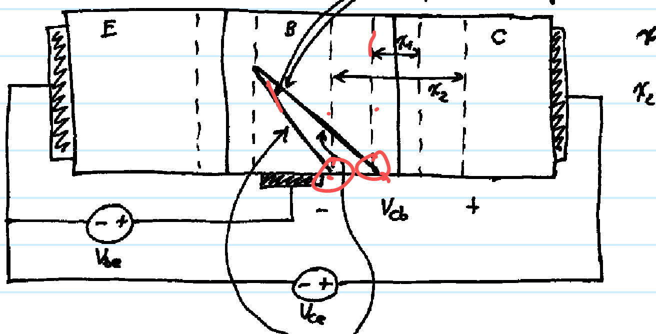
EE 140 | BJT Early Effect | CTN | 6

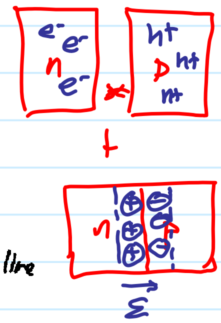
What is happening physically?

minority carrier charge densities

$x_1 \triangleq$ depl. region width for $V_{ce} = V_{ce1}$

$x_2 \triangleq$ depl. region width for $V_{ce} = V_{ce2} > V_{ce1}$





① Case: $V_{ce} = V_{ce1} \rightarrow x_1 \rightarrow I_{c1} \propto$ slope of this ~~curve~~ line

② Now, increase $V_{ce1} \rightarrow V_{ce2} \rightarrow V_{cb}\uparrow \rightarrow x\uparrow$ to $x_2 \rightarrow I_{c2} \propto$ slope of this line

$\therefore I_{c2} > I_{c1}$

Thus, $V_{ce}\uparrow \rightarrow I_c\uparrow$ due to $x_{depl.}\uparrow$

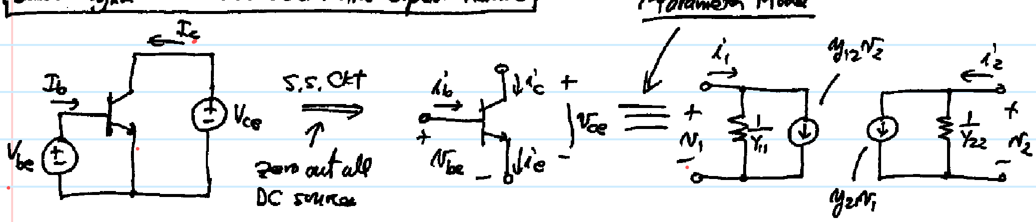
Result: $I_c = f(I_b, V_{ce})$ in forward-active!

$$I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1 + \frac{V_{ce}}{V_A}\right]$$

This, then, is a more accurate I_c equation.

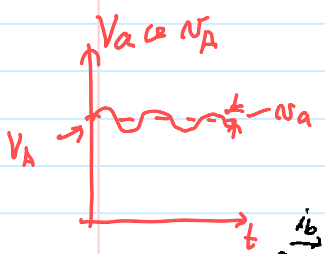
Small-Signal Models for Forward-Active Bipolar Xsistors

Y-Parameter Model



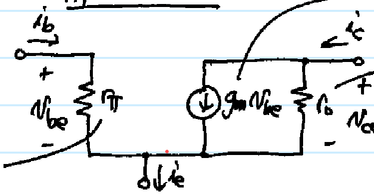
$$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0} \qquad y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$$

$$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0} \qquad y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$$



If only interested in the forward direction

Hybrid-π Model



$g_m = \frac{i_c}{v_{be}}$

$r_o = \frac{v_{ce}}{i_c}$

$r_\pi = \frac{v_{be}}{i_b}$

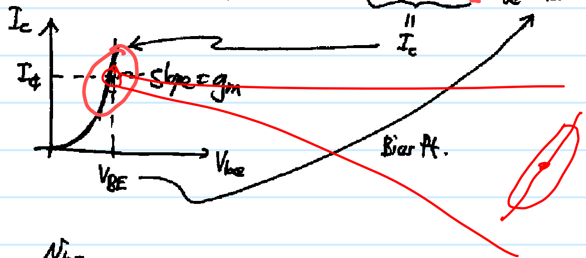
Specified by the bias point.

EE 140 | BJT Small-Signal Model | CTN | 7

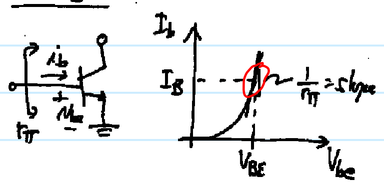
Determine the S.S. elements:

$$g_m = \frac{i_c}{v_{be}} = \frac{\partial I_C}{\partial V_{be}}\Big|_{Q\text{-pt.}} = \frac{\partial}{\partial V_{be}}\Big[\underbrace{I_S \exp\left(\frac{V_{be}}{V_T}\right)}_{= I_C}\Big]\Big|_{V_{be}=V_{BE}} = \frac{I_S}{V_T}\overbrace{\exp\left(\frac{V_{BE}}{V_T}\right)}^{I_C} \Rightarrow \boxed{g_m = \frac{I_C}{V_T}}$$

Note: function of the DC operating pt.



$r_\pi = \frac{v_{be}}{i_b}$:



$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{g_m} = \frac{\beta}{\frac{I_C}{V_T}} = \frac{V_T}{I_B}$$

$$\therefore \boxed{r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

Again, function of the DC operating pt.

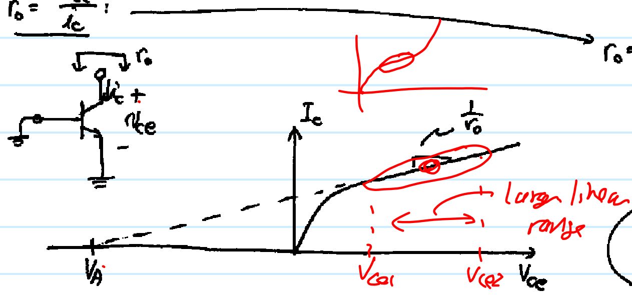
$r_o = \frac{v_{ce}}{i_c}$:

$$r_o = \frac{\partial V_{ce}}{\partial I_C}\Big|_Q = \left[\frac{\partial I_C}{\partial V_{ce}}\Big|_{Q\text{-pt.}}\right]^{-1}$$

$$= \left[\frac{\partial}{\partial V_{ce}}\left(\left[I_S \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1+\frac{V_{ce}}{V_A}\right]\right)\Big|_{V_{be}=V_{BE}}\right]^{-1}$$

$$= \left[\frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A}\right]^{-1} = \left[\frac{I_C}{V_A+V_{CE}}\right]^{-1} = \frac{V_A+V_{CE}}{I_C}$$

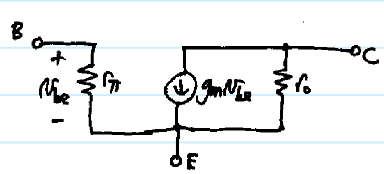
$$I_S \exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_C}{1+\frac{V_{CE}}{V_A}}$$



$$\therefore \boxed{r_o = \frac{V_A+V_{CE}}{I_C} \approx \frac{V_A}{I_C}}$$

$[V_A \gg V_{CE}]$

... and thus, we have the hybrid-π model:



SPICE: BF

$$\boxed{\begin{aligned} r_\pi &= \frac{\beta}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{V_A+V_{CE}}{I_C} \approx \frac{I_C}{V_A} \end{aligned}}$$

SPICE: VAF

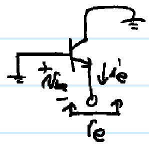
Remarks:

① g_m is independent of $\underbrace{\text{device specifics}}_{\text{i.e., } \beta, I_S}$; depends only on temperature (thru V_T) and biasing I_C ← quite different from MOS, as we'll see

② small-signal model valid for $v_{be} \ll V_T$ ← $\approx$ 26mV @ 300K

EE 140 | BJT SS Model | CTN | 8

What about emitter resistance?

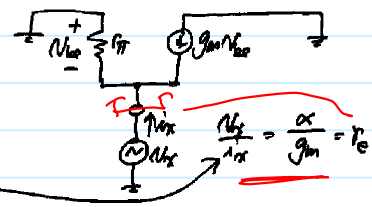


$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$\frac{1}{g_m}$

$$\Rightarrow \boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

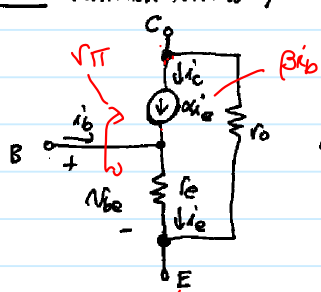
Note that although it's not explicitly shown in the hybrid-π model, r_e is present.
$\Rightarrow$ i.e., if you analyze this, you find that



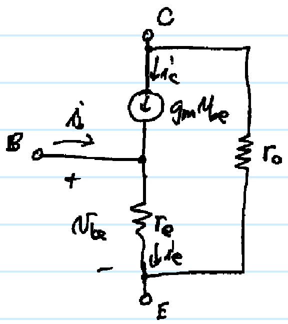
$$\frac{v_x}{i_x} = \frac{\alpha}{g_m} = r_e$$

To explicitly show emitter resistance, use the T-model:

T-Model: (Common Base Model)



or



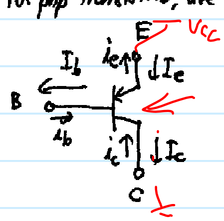
where as before:

$$g_m = \frac{I_C}{V_T}$$

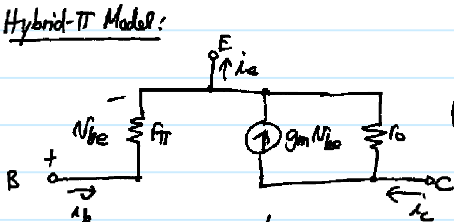
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

Small-Signal Models for forward-active pnp Transistors

For pnp transistors, use the same small-signal models as npn with no relative change in polarities!



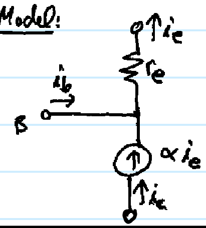
Hybrid-π Model:



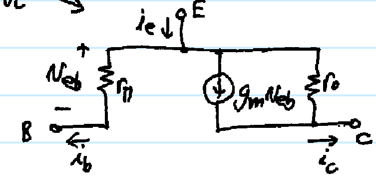
(same as npn model ... just upside-down)

Note that in these S.S. models, the same current directions as used for npn are used here
$\Rightarrow$ i.e., no change in S.S. polarities
(large-signal directions, however, can be as before)

T-Model:

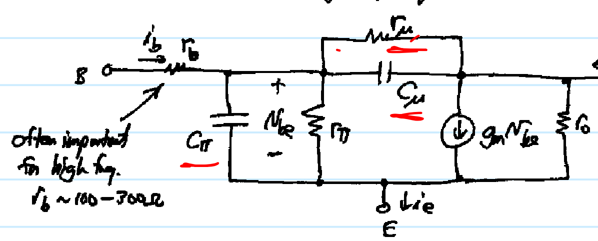


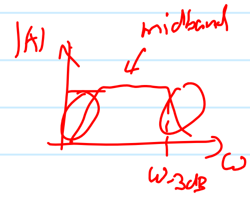
or



EE 140 | C_μ, C_π | CTN | 9

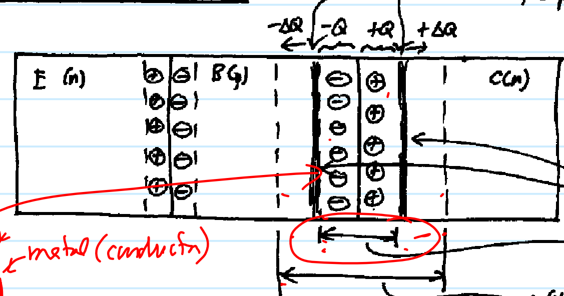
More Complete Hybrid-Π Model (adding frequency effects & 2nd order effects)



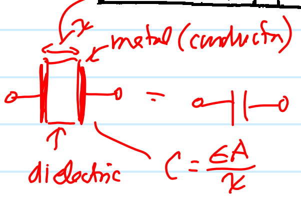


C_μ – Base-to-Collector Capacitance

charge modulates here ∴ these are the plates of a capacitor → C_μ



S.S.
if have V_{cb}: $V_{CB1} + v_{cb}$, then Δq gets modulated over these small regions ⇒ thus, these are effectively the plates of a capacitor!



$x_d(V_{CB2})$, where $V_{CB2} > V_{CB1}$

if change to a different bias V_{CB2}, get new capacitor plate locations → $C_\mu \approx \frac{\epsilon_s}{x_d(V_{CB2})}$

$C_\mu = f(V_{CB})$!

$$\boxed{C_\mu = \frac{C_{\mu o}}{\sqrt{1+\frac{V_{CB}}{\phi_j}}}}$$

$= f(V_{CB})$

where $C_{\mu o}$ = capacitance for $V_{CB}=0$

ϕ_j = function of the built-in potential between p and n-type semiconductors $\Big\} = \frac{kT}{q}\ln\left(\frac{N_{DB}N_{AC}}{n_i^2}\right)$

n_i: $1.45\times10^{10}\,cm^{-3}$

In general:

$$C_\mu = \frac{C_{\mu o}}{\left(1+\frac{V_{CB}}{\phi_j}\right)^m}, \text{ where } m = \tfrac{1}{2} \text{ or } \tfrac{1}{3} \text{ depending upon how abrupt the junction is}$$

In SPICE: CJC ($C_{\mu o}$), VJC (ϕ_j), MJC (m)

Detailed Derivation: [FYI]

$$x_d \cong x_a = \left[\frac{2\epsilon(\psi_o+V_{CB})}{qN_A\left(1+\frac{N_A}{N_D}\right)}\right]^{1/2} \longrightarrow Q = qN_A A x_d = A\left[\frac{2\epsilon q N_A(\psi_o+V_{CB})}{1+\frac{N_A}{N_D}}\right]^{1/2}$$

$[N_A \ll N_D]$

$$C_j = \frac{dQ}{dV_{CB}}\bigg|_{V_{CB}} = \left[\frac{2\epsilon q N_A}{1+\frac{N_A}{N_D}}\right]^{1/2}\frac{A}{2}(\psi_o+V_{CB})^{-1/2} = \boxed{A\left[\frac{q\epsilon N_A N_D}{2(N_A+N_D)}\right]^{1/2}\frac{1}{\sqrt{\psi_o+V_{CB}}} = C_j\Big|_{V_{CB}}}$$

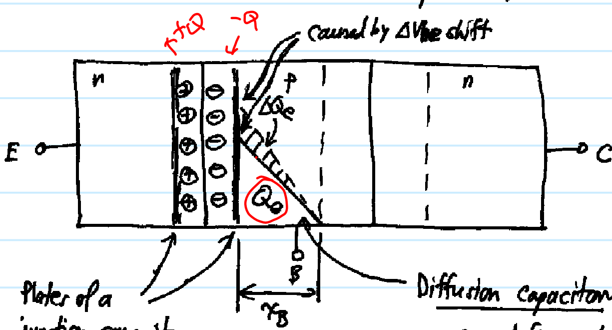
$$C_j = \frac{\epsilon_s A}{x_d(V_{CB})}$$

EE 140 | C_π | CTN | 10

C_π – Base-to-Emitter Capacitance

Two components comprise C_π: ① Junction capacitance, C_{je}

② Diffusion capacitance, C_b



$Q = CV \rightarrow C = \frac{Q}{V}$

$V = IR$

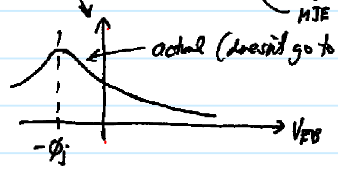
Plates of a junction capacitor:

$$C_{je} = \frac{C_{jeo}}{\left(1+\frac{V_{EB}}{\psi_o}\right)^m}$$

C_{jeo} ← bias level determines where the plates are

In SPICE: CJE, VJE, MJE

Remember to say this fn



GTM use this!

$C_\pi = C_b + C_{je}$ ($\approx 2C_{jeo}$)

$$\boxed{C_\pi = \tau_F g_m + \frac{C_{jeo}}{\left(1+\frac{V_{EB}}{\psi_o}\right)^m}}$$

Diffusion capacitance: (or Base Charging Capacitance)

⇒ can define a base transit time:

$$\tau_F = \frac{Q_e}{I_C} = \frac{x_B^2}{2D_n}$$ } avg. time spent by carriers in crossing base

think of I_C as the rate of xfer of charge through the base

$Q_e = \tau_F I_C$

$\Delta Q_e = \tau_F \Delta I_C$

Switch to s.s. parameters (variables):

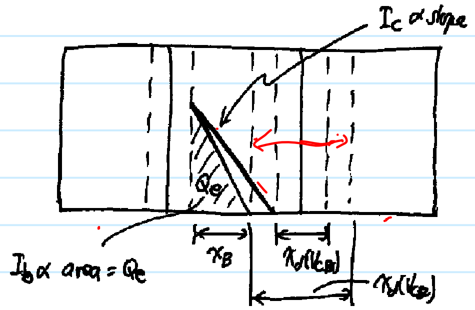
$q_e = \tau_F i_c$

SPICE: TF

$$q_e = C_b v_{be} \rightarrow C_b = \frac{q_e}{v_{be}} = \tau_F \frac{i_c}{v_{be}} = \boxed{\tau_F g_m = \tau_F \frac{I_C}{V_T} = C_b}$$

$\therefore \boxed{C_b \propto I_C}$

Collector-to-Base Feedback Resistor, r_μ

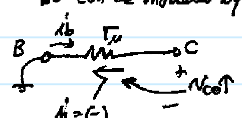


Remember, recombination base current $I_{rB} = \frac{Q_e}{\tau_b}$.

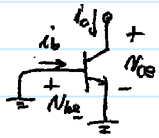
$\therefore v_{ce}\uparrow \rightarrow x_B\downarrow \rightarrow Q_e\downarrow \rightarrow i_b\downarrow$

$\searrow i_c\uparrow$ (due to Early effect)

$v_{ce}\uparrow \rightarrow i_b\downarrow$ can be modeled by an r_μ connecting C-to-B



EE 140 | r_π CTN (11)



⇒ here, $v_{be}=0 \rightarrow v_{ce}=v_{cb}$

∴ $\frac{i_c}{v_{ce}} = \frac{1}{r_o} = \frac{i_c}{v_{cb}} = \frac{\beta i_b}{v_{cb}} \rightarrow \frac{v_{cb}}{i_b} = \boxed{\beta r_o = r_\mu}$

(V_{be} = const)

assuming all of i_b is recombination current

In general, base recombination current is only part of the total base current and is the only component dependent on V_{bc} ⇒ thus, $\boxed{r_\mu > \beta_o r_o} \rightarrow r_\mu = 2\text{-}10\,\beta_o r_o$

lateral pnp   npn → I_b is 10% recomb.

where base recomb. more significant

Complete Forward-Active BJT S.S. Model (including parasitics)

⇒ Actual integrated BJT:

should draw this on the board

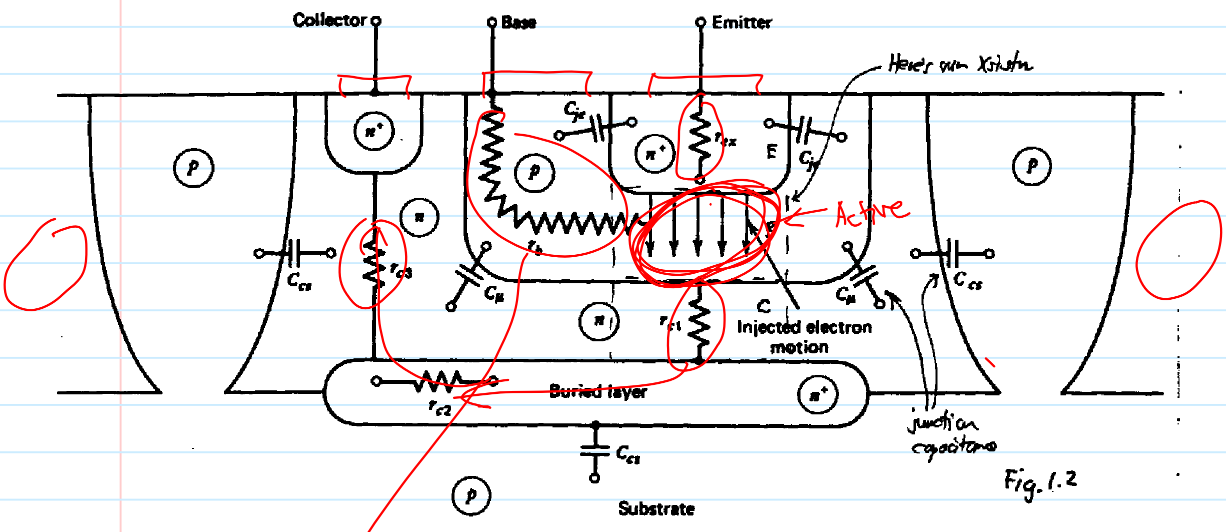
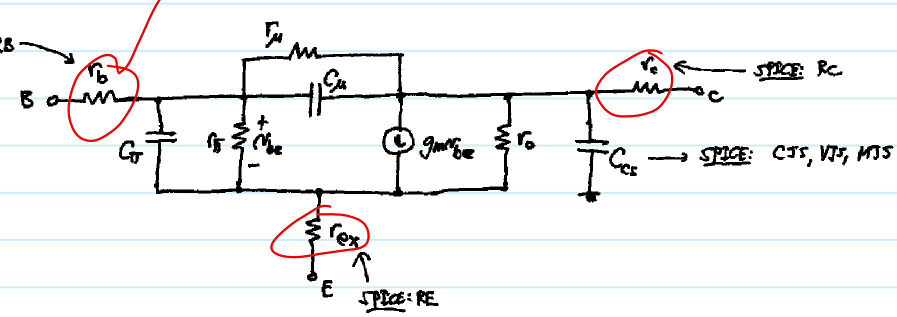


Fig. 1.2



EE 140 | BJT Layout | CTN | 12

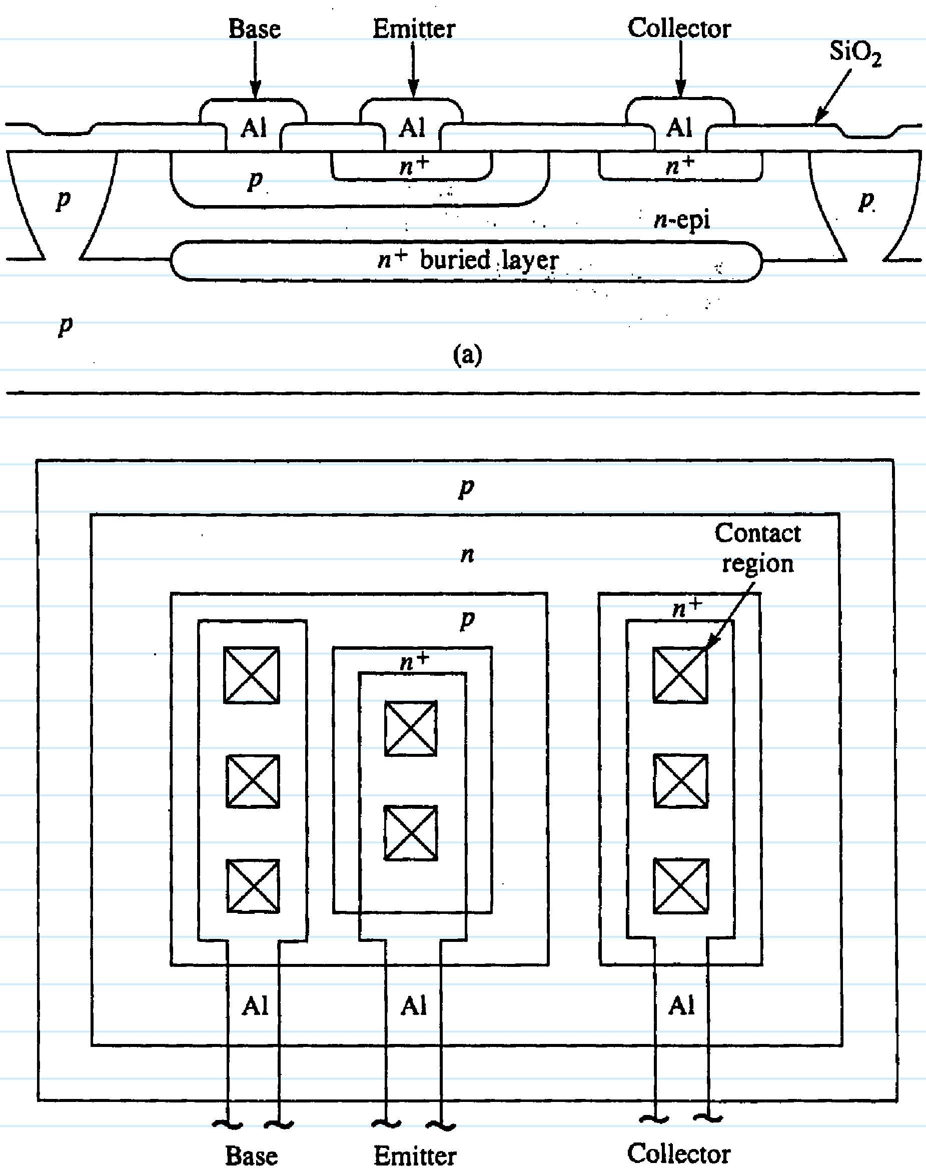
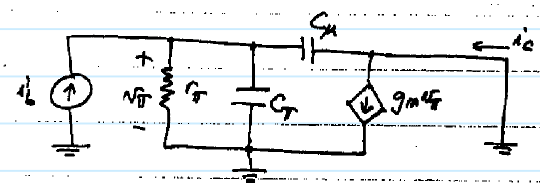


Fig. 1.1

EE 140 f_T CTN 13

$\boxed{f_T \text{ (unity gain freq. for } \beta)}$

Find $\beta(j\omega)$: (β as a function of freq.)



Find $\left.\frac{i_c}{i_b}\right|_{v_c=0}$: $v_\pi = i_b \left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu} \right)$

$[g_m >> sC_\mu]$

$i_c = g_m v_\pi - sC_\mu v_\pi = (g_m - sC_\mu) v_\pi \approx g_m v_\pi$

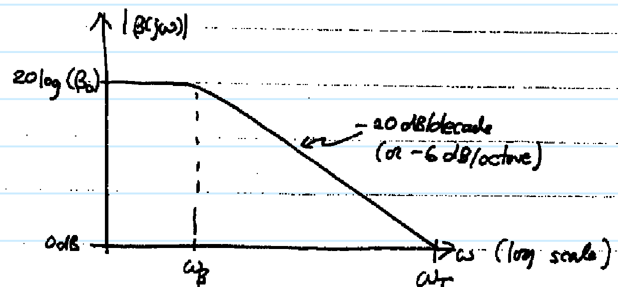
$i_c = g_m \left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu} \right) i_b$

$$\frac{i_c}{i_b} = \frac{g_m}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)} = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)} = \frac{\beta_o}{1 + s r_\pi (C_\pi + C_\mu)}$$

$[\beta_o = g_m r_\pi]$ (low freq. β)

$$\boxed{\beta(j\omega) = \frac{\beta_o}{1 + \frac{j\omega}{\omega_\beta}}} \qquad \boxed{\omega_\beta = \frac{1}{r_\pi (C_\mu + C_\pi)}}$$

Plot $|\beta(j\omega)|$: (Bode plot)



ω_T ≜ defined as the ~~point~~ radian frequency where $|\beta(j\omega)| = \beta_o$

For ω large: (e.g. ω close to ω_T)

$$|\beta(j\omega_T)| \approx \frac{\beta_o}{\omega_T r_\pi (C_\pi + C_\mu)} = 1 \longrightarrow \boxed{\omega_T = \frac{g_m}{C_\pi + C_\mu}} \Rightarrow f_T = \frac{\omega_T}{2\pi}$$

is a figure of merit for the frequency performance of a transistor.

Also, note that $\boxed{\omega_T = \beta_o \omega_\beta}$

f_T = 100 MHz $\longrightarrow$ 15 GHz for bipolar Xistors.

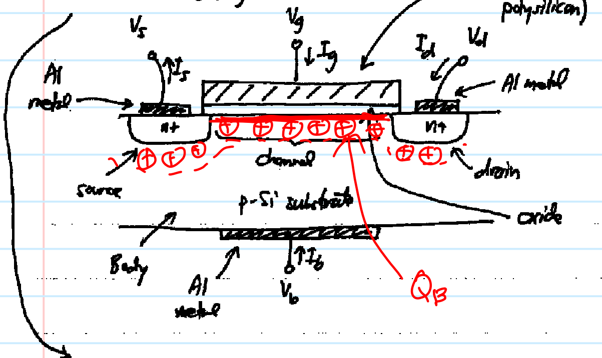
$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$$

EE 140 | MOS Transistors | CTN | 14

MOS Transistors

Physical Structure & Device Symbols – (cross-sectional view) → of NMOS device

NMOS Xsistor Device Symbol



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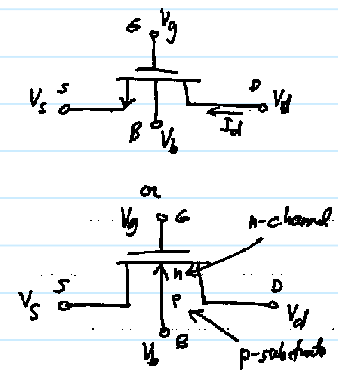
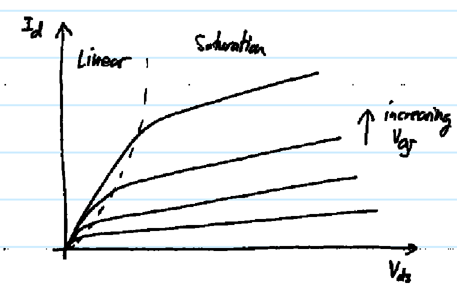
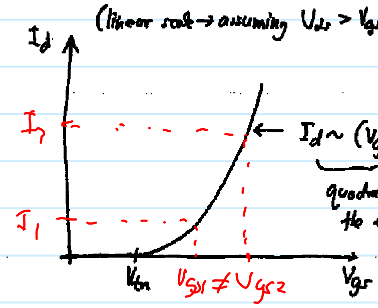


But first start w/ a perspective view: (This also defines dimensions)
↳ use the viewgraph on next page → pg. 14a

IV Characteristics (NMOS)



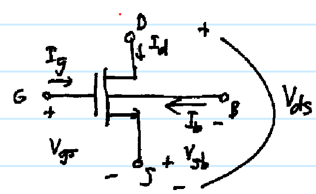
(linear scale → assuming $V_{ds} > V_{gs} - V_{tn}$ → saturation)



← $I_d \sim (V_{gs} - V_{tn})^2$

quadratic not as fast as the exponential of npn
∴ in biasing for MOS, can't just assume a $V_{GS(on)}$
↓
rather, must calculate!

NMOS Xsistor Mathematical Model



① Cut-Off Region: $(V_{gs} \leq V_t)$

$I_g = I_b = 0$; $I_d = 0$

② Linear (or Triode) Region: $(V_{gs} - V_{tn} \geq V_{ds} \geq 0)$

$I_g = I_b = 0;\ I_d = \mu_n C_{ox} \frac{W}{L}\left(V_{gs} - V_{tn} - \frac{V_{ds}}{2}\right)V_{ds}$

Body Factor → $\gamma = \frac{1}{C_{ox}}\sqrt{2q\epsilon_s N_{sub}}$ ← substrate doping conc. $\qquad = k_n\left(V_{gs} - V_{tn} - \frac{V_{ds}}{2}\right)V_{ds}$

permittivity in Si

General:

$k_n = k_n' \frac{W}{L} = \mu_n C_{ox} \frac{W}{L}$

$I_g = I_b = 0$ for all regions (at least for dc)

$V_{tn} = f(V_{SB}) = V_{to} + \gamma\left(\sqrt{V_{sB} + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$

③ Saturation Region: $(V_{ds} \geq V_{gs} - V_{tn} \geq 0)$

$I_g = I_b = 0;\ I_d = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{gs} - V_{tn})^2(1 + \lambda V_{ds})$

$= \frac{1}{2}k_n(V_{gs} - V_{tn})^2(1 + \lambda V_{ds})$

$\mu_n \triangleq e^-$ mobility in the channel

$C_{ox} \triangleq$ gate oxide capacitance per unit area

EE 140 | MOS Transistor Structure | CTN | 15

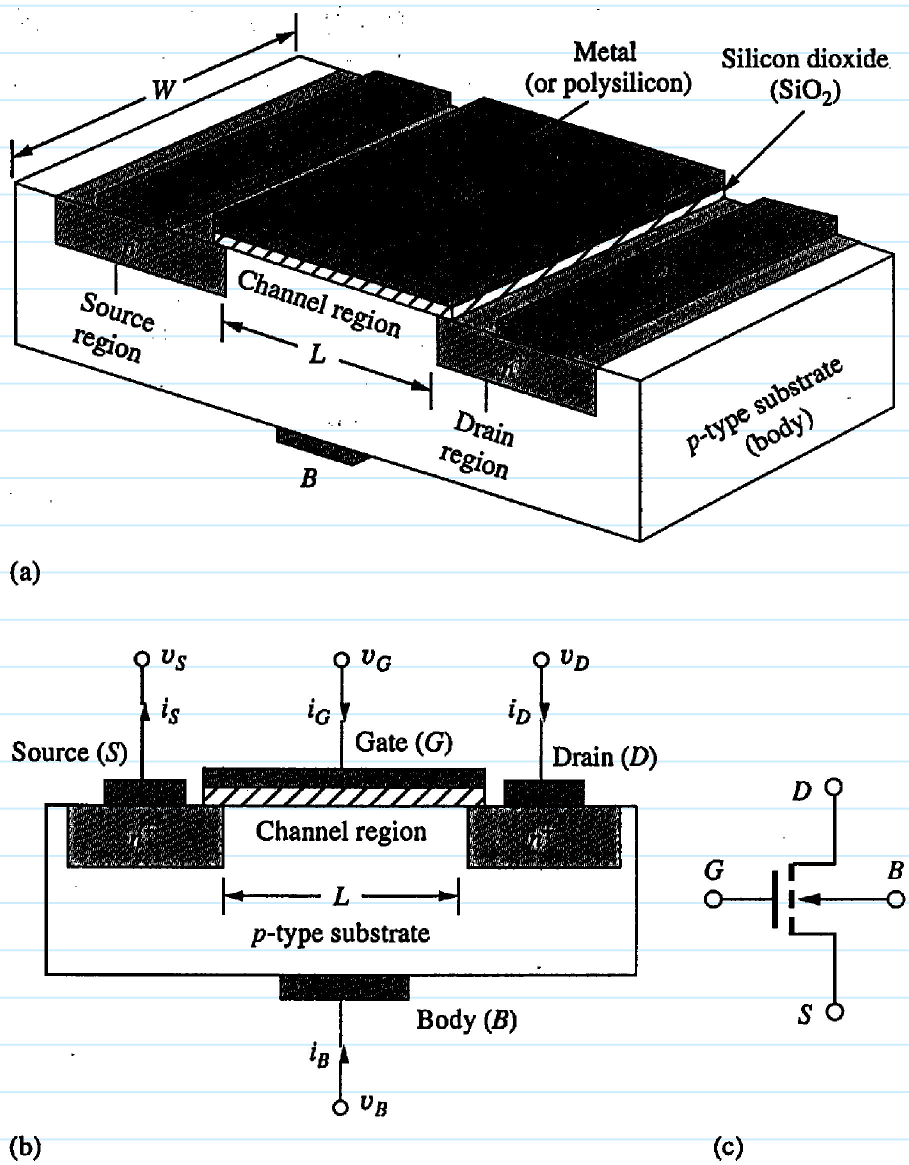
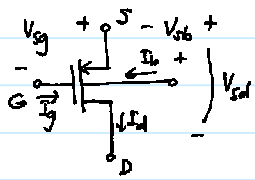


Fig. 2.1

EE 140 | PMOS | CTN | 16

PMOS Xstr Mathematical Model



① Cut-Off Region: $(V_{sg} \leq -V_{tp})$ or $(|V_{gs}| \geq |V_{tp}|)$

$I_{sd} = 0$

② Linear (or Triode) Region: $(V_{sg} + V_{tp} \geq V_{sd} \geq 0$; or $|V_{gs}| - |V_{tp}| \geq |V_{ds}| \geq 0)$

$$I_{sd} = k_p\left(V_{sg} + V_{tp} - \frac{V_{sd}}{2}\right)V_{sd} = \mu_p C_{ox}\frac{W}{L}\left(V_{sg} + V_{tp} - \frac{V_{sd}}{2}\right)V_{sd}$$
$$= \mu_p C_{ox}\frac{W}{L}\left(|V_{gs}| - |V_{tp}| - \frac{|V_{ds}|}{2}\right)|V_{ds}|$$

For all regions:

$k_p = k_p'\frac{W}{L} = \mu_p C_{ox}\frac{W}{L}$

$I_g = 0$ and $I_b = 0$ (at dc)

$V_{tp} = V_{to} - \gamma\left(\sqrt{V_{BS} + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$

③ Saturation Region: $(V_{sd} \geq V_{sg} + V_{tp} \geq 0$; $|V_{ds}| \geq |V_{gs}| - |V_{tp}| \geq 0)$

$$I_{sd} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{sg} + V_{tp})^2(1 + \lambda V_{sd}) = \frac{1}{2}k_p(V_{sg} + V_{tp})^2(1 + \lambda V_{sd})$$
$$= \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(|V_{gs}| - |V_{tp}|)^2(1 + \lambda|V_{ds}|)$$

$\mu_p \triangleq h^+$ mobility in the channel

$C_{ox} \triangleq$ gate oxide capacitance per unit area

Threshold Voltage

$$\boxed{V_t = \phi_{ms} - \psi_s - \frac{Q_B}{C_{ox}} - \frac{Q_{ss}}{C_{ox}}}$$

, where ϕ_{ms} = work function difference [in V] between gate material and bulk Si

ψ_s = surface potential in the Si @ onset of strong inversion

$= 2\phi_f$ for uniformly doped substrate ($\phi_f \sim 0.3$ V)

Q_{ss} = oxide charge per unit area at the oxide-Si interface [C/cm^2]

Q_B = charge stored per unit area in the depletion region (at onset of inversion)

$\Rightarrow |Q_B| = \sqrt{2q\epsilon_s N_B(2|\phi_f| + |V_{SB}|)}$ [C/cm^2]

conc. in bulk — reverse bias

C_{ox} = gate oxide capacitance per unit area [F/cm^2]

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Case: $V_{SB}=0 \Rightarrow V_t(V_{SB}=0) = V_{to} = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{Bo}}{C_{ox}}$, where

$$Q_{Bo} = \sqrt{2q\epsilon_{si}N_B(2|\phi_f| + |V_{SB}|)}$$

Then:

$$V_t = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_B}{C_{ox}}$$

$$= \underbrace{\phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{Bo}}{C_{ox}}}_{V_{to}} - \frac{Q_B - Q_{Bo}}{C_{ox}}$$

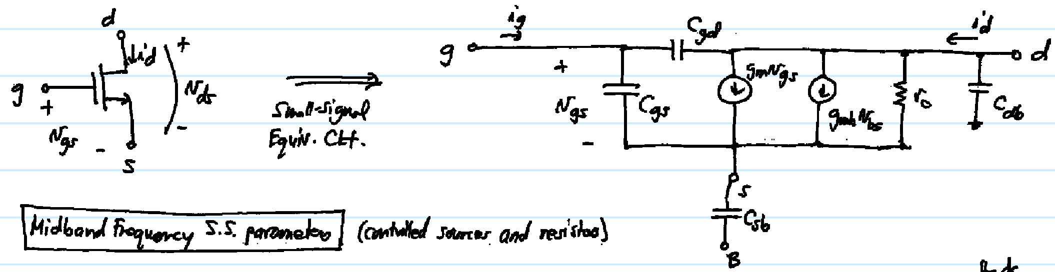
$$\boxed{V_t = V_{to} - \gamma\left(\sqrt{2|\phi_f| + |V_{SB}|} - \sqrt{2\phi_f}\right), \quad \gamma = \frac{1}{C_{ox}}\sqrt{2q\epsilon_{si}N_B}}$$

Signs in the V_t Equation:

Parameter	NMOS	PMOS
Substrate	p-type	n-type
ϕ_{ms}: metal gate	−	−
n+ Si gate	−	−
p+ Si gate	+	+
ϕ_f	−	+
Q_{Bo} (or Q_B)	−	+
Q_{ss}	+	+
γ	−	+
C_{ox}	+	+

EE 140 | MOS Small-Signal Model | CTN | 18

MOS Small-Signal Model (for NMOS) → in saturation



Midband Frequency S.S. parameters (controlled sources and resistors)

Transconductance, g_m:

$$g_m = \frac{i_d}{v_{gs}} = \frac{\partial I_d}{\partial V_{gs}}\Big|_{Q\text{-}pt} = \frac{\partial}{\partial V_{gs}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})^2\right)\Big|_{Q\text{-}pt} = \mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})\Big|_{V_{gs}=V_{GS}}$$

(the dc V_{GS})

$$\therefore \boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{tn}) = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}}$$

$$\left[I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{tn})^2 \rightarrow (V_{GS}-V_{tn}) = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} \right]$$

$$\frac{i_d}{v_{bs}} = g_{mb} = \frac{-i_d}{v_{sb}} = -\frac{\partial I_d}{\partial V_{sb}}\Big|_{Q\text{-}pt} = -\left(\frac{\partial I_d}{\partial V_{tn}}\cdot\frac{\partial V_{tn}}{\partial V_{sb}}\right)\Big|_{Q\text{-}pt}$$

$$\frac{\partial I_d}{\partial V_{tn}}\Big|_{Q\text{-}pt} = -\frac{\partial I_d}{\partial V_{gs}}\Big|_{Q\text{-}pt} = -g_m \quad ; \quad \frac{\partial V_{tn}}{\partial V_{sb}}\Big|_{Q\text{-}pt} = \frac{\partial}{\partial V_{sb}}\left[V_{to} + \gamma\left(\sqrt{V_{sb}+2|\phi_f|} - \sqrt{2|\phi_f|}\right)\right]\Big|_{Q\text{-}pt} = \boxed{\frac{\gamma}{2\sqrt{V_{SB}+2|\phi_f|}} = \eta}$$

$$\therefore \boxed{g_{mb} = \eta g_m}$$

often neglected!

Note: $V_{SB}\uparrow \rightarrow V_T\uparrow \rightarrow \eta\downarrow \rightarrow I_d\downarrow$

↳ g_{mb} is minimized by maximizing V_{SB}!

Output Resistance, r_o: $\left(= \frac{1}{g_{ds}}\right)$

explain what λ is!

$$\Rightarrow \text{output conductance} = g_{ds} = \frac{i_d}{v_{ds}} = \frac{\partial I_d}{\partial V_{ds}}\Big|_{Q\text{-}pt} = \frac{\partial}{\partial V_{ds}}\Big(\overbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})^2}^{I_{dsat}}(1+\lambda V_{ds})\Big)\Big|_{Q\text{-}pt}$$

$$= \lambda I_{dsat} = \frac{\lambda I_D}{1+\lambda V_{DS}} \approx \lambda I_D = g_{ds}$$

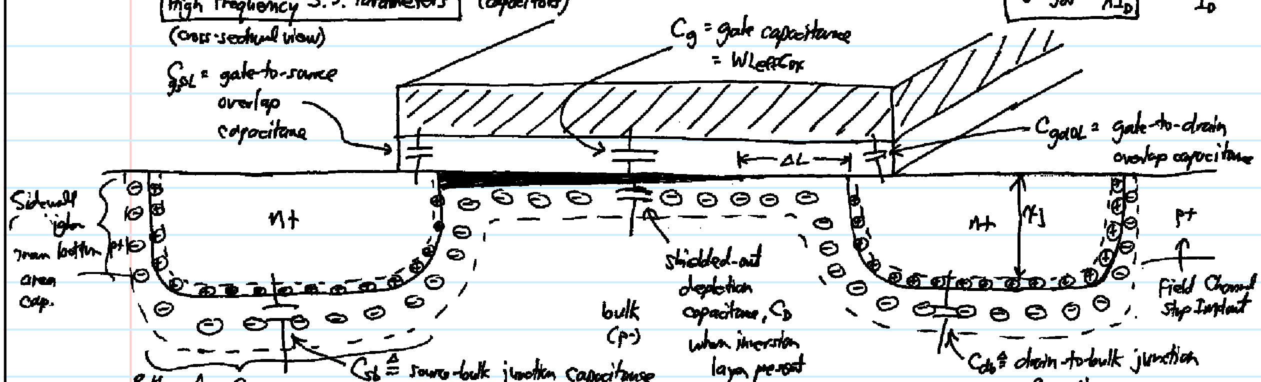
$[1 \gg \lambda V_{DS}]$

$$\therefore \boxed{r_o = g_{ds}^{-1} = \frac{1}{\lambda I_D}} = \frac{\frac{1}{\lambda}+V_{DS}}{I_D}$$

(if V_{DS} is very large)

High Frequency S.S. Parameters (capacitors)

(cross-sectional view)



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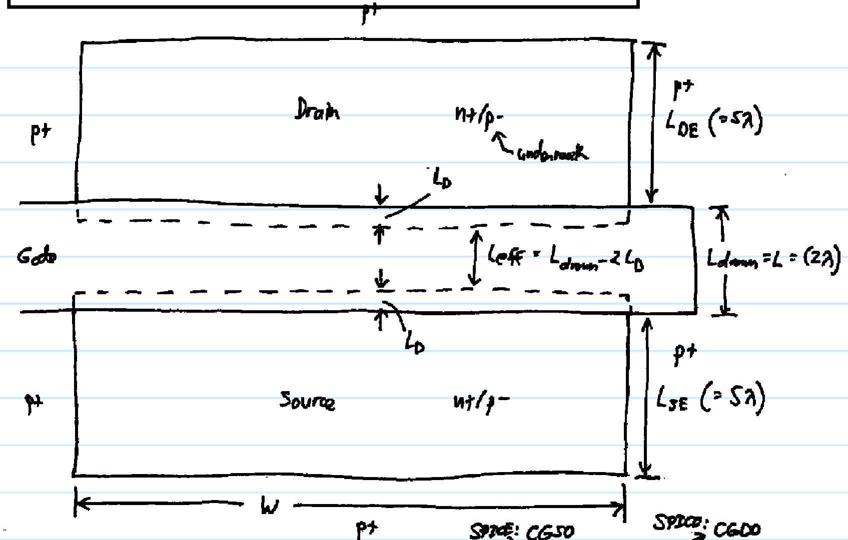
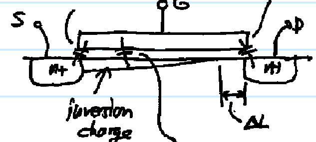
(layout view)



(still considering saturation region)

In saturation, the inversion charge is not present near the drain:



$\frac{2}{3}C_g = \frac{2}{3}WL_{eff}C_{ox}$ — obtained by integrating the charge over the gate length

Gate-to-Source Capacitor, C_{gs}:

$C_{gs} = C_{oL} + \frac{2}{3}WL_{eff}C_{ox}$ (inversion charge integrated)

Gate-to-Drain Capacitor, C_{gd}:

$C_{gd} = C_{oL}$ (no inversion charge near the drain in saturation)

Source/Drain Junction Capacitance, C_{sb} & C_{db}: (must include these in SPICE simulations!)

⇒ there are depletion capacitors associated w/ the drain-to-bulk and source-to-bulk pn junctions

⇒ bottom-side capacitance per unit area is different from that at sidewalls due to higher bulk doping at the sidewalls
(there is higher doping in the field areas to prevent channels from forming under interconnect wires)

⇒ take drain capacitance as an example:

SPICE: CJ

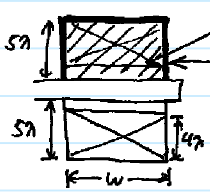
$C_{j0} = \sqrt{\frac{q\epsilon_s N_B}{2|\phi_j|}} \rightarrow \left(\frac{q\epsilon_s N_B}{2|\phi_j|}\right)^m$ — N_B: bulk doping level; $\left(\frac{q\epsilon_s N_A N_D}{2(N_A+N_D)\psi_0}\right)^{1/2}$

C_{j0}: depl. cap. per unit area @ bottom-side w/ $V_{SB} = 0V$

$$C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{\psi_0}}}, \quad C_{db0} \triangleq \text{depletion capacitance with } V_{DB} = 0V$$

$= (\text{junction bottom-side area})\,C_{j0} + (\text{junction outside perimeter})\,C_{jsw}$

$= W(5\lambda)C_{j0} + (W + 2(5\lambda))C_{jsw}$



C_{jsw}: depletion cap. along sidewalls per unit length for $V_{DB} = 0V$

$C_{jsw} = \sqrt{\frac{q\epsilon_s N_{cs}}{2|\phi_j|}} \times x_j \rightarrow \left(\frac{q\epsilon_s N_{cs}}{2|\phi_j|}\right)^m$

N_{cs}: channel-stop implant doping level; x_j: S/D junction depth

