Lecture 6: Frequency Response Inspection Analysis

- **Announcements**:
- HW#2 due tomorrow at 8 a.m.
- Lab#1 is online; HW#3 online soon
- **Lecture Topics**:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
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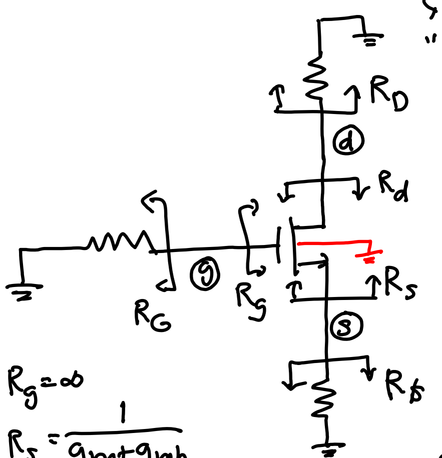
- **Last Time**:

$\boxed{\text{MOS Inspection Formulas w/ Substrate Grounded}}$ or as well as tied to V_{DD}



only difference from "substrate tied to source" case is that g_m is replaced by g_m+g_{mb} in some of the formulas — particularly ones where the source is involved!

$R_g = \infty$

$R_s = \dfrac{1}{g_m+g_{mb}}$

$R_d = r_o\left[1+(g_m+g_{mb})R_S\right]$

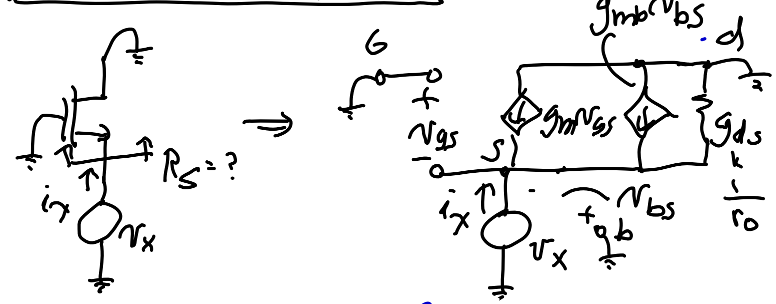
$\dfrac{v_s}{v_g} = \dfrac{g_m R_S}{1+(g_m+g_{mb})R_S}$

$\dfrac{v_d}{v_g} = -G_m R_{d}$, $G_m = \dfrac{g_m}{1+(g_m+g_{mb})R_S}$

$\dfrac{v_d}{v_s} = -G_m R_{d}$, $G_m = -(g_m+g_{mb})$

Remark. When the substrate is tied to the source, $g_{mb}=0$.

$\boxed{\text{Effect of } g_{mb} \text{ on Impedance}}$



$$\left[\begin{aligned} v_{gs} &= -v_x = v_{bs} \\ v_{ds} &= -v_x \end{aligned}\right]$$

$R_s = \dfrac{1}{g_m+g_{mb}+g_{ds}} = \dfrac{1}{g_m} \| \dfrac{1}{g_{mb}} \| \dfrac{1}{g_{ds}}$

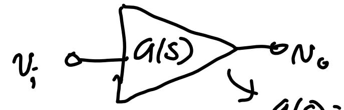
$r_o \gg \frac{1}{g_m}$ or $\frac{1}{g_{mb}}$

$$R_s \approx \frac{1}{g_m} \| \frac{1}{g_{mb}} = \frac{1}{g_m+g_{mb}}$$

If continue to analyze all quantities,

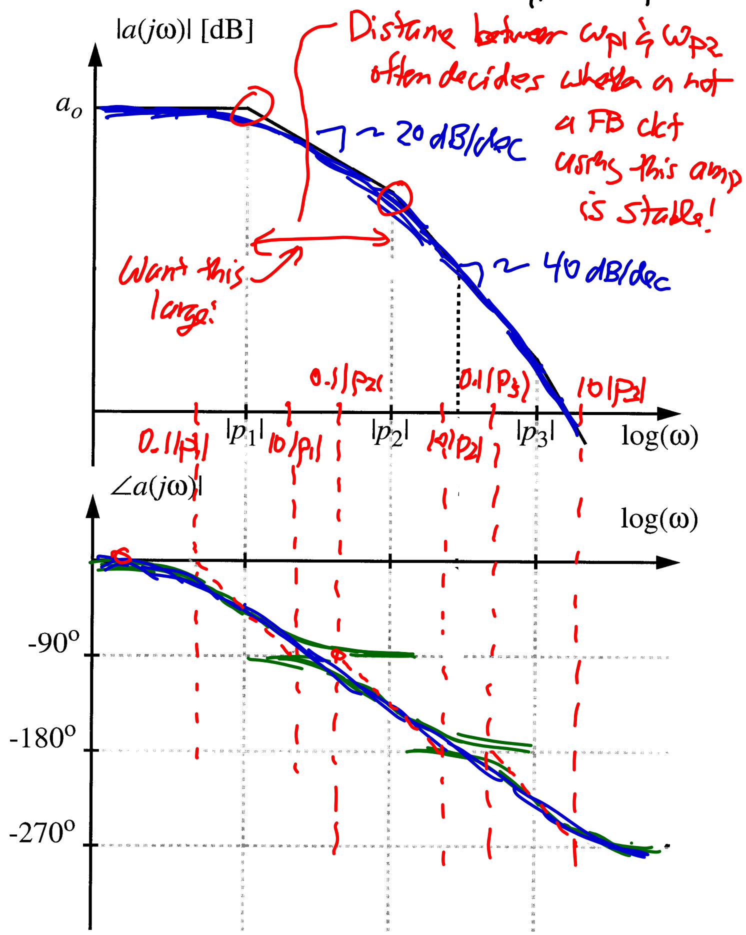
"g_m" → "g_m+g_{mb}" (for all denominators) (but not some numerators)

Poles & Zeros → Bode Plots



complex variable → $j\omega$

$$a(s) = \frac{v_o}{v_i}(s) = \frac{a_o}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$



Freq. Response

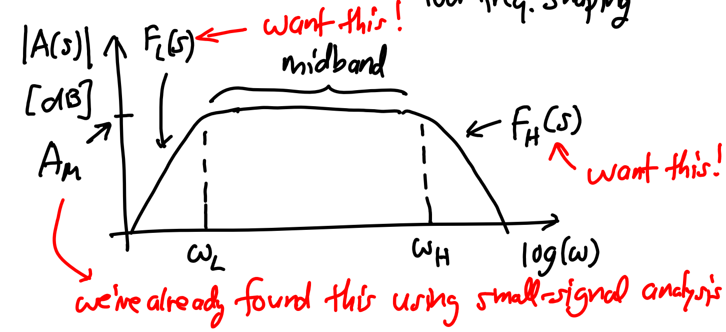
Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s) \qquad s = j\omega$$

midband gain

low freq. shaping

high frequency shaping

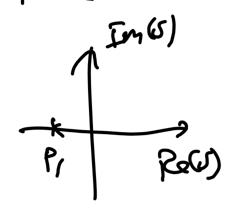


High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones
$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pnp}$

(ii) $F_H(s) \cong \dfrac{1}{1+\frac{s}{\omega_H}}$

(iii) $b_1 \cong \dfrac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \dfrac{1}{b_1} = \dfrac{1}{\sum_j \tau_{jo}} = \dfrac{1}{\sum_j C_j R_{jo}}}$

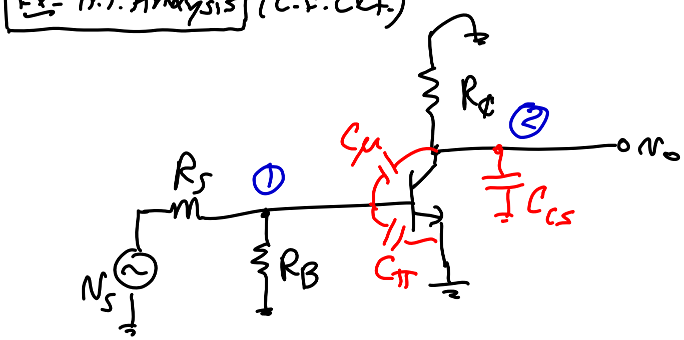
When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

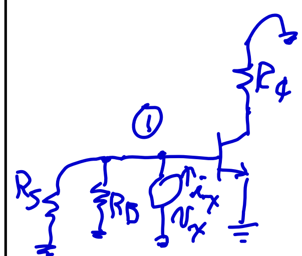
(just FYI)

- Now, go to inspection formula sheet and go over how to use the frequency response parts

Ex: H.F. Analysis (C.E. Ckt.)



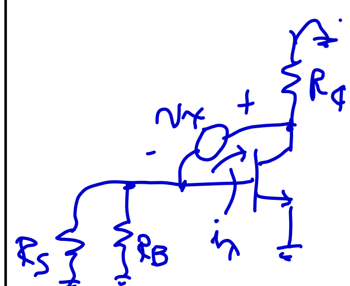
Find $\tau_{\pi o}$:



$$R_{①} = \frac{v_x}{i_x} = R_S \| R_B \| r_\pi$$

$$\tau_{\pi o} = C_\pi (R_S \| R_B \| r_\pi)$$

Find $\tau_{\mu o}$:



$$R_{\mu o} = \frac{v_x}{i_x} = R_{①} + R_{②} + g_m R_{①} R_{②}$$

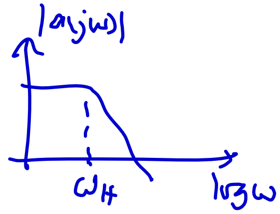
$$R_{\mu o} = (R_S \| R_B \| r_\pi) + (\cancel{r_o} \| R_C) + g_m (R_S \| R_B \| r_\pi)(\cancel{r_o} \| R_C)$$

$$\tau_{\mu o} = C_\mu R_{\mu o}$$

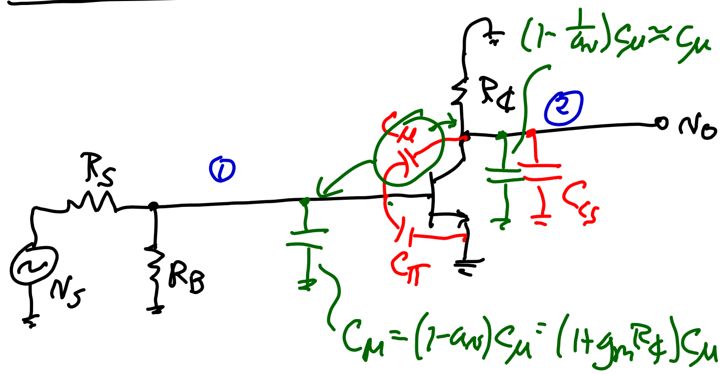
Find $\tau_②$: $\tau_② = C_{cs}(r_o \| R_\ell) \approx C_{cs}R_\ell$

$[r_o \gg R_\ell]$

$$\omega_H = \frac{1}{\tau_① + \tau_{\mu o} + \tau_②}$$



Now, use Miller's Theorem:



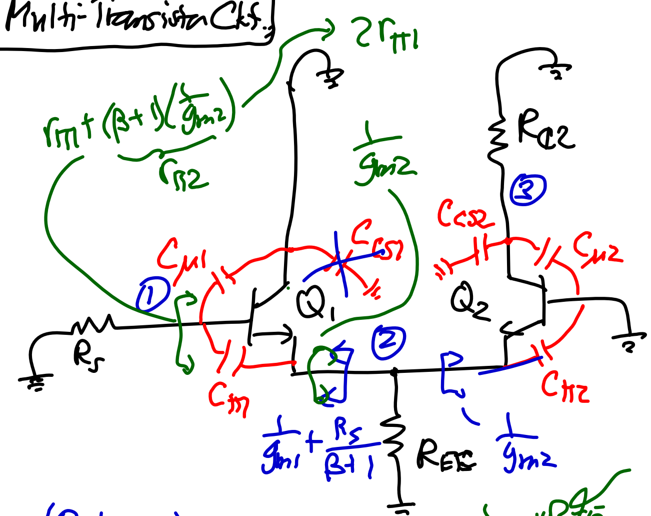
$\tau_① = (R_S \| R_B \| r_\pi)(C_\pi + C_\mu(1+g_mR_\ell))$

$\tau_② = R_\ell(C_\mu + C_{cs})$

$$\omega_H = \frac{1}{\tau_① + \tau_②}$$

Exactly the same as before!

Ex. Multi-Transistor Ckt.



$\tau_① = (R_S \| 2r_{\pi1})C_{\mu1}$

$\frac{1}{g_{m2}} \| R_{EE}$

$\tau_\pi = C_{\pi1}\left(r_{\pi1} \,\Big\|\, \left[\frac{R_S + \frac{1}{g_{m2}}}{1 + g_{m1}(\frac{1}{g_{m2}})}\right]\right)$

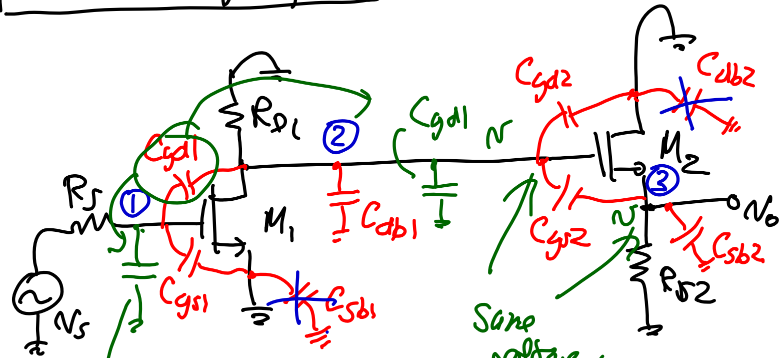
$[R_{EE}$: very large$]$

$\tau_② = C_{\pi2}\left[\left(\frac{1}{g_{m1}} + \frac{R_S}{\beta+1}\right) \Big\| \frac{1}{g_{m2}} \Big\| \cancel{R_{EE}}\right]$

$\tau_③ = (C_{cs2} + C_{\mu2})R_\ell$

$$\omega_H = \frac{1}{\tau_① + \tau_② + \tau_③ + \tau_{\pi1}}$$

Ex. MOS Two-Stage Amplifier



Same voltage (w/ unity gain)

$C_M = C_{gd1}(1+g_{m1}R_{D1})$

v C_{gs2} v

$+ \ \Delta v = 0V \ -$

No voltage drop across C_{gs2} ∴ it's effectively not there!

$$\tau_1 = \left[C_{gd1}(1+g_{m1}R_{D1}) + C_{gs1}\right] R_S$$

$$\tau_2 = \left[C_{db1} + C_{gd1} + C_{gd2}\right](\cancel{r_{o1}} \,\|\, R_{D1})$$

$$\tau_3 = C_{sb2}\left(R_{S2} \,\|\, \frac{1}{g_{m2}+g_{mb2}}\right)$$

$$\cancel{\tau_{gs2} = C_{gs2}\left(\frac{R_{D1}+R_{S2}}{1+(g_{m2}+g_{mb2})R_{S2}}\right)}$$

$$\omega_H = \frac{1}{\tau_1 + \tau_2 + \tau_3 + \cancel{\tau_{gs2}}}$$