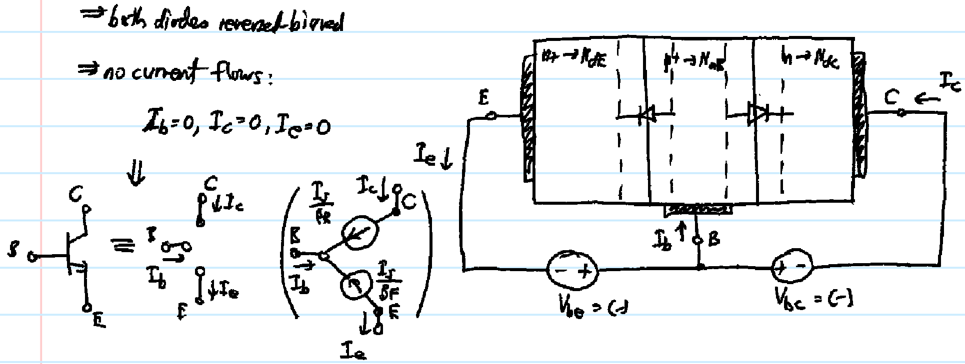
① Cut-Off Region - (npn transistor)

⇒ both diodes reverse-biased

⇒ no current flows:

$I_b = 0, I_c = 0, I_e = 0$

⇓



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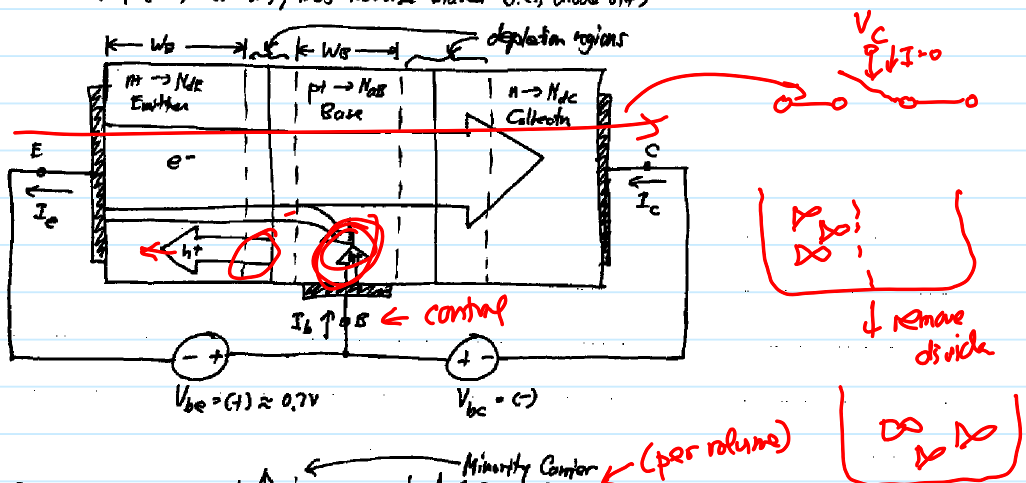
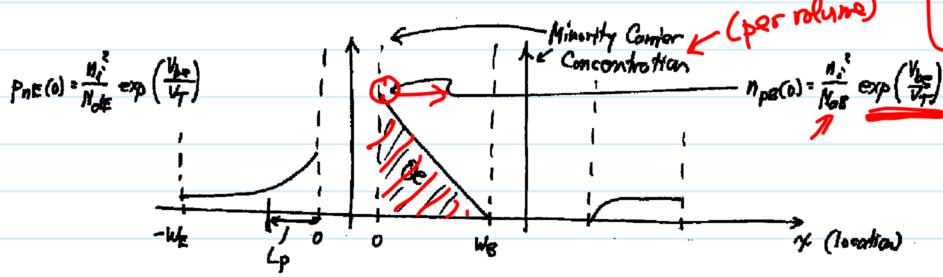
② Forward-Active Region – (npn transistor)

⇒ BEJ Forward-Biased (i.e., diode on), BCJ Reverse-Biased (i.e., diode off)





Forward biasing of the BEJ generates three current components:

① e⁻'s injected from emitter to base: $I_{nB} = -A J_{nB}^{diff}$

② h⁺'s injected from base to emitter: $I_{pE} = A J_{pE}^{diff}$

③ recombination of e⁻'s & h⁺'s in base: I_{rB}

$I_c = I_{nB} = ①$

$I_e = I_{nB} + I_{pE} + I_{rB} = ① + ② + ③$

$I_b = I_{pE} + I_{rB} = ② + ③$

$$I_{nB} = -A J_{nB}^{diff} = -A q D_{nB} \underbrace{\frac{dn_{pB}(x)}{dx}}_{\text{slope}} = -qAD_{nB} \frac{[n_{pB}(W_B) - n_{pB}(0)]}{W_B} = \boxed{qAD_{nB} \frac{n_i^2}{N_{aB} W_B} \exp\left(\frac{V_{be}}{V_T}\right) = ①} *$$

D_{nB}: diffusion constant for e⁻'s in B

$$\left[\begin{array}{l} n_{pB}(W_B) = \frac{n_i^2}{N_{aB}} \exp\left(\frac{V_{bc}}{V_T}\right) \approx 0 \\ n_{pB}(0) = \frac{n_i^2}{N_{aB}} \exp\left(\frac{V_{be}}{V_T}\right) \end{array}\right]$$

$$I_c = I_s \exp\left(\frac{V_{be}}{V_T}\right)$$

D_{pE}: diffusion constant for h⁺'s in E

$$I_{pE} = A J_{pE}^{diff} = A q D_{pE} \underbrace{\frac{dp_{nE}(x)}{dx}}_{\text{slope}} = qAD_{pE} \frac{[p_{nE}(0) - p_{nE}(-W_E)]}{W_E} = \boxed{qAD_{pE} \frac{n_i^2}{N_{dE} W_E} \exp\left(\frac{V_{be}}{V_T}\right) = ②} *$$

$$\left[\begin{array}{l} p_{nE}(0) = \frac{n_i^2}{N_{dE}} \exp\left(\frac{V_{be}}{V_T}\right) \\ p_{nE}(-W_E) \approx 0 \end{array}\right]$$

could also replace by diffusion length, L_p (for h⁺ in n-type material)

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$$I_{rB} = \frac{Q_e}{\tau_b} = \frac{1}{\tau_b}\left[\frac{1}{2} n_{pB}(0) W_B qA\right] = \boxed{\frac{1}{2}\frac{n_i^2 W_B q A}{N_{aB}\tau_b}\exp\left(\frac{V_{BE}}{V_T}\right) = ③} \quad *$$

minority-carrier charge in base

minority carrier lifetime in base

Define Forward Current Gain $= \beta_F$:

$$\beta_F = \frac{I_C}{I_b} = \frac{①}{③+②} = \frac{\frac{qAD_{nB}n_i^2}{N_{aB}W_B}}{\frac{1}{2}\frac{n_i^2 W_B qA}{N_{aB}\tau_b} + \frac{qAD_{pE}n_i^2}{N_{dE}W_E}} = \left[\frac{W_B^2}{2\tau_b D_{nB}} + \frac{D_{pE}W_B N_A}{D_{nB}W_E N_D}\right]^{-1}$$

$W_E \to L_p$; $N_A \to N_{aB}$; $N_D \to N_{dE}$; β

To maximize β_F, want: ① W_B = small

② $N_{dE} \gg N_{aB}$ (this is why emitter is n+) → also leads to $D_{pE} \ll D_{nB}$ which we also want

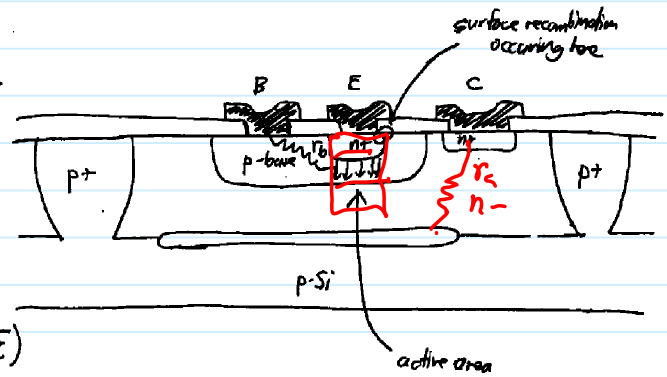
③ τ_b = large (base Si must be free of impurities/defects to prevent recombination)

More Complete Expression for β_F:

$$\beta_F = \underbrace{\frac{N_{aB}W_B}{D_{nB}}\cdot\frac{D_{pE}}{N_{dE}L_{pE}}}_{\text{Injection Efficiency}} + \underbrace{\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}_{\text{Volume Recombination}} + \underbrace{s\left(\frac{A_s}{A_E}\right)\left(\frac{W_B}{D_{nB}}\right)}_{\text{Surface Recombination}} + \underbrace{\frac{W_E N_{aB} W_B}{2D_{nB}\tau n_i}e^{-\frac{V_{be}}{2V_T}}}_{\text{Recombination in the BE Depletion Region}}$$

← significant @ low current levels

where:
- s = Surface Recombination Velocity
- D_i = Diffusion constant
- n_i = intrinsic carrier concentration
- N_i = carrier concentration
- A_E = total emitter area
- A_s = sidewall emitter area
- τ = minority carrier lifetime
- L_i = diffusion length ($L_i = \sqrt{D_i \tau}$)
- W_B = active base width



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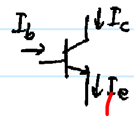
So β relates I_b & I_c. To relate I_c & I_e, use KCL:



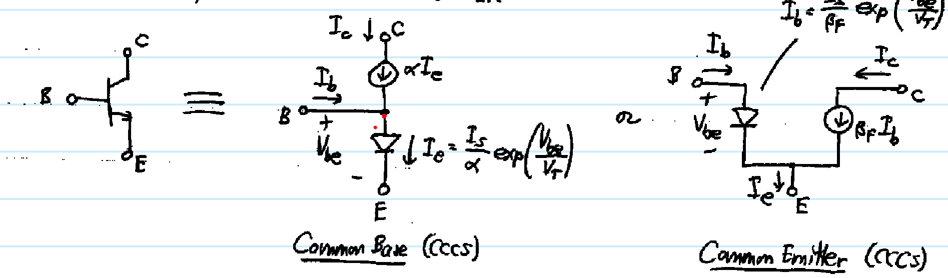
$$I_e = I_c + I_b = I_c + \frac{I_c}{\beta} = \left(1+\frac{1}{\beta}\right) I_c$$

$$\Rightarrow I_c = \left(\frac{1}{1+\frac{1}{\beta}}\right) I_e = \left(\frac{\beta}{\beta+1}\right) I_e = \alpha I_e, \text{ where } \boxed{\alpha = \frac{\beta}{\beta+1}} \Rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha}}$$

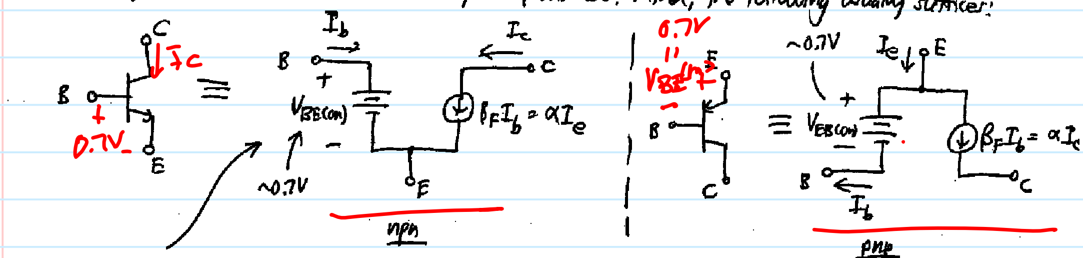
$\frac{100}{101} = 0.99$

Equivalent Large Signal Ckt. Models for Forward-Active BJTs

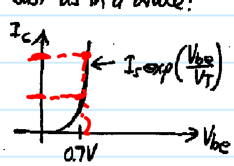
There are several of them. The most useful ones are:



But usually one doesn't have to use those complicated models. Rather, the following usually suffices:



Just as in a diode:



You should already be used to using approximate models like this
$\Rightarrow$ the more complicated models are a waste of time in comparison

③ Reverse-Active Region –

$\Rightarrow$ very similar to forward-active region except now: BEJ reverse-biased
BCJ forward-biased

$\Rightarrow$ one important difference: $\beta_R \propto \frac{N_{dC} W_C D_{nB}}{N_{aB} W_B D_{pE}}$ (L_{pC}) $\rightarrow$ $N_{dC} \ll N_{aB} \rightarrow D_{nB} \ll D_{pC}$ (since collector is n−)

$\therefore \beta_R$ is much smaller than β_F

$\Rightarrow$ poor device performance

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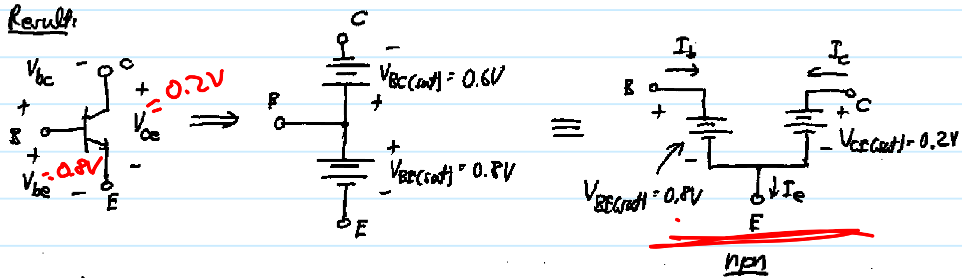
④ Saturation Region:-

BEJ forward-biased → $V_{BE(on)}^{sat} \sim 0.8V$ (higher than 0.7V in saturation)

BCJ forward-biased → $V_{BC(on)}^{sat} \sim 0.6V$

Result:



⇒ currents are determined by the attached elements & KCL:

$I_e = I_b + I_c$; no longer have $I_b = \frac{I_c}{\beta}$ or $I_c = \alpha I_e$

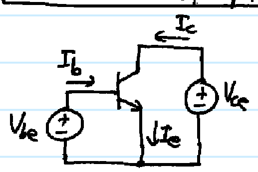
These no longer apply when BJT is in saturation.

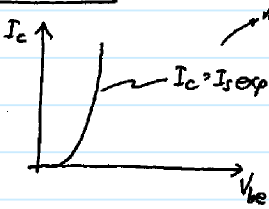
When determining DC operating point:

Pass { ① Assume forward-active → check for cut-off (enough V_{be}?)
② Determine V_{ce}.

③ If $V_{ce} > V_{CE(sat)} = 0.2V$, then ok (i.e., it's forward-active) ... otherwise, must do the analysis over assuming saturation.

IV Characteristics of Bipolar Junction Transistors





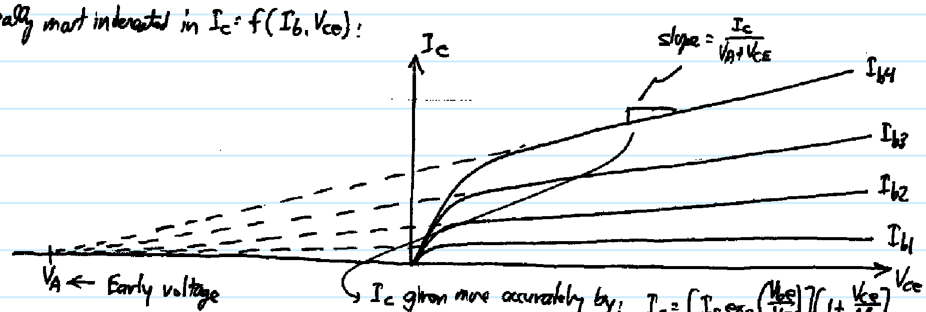
$I_c = I_s \exp\left(\frac{V_{be}}{V_T}\right)$ (a diode-like characteristic)

nonlinear → not easy to analyze since we can't use linear system theory → thus need to linearize!

digital ckts → piecewise lin.

analog ckts → small-signal models

→ really most interested in $I_c = f(I_b, V_{ce})$:



I_c given more accurately by: $I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left(1 + \frac{V_{ce}}{V_A}\right)$

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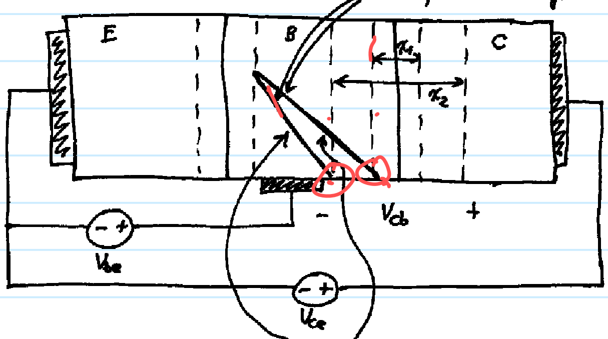
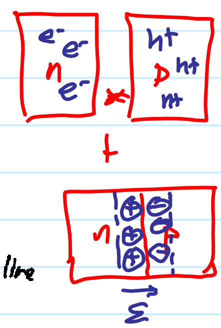
What is happening physically?



$x_1 \triangleq$ depl. region width for $V_{ce} = V_{ce1}$

$x_2 \triangleq$ depl. region width for $V_{ce} = V_{ce2} > V_{ce1}$



① Case: $V_{ce} = V_{ce1} \rightarrow x_1 \rightarrow I_{c1} \propto$ slope of this line

② Now, increase $V_{ce1} \rightarrow V_{ce2} \rightarrow V_{cb}\uparrow \rightarrow x\uparrow$ to $x_2 \rightarrow I_{c2} \propto$ slope of this line

$\therefore I_{c2} > I_{c1}$

Thus, $V_{ce}\uparrow \rightarrow I_c\uparrow$ due to $x_{depl.}\uparrow$

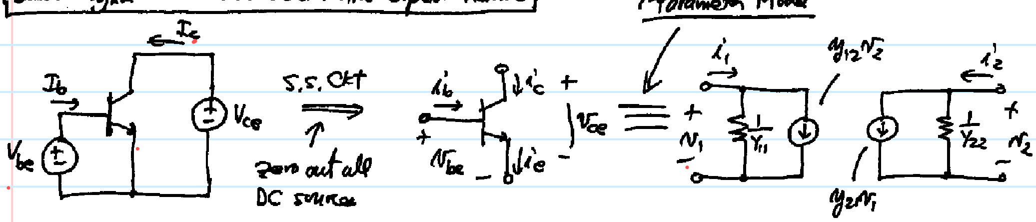
Result: $I_c = f(I_b, V_{ce})$ in forward-active!

$$I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1 + \frac{V_{ce}}{V_A}\right]$$

This, then, is a more accurate I_c equation.

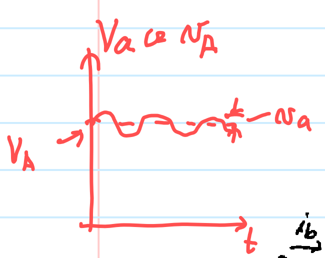
Small-Signal Models for Forward-Active Bipolar Xsistors

Y-Parameter Model



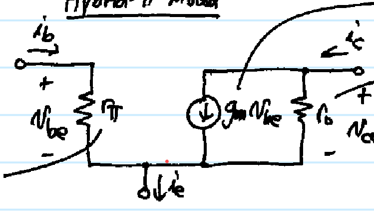
$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$ $\quad y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$

$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$ $\quad y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$



If only interested in the forward direction

Hybrid-π Model



$g_m = \frac{i_c}{v_{be}}$

$r_o = \frac{v_{ce}}{i_c}$

$r_\pi = \frac{v_{be}}{i_b}$

Specified by the bias point.

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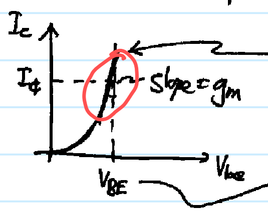
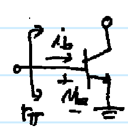
Determine the S.S. elements:

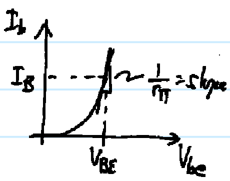
$$g_m = \frac{i_c}{v_{be}} = \left.\frac{\partial I_C}{\partial V_{be}}\right|_{Q\text{-pt.}} = \frac{\partial}{\partial V_{be}}\Big[\underbrace{I_s \exp\left(\frac{V_{be}}{V_T}\right)}_{= I_C}\Big]\Big|_{V_{be}=V_{BE}} = \frac{I_s}{V_T}\underbrace{\exp\left(\frac{V_{BE}}{V_T}\right)}_{I_C} \Rightarrow \boxed{g_m = \frac{I_C}{V_T}}$$

Note: function of the DC operating pt.



$r_\pi = \frac{v_{be}}{i_b}$:





$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{g_m} = \frac{\beta}{\frac{I_C}{V_T}} = \frac{V_T}{I_B}$$

$$\therefore \boxed{r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

Again, function of the DC operating pt.

$r_o = \frac{v_{ce}}{i_c}$:

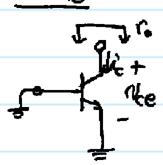
$$r_o = \left.\frac{\partial V_{ce}}{\partial I_c}\right|_Q = \left[\left.\frac{\partial I_c}{\partial V_{ce}}\right|_{Q\text{-pt.}}\right]^{-1}$$

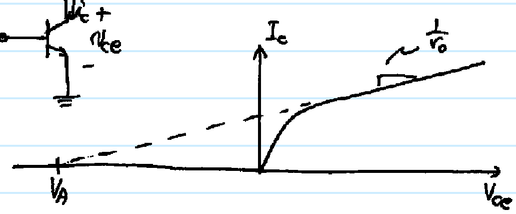
$$= \left[\frac{\partial}{\partial V_{ce}}\left(\left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1+\frac{V_{ce}}{V_A}\right]\right)\Big|_{V_{be}=V_{BE}}\right]^{-1}$$

$$= \left[\frac{I_s \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A}\right]^{-1} = \left[\frac{I_C}{V_A+V_{CE}}\right]^{-1} = \frac{V_A+V_{CE}}{I_C}$$

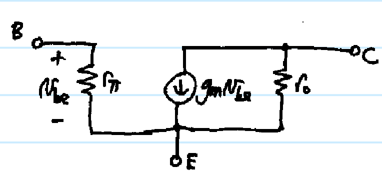
$$\frac{I_C}{1+\frac{V_{CE}}{V_A}}$$

$$\therefore \boxed{r_o = \frac{V_A+V_{CE}}{I_C} \approx \frac{V_A}{I_C}} \quad [V_A \gg V_{CE}]$$





... and thus, we have the hybrid-π model:



SPICE: BF

$$\boxed{\begin{aligned} r_\pi &= \frac{\beta}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{V_A+V_{CE}}{I_C} \approx \frac{I_C}{V_A} \end{aligned}}$$

SPICE: VAF

Remarks:

① g_m is independent of device specifics (i.e., β, I_s); depends only on temperature (thru V_T) and biasing I_C ← quite different from MOS, as we'll see

② small-signal model valid for $v_{be} \ll V_T$ ← ≈ 26mV @ 300K