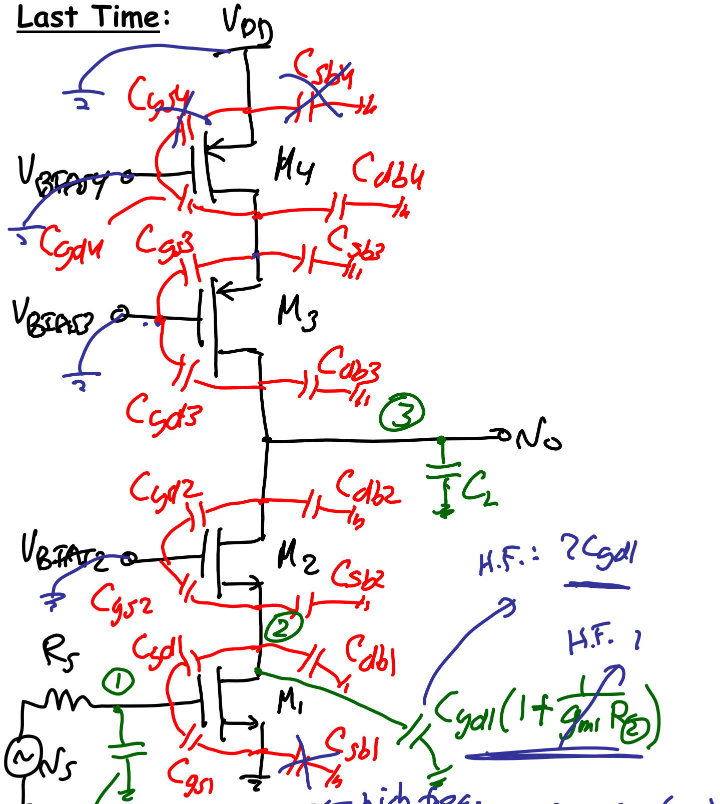
Lecture 9: Current Sources

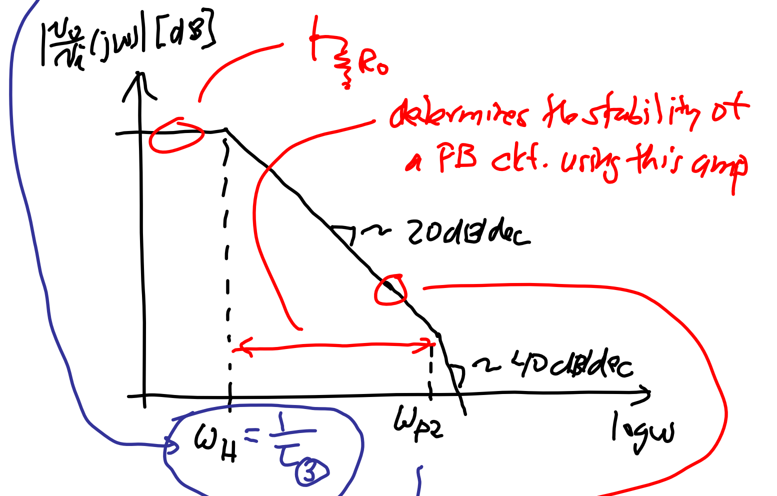
- **Announcements**:
  - HW#4 online
- **Lecture Topics**:
  - Analysis of actively loaded circuits (continued)
  - Current Sources
  - Widlar Current Source
  - Supply & Temperature Independent Biasing
- ------------------------------------
- **Last Time**:



H.F. ← high freq.

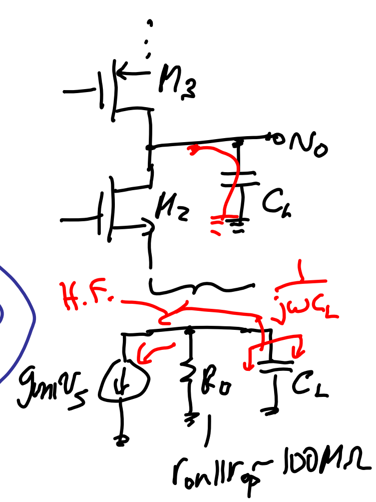
$C_M=(1+g_mR_{②})C_{gd1}$ $\quad R_{②}=\frac{1}{g_{m2}+g_{mb2}}$ $\rightarrow C_M=C_{gd1}(1+1)=2C_{gd1}$

$\tau_{③}=(C_{gd2}+C_{gd3}+C_{db2}+C_{db3}+C_L)R_o$



$\omega_{p2}=\frac{1}{\tau_{②}}$

$\tau_{②}=\left(2C_{gd1}+C_{db1}+C_{sb2}+C_{gs2}\right)\left(\frac{1}{g_{m2}+g_{mb2}}\right)$

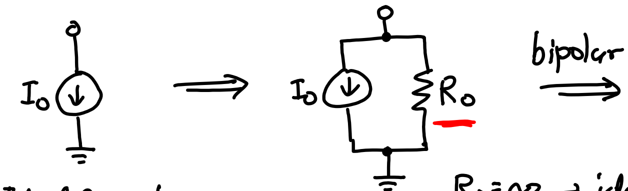


$a_v(s)=a_o\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)\left(\frac{1}{1+\frac{s}{\omega_{p2}}}\right)$

$s=j\omega$ $\quad$ midband gain

Transistor Current Sources

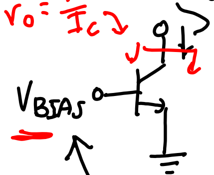
How can a transistor implement a current source?



Bipolar: Forward Active Xsistor Current Source

$I_o = I_C = I_s \exp\left(\frac{V_{BIAS}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right)$

$R_o = r_o = \frac{V_A}{I_C}$

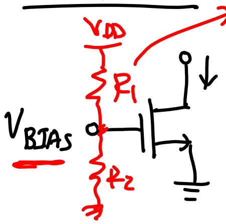


Very stable if V_{BIAS} = const. & V_A = large

Note that V_{BIAS} must be very accurate due to exponential ↳ to several sig. figs.

↳ e.g., V_{BIAS} = 0.68745V

MOS: Saturated MOS Xsistor Current Source



$I_o = I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{BIAS} - V_t)^2(1 + \lambda V_{DS})$

Again, very stable if V_{BIAS} = const. & λ = small

Problem: hard to specify R_1 & R_2 so exactly that they get the voltage to this precision

Problem: need to generate a stable V_{BIAS}

We now focus on methods for generating V_{BIAS}.

But how do we get this degree of precision using a transistor ckt?

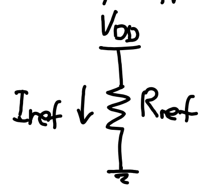
Solution: ↴

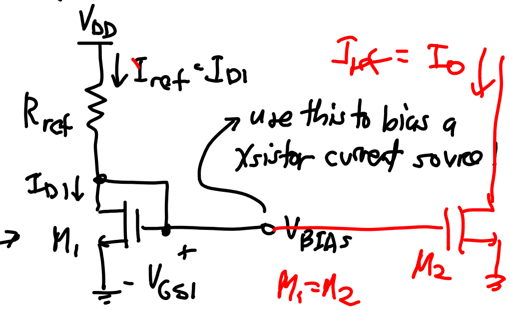
Replica Biasing (a simple & effective approach)

1. Generate the desired current.
2. Push the current through a Xsistor and allow it to reach a stable bias pt.
3. Use this stable bias pt. as V_{BIAS}

↳ this can be very precise!

One simple approach:





use this to bias a Xsistor current source

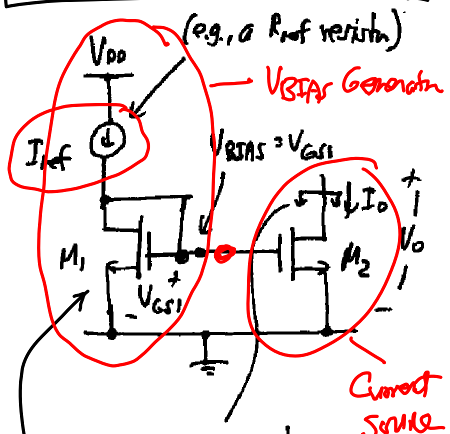
A diode-connected Xsistor is always in saturation and will basically bias itself to support the needed current!

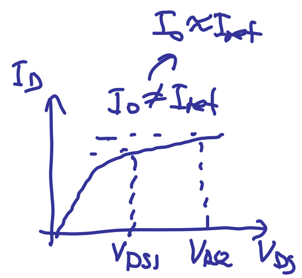
$I_{ref} = I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_t)^2(1 + \lambda V_{DS1})$

↑ V_{BIAS}

↳ Now, can distribute this V_{BIAS} to the gates of many MOS transistor current sources!

Ex. Simple MOS Current Source





$R_o = r_o = \frac{1}{\lambda I_o}$

In general, $V_{DS1} \neq V_{DS2}$ (V_o), but if λ = small, then little difference in I_{D1} & I_{D2}

Diode-connected Xsistor → saturation:

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{BIAS} - V_t)^2 (1 + \lambda V_{DS1})$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{BIAS} - V_t)^2 (1 + \lambda V_{DS2})$$

① Case: matched M_1 & M_2 ⟹ $I_o = I_{ref}$

② Case: M_1 & M_2 scaled w/r to each other

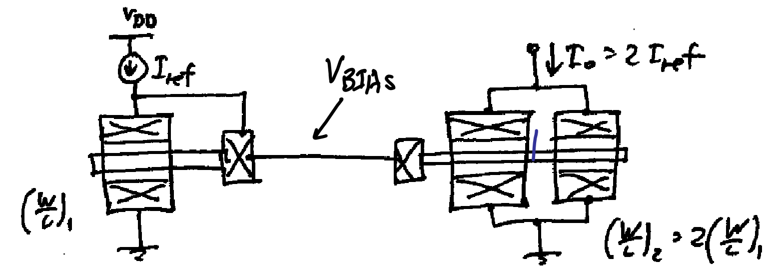
⟹ $\boxed{I_o = I_{ref}\frac{(W/L)_2}{(W/L)_1}}$

⟹ use $L_1 = L_2$ for better accuracy, then:

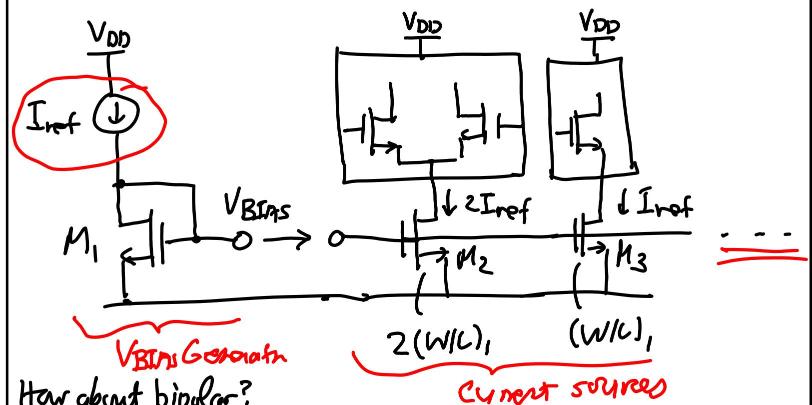
$$\boxed{\frac{I_o}{I_{ref}} = \frac{W_2}{W_1}}$$

Note: For better accuracy, should use multiple copies of one device when scaling current → reduces edge effects!

Ex: Layout for a Doubling Current Source

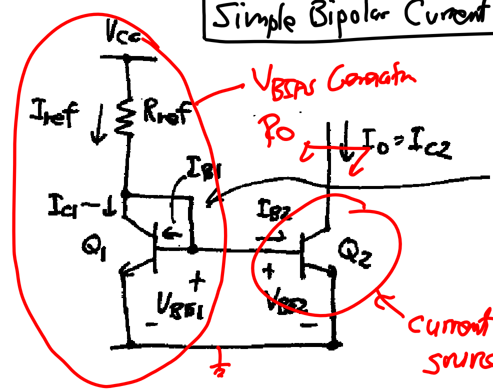


A single V_{BIAS} generator can now serve numerous current sources:



How about bipolar?

Simple Bipolar Current Source



KCL here to get $I_{C2} = f(I_{ref})$

Assume Q_1 & Q_2 are matched.

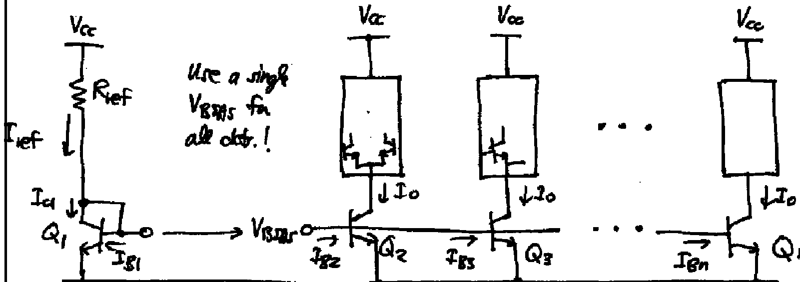
$V_{BE1} = V_{BE2} \longrightarrow I_{C1} = I_{C2} = I_O$ (neglecting V_A's) i.e., $V_A = \infty$

KCL: $I_{ref} = I_{C1} + I_{B1} + I_{B2} = I_{C1}\left(1 + \frac{2}{\beta}\right)$

$\therefore I_{C1} = I_{C2} = \boxed{I_o = \frac{I_{ref}}{1 + \frac{2}{\beta}}} \longrightarrow I_o \approx I_{ref}$ can say error $\sim \frac{2}{\beta}$

and $\boxed{I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R_{ref}}}$ $\boxed{R_o = r_{o2}}$

Again, a single V_{BIAS} generator can serve many current sources throughout the IC chip:



$I_{ref} = I_{C1} + I_{B1} + I_{B2} + I_{B3} + \cdots + I_{Bn}$

[Identical Xsistors] $\Longrightarrow$ $I_{ref} = I_{C1}\left(1 + \frac{n}{\beta}\right)$

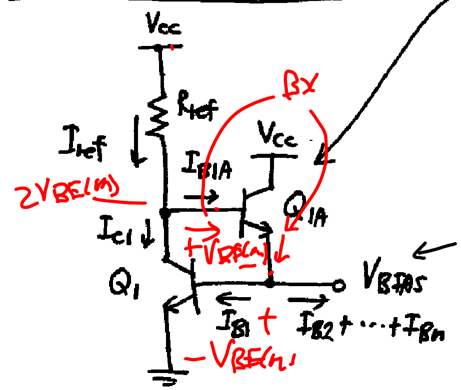
$\Longrightarrow \boxed{I_o = I_{C1} = \frac{I_{ref}}{\left(1 + \frac{n}{\beta}\right)}}$

Problem: error $\sim \frac{n}{\beta}$ increases as n (Io deviates from Iref, and the deviation depends on n.)

How can one reduce the error?

To reduce the error term, use a

Buffered V_{BIAS}-Generator



Add a buffer Xsistor to attenuate base currents from Xsistor current sources.

This can now drive the base currents of many bipolar transistor current sources (i.e., active loads).

$I_{ref} = I_{C1} + I_{B1A}$

$I_{B1A} = \frac{I_{B1} + I_{B2} + \cdots I_{Bn}}{\beta + 1} = \frac{nI_{C1}}{\beta(\beta+1)}$

[Assuming identical Xsistors]

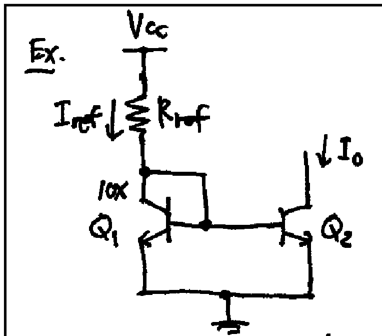
$I_{ref} = I_{C1}\left(1 + \frac{n}{\beta(\beta+1)}\right)$

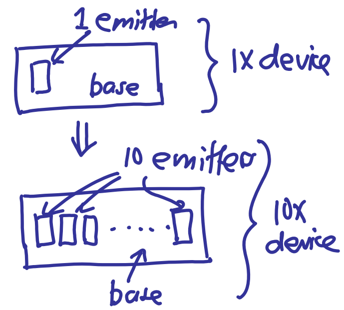
$\longrightarrow \boxed{I_o = I_{C2} = \frac{I_{ref}}{1 + \frac{n}{\beta(\beta+1)}} \approx I_{ref}\left(1 - \frac{n}{\beta^2}\right)}$

Note: Now, $\boxed{I_{ref} = \frac{V_{CC} - 2V_{BE(on)}}{R_{ref}}}$

Problem: For power savings reasons, oftentimes very small bias currents are needed, on the order of 5μA. This might force too large an R_{ref} in the above bipolar V_{BIAS} generator.

Ex.





i.e., larger emitter area: ↺ Layout like this to get more accurate ratio.

If Q_1 is 10X larger than Q_2.

$\therefore I_{S1} = 10 I_{S2} \rightarrow I_o \cong I_{ref}/10$

$\therefore I_o = \dfrac{(V_{cc} - V_{BE(on)})}{10 R_{ref}} \longrightarrow R_{ref} = \dfrac{V_{cc} - V_{BE(on)}}{10 I_o}$

this helps to lower R_{ref}, but is it enough?

Ex. $I_o = 5\mu A$, $V_{cc} = 30V$

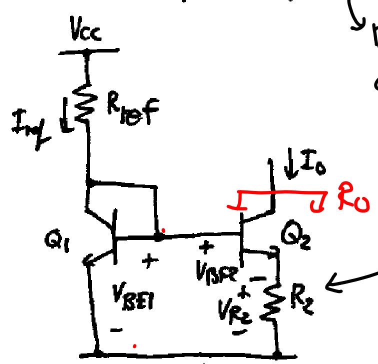
$R_{ref} \cong \dfrac{30}{5\mu} = \underline{600k\Omega}$ ← That's way too big!

(Yes, there's only one of them on the chip, but this takes up too much space!)

The Low Current Solution: $\boxed{\text{Widlar Current Source}}$

⇒ scale $I_{C2} = I_o$ by reducing V_{BE2} (relative to V_{BE1}):

Do this by emitter degenerating Q_2 via R_2



$V_{BE1} = V_{BE2} + V_{R_2} = V_{BE2} + \frac{1}{\alpha} I_{C2} R_2 \approx V_{BE2} + I_{C2} R_2$

$I_{C2} R_2 = V_{BE1} - V_{BE2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}}$

$I_{C2} R_2 = V_T \ln \frac{I_{C1}}{I_{C2}}$ [Assuming Q_1 & Q_2 are matched.]

$$\boxed{I_o R_2 = V_T \ln \frac{I_{ref}}{I_o}}$$

Rule of Thumb:

$V_{R_2} = I_{C2} R_2$	$I_{C2} = I_o$
18mV	$\frac{1}{2} I_{ref}$
42mV	$\frac{1}{5} I_{ref}$
60mV	$\frac{1}{10} I_{ref}$
120mV	$\frac{1}{100} I_{ref}$

5μA example again

Ex. scale by 100X using Widlar source

$V_{BE1} - V_{BE2} = 120mV \longrightarrow R_2 = \dfrac{120mV}{5\mu A} = \underline{24k\Omega}$

$I_{ref} = 500\mu A \rightarrow R_{ref} = \dfrac{30V}{500\mu A} = \underline{60k\Omega}$

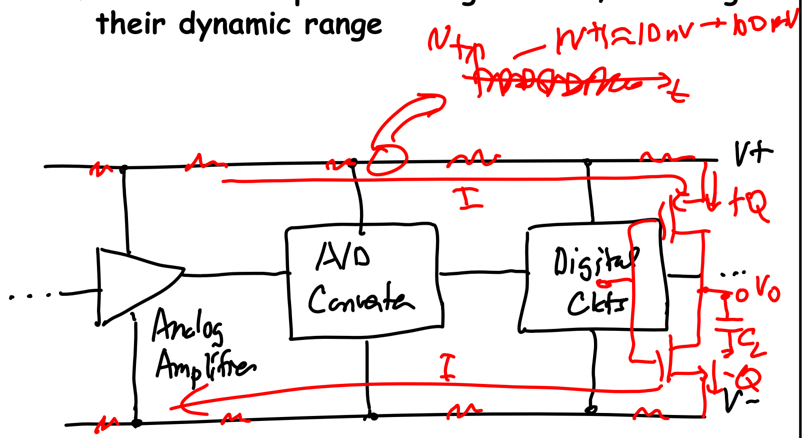
More reasonable than 600kΩ before.
If want smaller, scale by 1000X instead.

Another advantage of the Widlar: larger R_o ∴ a more ideal current source:

$R_o = r_{o2}(1 + g_{m2} R_2)$

Supply & Temperature Independent Biasing

- Why is it necessary?
- For battery-operated systems, battery voltages vary over time
  - Amplifier gains change
  - Power consumption changes
  - Frequency of oscillators changes
  - In summary: long-term stability degrades
  - Large uncertainty in biasing translates to overdesign that wastes power
- Same issues as above when temperature varies with time
- Short-term supply variations
  - In mixed signal circuits, i.e., both analog and digital together, digital switching generates noise on the supply lines
  - Noise can couple to analog circuits, reducing their dynamic range

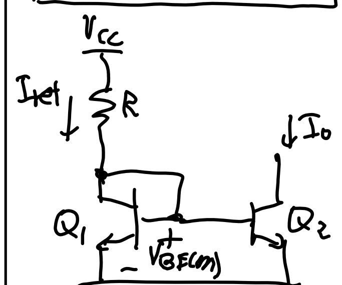


Definition. Sensitivity of Y to X

$$S_x^y = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{x}{y}\frac{\Delta y}{\Delta x} = \frac{x}{y}\frac{\partial y}{\partial x}$$

For supply independence, we want $S_{V_{CC}}^{I_o} = 0$.

Simple Current Source



Neglecting base currents:

$$I_o \approx I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R}$$

$$I_o \approx \frac{V_{CC}}{R} \quad [V_{CC} \gg V_{BE(on)}]$$

Thus:

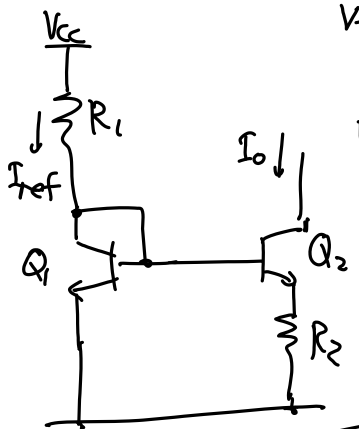
$$S_R^{I_o} = \frac{R}{I_o}\frac{\partial I_o}{\partial R} = \frac{R^2}{V_{CC}}\left(-\frac{V_{CC}}{R^2}\right) \Rightarrow S_R^{I_o} = -1$$

$$S_{V_{CC}}^{I_o} = \frac{V_{CC}}{I_o}\frac{\partial I_o}{\partial V_{CC}} = R\left(\frac{1}{R}\right) \Rightarrow S_{V_{CC}}^{I_o} = 1$$

∴ 10% change in V_{CC} leads to 10% change in I_o!

(terrible!)

$\boxed{\text{Widlar Current Source}}$ (Any better?)



$$V_T \ln\frac{I_{ref}}{I_o} = I_o R_2$$

$$V_T(\ln I_{ref} - \ln I_o) = I_o R_2$$

Differentiate w.r.t. V_{cc}:

$$V_T\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}} - \frac{1}{I_o}\frac{\partial I_o}{\partial V_{cc}}\right) = R_2\frac{\partial I_o}{\partial V_{cc}}$$

↓ rearrange

$$\frac{\partial I_o}{\partial V_{cc}} = \frac{\frac{V_T}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}}{\left(R_2+\frac{V_T}{I_o}\right)}$$

$$\therefore\ S^{I_o}_{V_{cc}} = \frac{V_{cc}}{I_o}\frac{\partial I_o}{\partial V_{cc}} = \frac{V_T\underbrace{\left(\frac{V_{cc}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}\right)}_{S^{I_{ref}}_{V_{cc}}}}{I_oR_2+V_T}$$

$$\Rightarrow \boxed{S^{I_o}_{V_{cc}} = \left(\frac{1}{1+\frac{I_oR_2}{V_T}}\right)S^{I_{ref}}_{V_{cc}}}$$

Since $I_{ref} = \frac{V_{cc}-V_{BE(on)}}{R_1} \simeq \frac{V_{cc}}{R_1} \Rightarrow S^{I_{ref}}_{V_{cc}} \approx 1$

$$\therefore\ \boxed{S^{I_o}_{V_{cc}} = \frac{1}{1+\frac{I_oR_2}{V_T}}}$$

For $I_{ref}=1mA$, $I_o=10\mu A$, $R_2=11.9k\Omega$, then

10% Δ in V_{cc} ⟶ 1.39% Δ in I_o

(better than a simple current source)

How can we do better? → use another voltage reference!

✓ ① $V_{BE(on)}$ → base emitter junction voltage

② V_Z → Zener diode

✓ ③ V_t → threshold voltage (MOS)

✓ ④ $V_T = \frac{kT}{q}$ → thermal voltage

✓ ⑤ E_g → bandgap