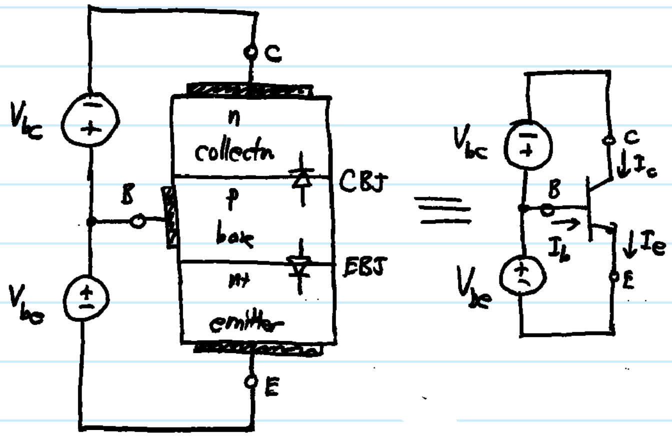
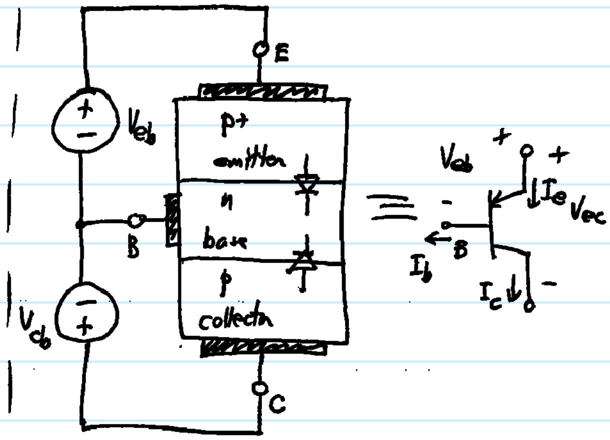
Modeling the Bipolar Junction Transistor (BJT)

⇒ physically, BJT's are just back-to-back pn junctions

npn bipolar Xsistor



pnp bipolar Xsistor



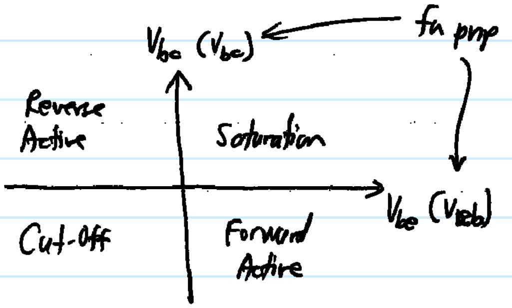
Regions of Bipolar Xsistor Operation

→ Key: R ≜ reverse-biased
F ≜ forward-biased

EBJ	CBJ	Mode
R	R	Cut-off (both diodes off)
F	R	Forward Active (widely used in analog amplifier ckts)
R	F	Reverse Active
F	F	Saturation

⇒ can also think of this in a convenient graphical sense:

→ for npn (pnp):



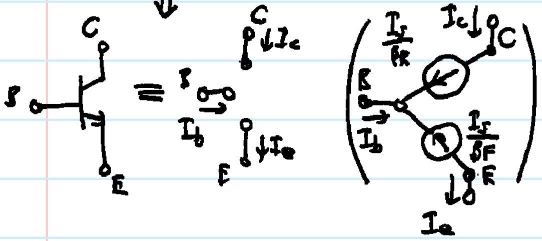
① Cut-Off Region – (npn transistor)

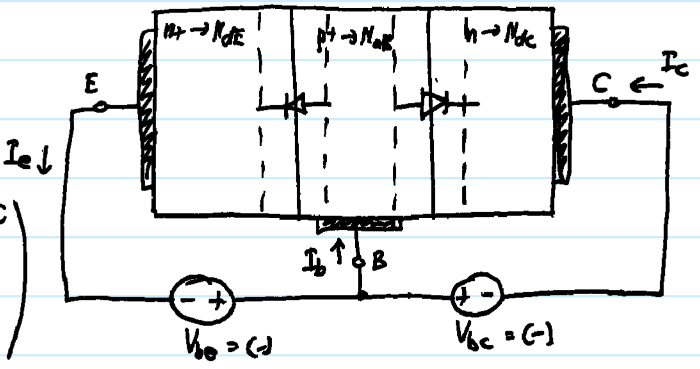
⇒ both diodes reversed-biased

⇒ no current flows:

$I_b=0, I_c=0, I_e=0$

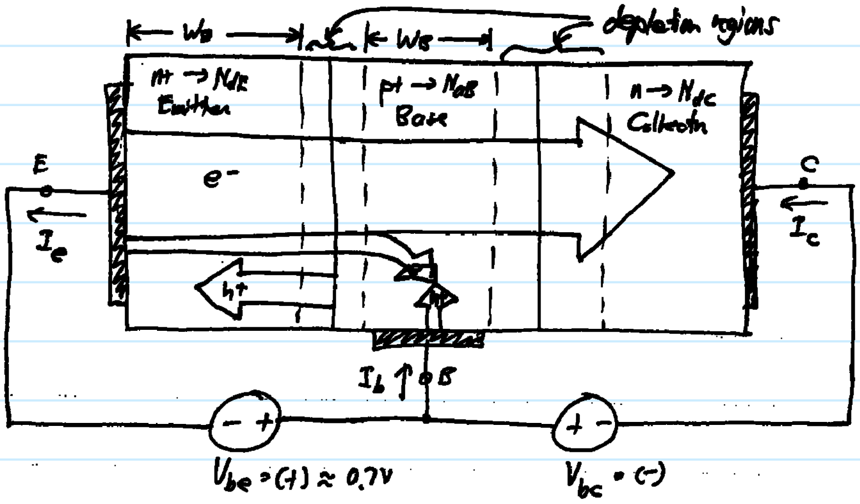
⇓

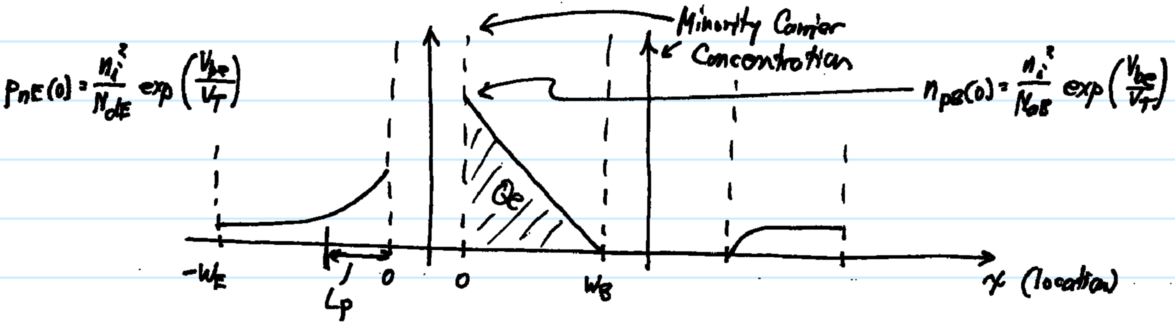




② Forward-Active Region - (npn transistor)

⇒ BEJ Forward-Biased (i.e., diode on), BCJ Reverse-Biased (i.e., diode off)





Forward biasing of the BEJ generates three current components:

① e^-'s injected from emitter to base : $I_{nB} = -AJ_{nB}^{diff}$

② h^+'s injected from base to emitter : $I_{pE} = AJ_{pE}^{diff}$

③ recombination of e^-'s & h^+'s in base : I_{rB}

$I_c = I_{nB} = ①$

$I_e = I_{nB} + I_{pE} + I_{rB} = ① + ② + ③$

$I_b = I_{pE} + I_{rB} = ② + ③$

$$I_{nB} = -AJ_{nB}^{diff} = -Aq\underset{\text{diffusion constant for } e^-\text{'s in } B}{D_{nB}}\underbrace{\frac{dn_p(x)}{dx}}_{\text{slope}} = -qAD_{nB}\frac{[n_{pB}(W_B) - n_{pB}(0)]}{W_B} = \boxed{qAD_{nB}\frac{n_i^2}{N_{aB}W_B}\exp\left(\frac{V_{be}}{V_T}\right) = ①} \;*$$

$$\left[\begin{array}{l} n_{pB}(W_B) = \frac{n_i^2}{N_{aB}}\exp\left(\frac{V_{bc}}{V_T}\right) \approx 0 \\ n_{pB}(0) = \frac{n_i^2}{N_{aB}}\exp\left(\frac{V_{be}}{V_T}\right) \end{array}\right]$$

$$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right)$$

diffusion constant for h^+'s in E

$$I_{pE} = AJ_{pE}^{diff} = Aq D_{pE}\underbrace{\frac{dp_n(x)}{dx}}_{\text{slope}} = qAD_{pE}\frac{[p_{nE}(0) - p_{nE}(-W_E)]}{W_E} = \boxed{qAD_{pE}\frac{n_i^2}{N_{dE}W_E}\exp\left(\frac{V_{be}}{V_T}\right) = ②} \;*$$

$$\left[\begin{array}{l} p_{nE}(0) = \frac{n_i^2}{N_{dE}}\exp\left(\frac{V_{be}}{V_T}\right) \\ p_{nE}(-W_E) \approx 0 \end{array}\right]$$

could also replace by diffusion length, L_p (for h^+ in n-type material) minority-carrier charge in base

$$I_{rB} = \frac{Q_e}{\tau_b} = \frac{1}{\tau_b}\left[\frac{1}{2} n_{pB}(0) W_B qA\right] = \boxed{\frac{1}{2}\frac{n_i^2 W_B qA}{N_B \tau_b}\exp\left(\frac{V_{be}}{V_T}\right) = ③} \quad *$$

minority carrier lifetime in base

Define Forward Current Gain = β_F:

$$\beta_F = \frac{I_c}{I_b} = \frac{①}{③+②} = \frac{\frac{qAD_{nB}n_i^2}{N_{aB}W_B}}{\frac{1}{2}\frac{n_i^2 W_B qA}{N_{aB}\tau_b} + \frac{qAD_{pE}n_i^2}{N_{dE}\underset{L_p}{W_E}}} = \left[\frac{W_B^2}{2\tau_b D_{nB}} + \frac{D_{pE}W_B \overset{N_{aB}}{N_A}}{D_{nB}\underset{L_p}{W_E}\underset{N_{dE}}{N_D}}\right]^{-1}$$

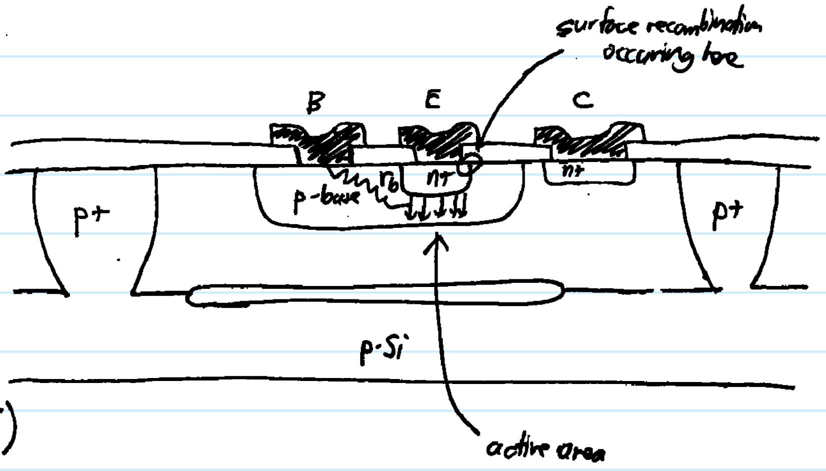
To maximize β_F, want:
① W_B = small
② $N_{dE} \gg N_{aB}$ (this is why emitter is n+) → also leads to $D_{pE} \ll D_{nB}$ which we also want
③ τ_b = large (base Si must be free of impurities/defects to prevent recombination)

More Complete Expression for β_F:

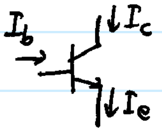
$$\beta_F = \underbrace{\frac{N_{aB}W_B}{D_{nB}}\cdot\frac{D_{pE}}{N_{dE}L_{pE}}}_{\text{Injection Efficiency}} + \underbrace{\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}_{\text{Volume Recombination}} + \underbrace{s\left(\frac{A_s}{A_E}\right)\left(\frac{W_B}{D_{nB}}\right)}_{\text{Surface Recombination}} + \underbrace{\frac{W_E N_{aB} W_B}{2D_{nB}\tau n_i} e^{-\frac{V_{be}}{2V_T}}}_{\text{Recombination in the BE Depletion Region}}$$

Recombination in the BE Depletion Region ← significant @ low current levels

where:
s = Surface Recombination Velocity
D_i = Diffusion constant
n_i = intrinsic carrier concentration
N_i = carrier concentration
A_E = total emitter area
A_s = sidewall emitter area
τ = minority carrier lifetime
L_i = diffusion length ($L_i = \sqrt{D_i \tau}$)
W_B = active base width



So β relates I_b & I_c. To relate I_c & I_e, use KCL:

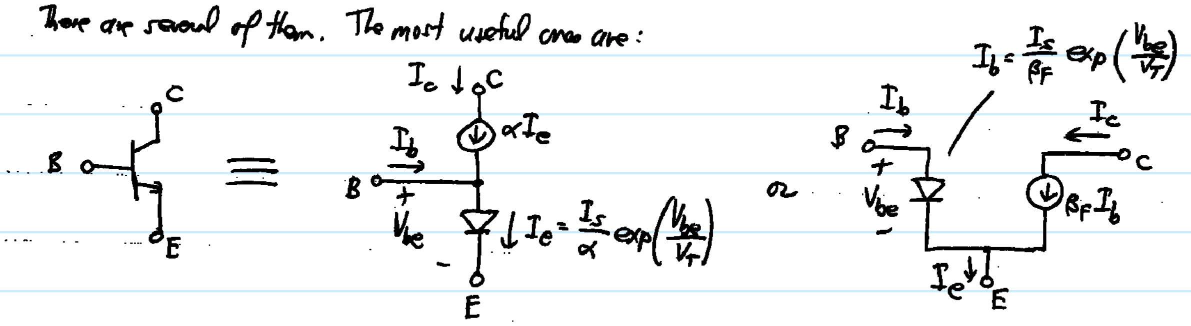


$$I_e = I_c + I_b = I_c + \frac{I_c}{\beta} = \left(1+\frac{1}{\beta}\right) I_c$$

$$\Rightarrow I_c = \left(\frac{1}{1+\frac{1}{\beta}}\right) I_e = \left(\frac{\beta}{\beta+1}\right) I_e = \alpha I_e, \text{ where } \boxed{\alpha = \frac{\beta}{\beta+1}} \Rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha}}$$

Equivalent Large Signal Ckt. Models for Forward-Active BJT's

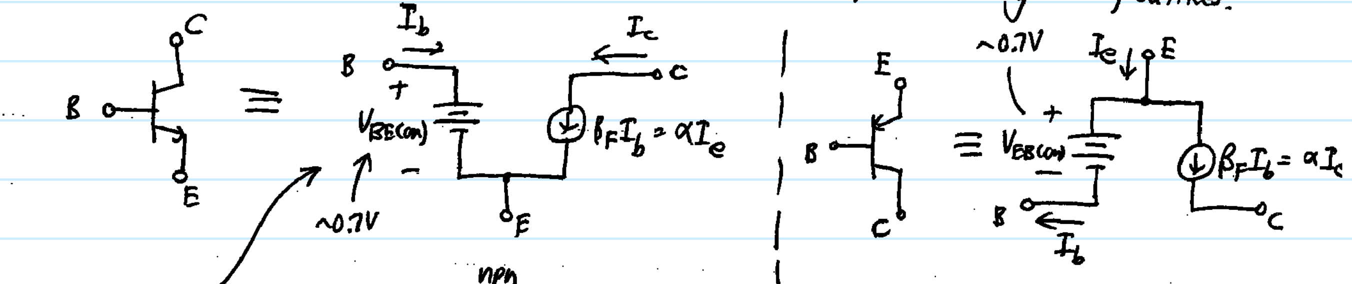
There are several of them. The most useful ones are:



Common Base (CCCS)

Common Emitter (CCCS)

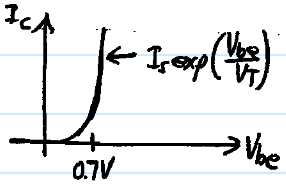
But usually one doesn't have to use those complicated models. Rather, the following usually suffices:



npn

pnp

Just as in a diode:



You should already be used to using approximate models like this

⇒ the more complicated models are a waste of time in comparison

③ Reverse-Active Region –

⇒ very similar to forward-active region except now: BEJ reverse-biased

BCJ forward-biased

⇒ one important difference: $\beta_R \propto \frac{N_{dC}\overset{L_{pC}}{\cancel{W_B}} D_{nB}}{N_{aB} W_B D_{pC}} \rightarrow N_{dC} \ll N_{aB} \rightarrow D_{nB} \ll D_{pC}$ since collector is n−

∴ β_R is much smaller than β_F

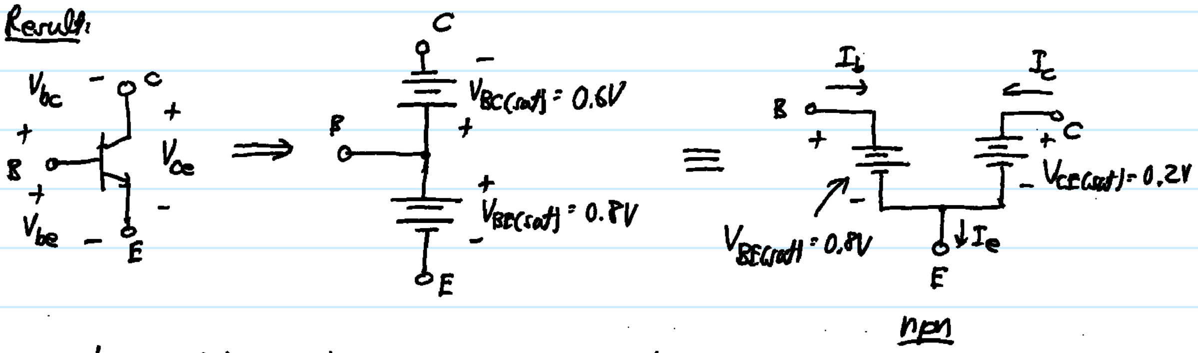
⇒ poor device performance

④ Saturation Region:

BEJ forward-biased → $V_{BE(on)}^{sat} \sim 0.8V$ (higher than 0.7V in saturation)

BCJ forward-biased → $V_{BC(on)}^{sat} \sim 0.6V$

Result:



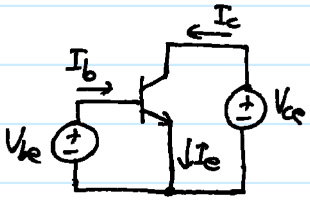
⇒ currents now determined by the attached elements & KCL:

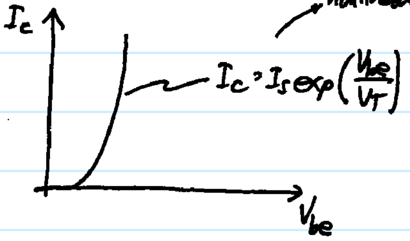
$$I_e = I_b + I_c \quad ; \text{ no longer have } \underbrace{I_b = \frac{I_c}{\beta} \text{ or } I_c = \alpha I_e}_{\text{These no longer apply when BJT is in saturation.}}$$

When determining DC operating point:

Pass
① Assume forward-active → check for cut-off (enough V_{be}?)
② Determine V_{ce}.
③ If $V_{ce} > V_{CE(sat)} = 0.2V$, then ok (i.e., it's forward-active) ... otherwise, must do the analysis over assuming saturation.

IV Characteristics of Bipolar Junction Transistors



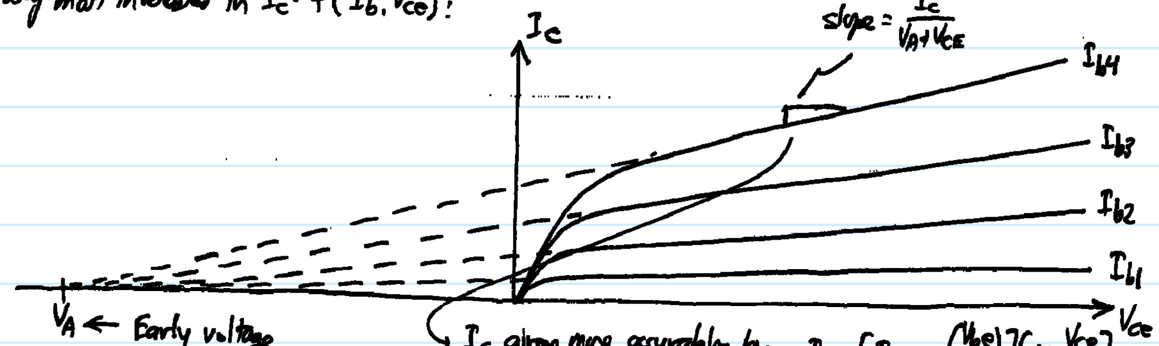


$I_c = I_s \exp\left(\frac{V_{be}}{V_T}\right)$ → nonlinear → not easy to work with since we can't use linear system theory → thus need to linearize!

(a diode-like characteristic)

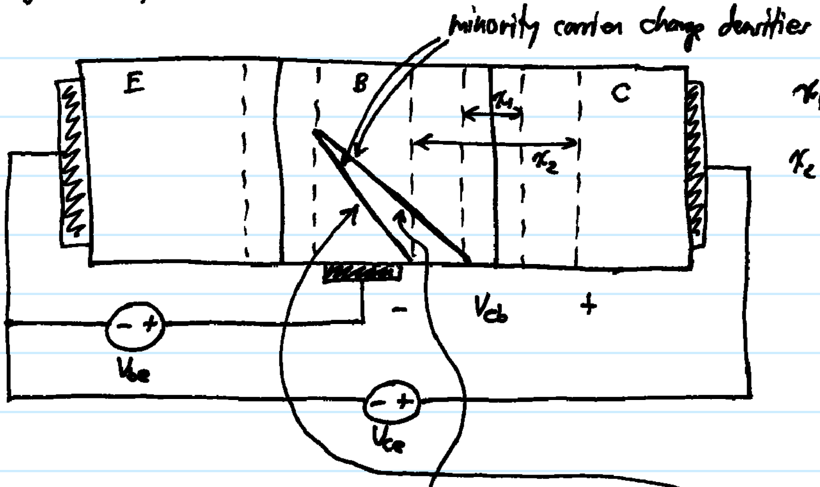
digital ckts. → piecewise lin.
analog ckt. → small-signal models

⇒ really most interested in $I_c = f(I_b, V_{ce})$:



I_c given more accurately by: $I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left(1 + \frac{V_{ce}}{V_A}\right)$

What is happening physically?



$x_1 \triangleq$ depl. region width for $V_{ce} = V_{ce1}$

$x_2 \triangleq$ depl. region width for $V_{ce} = V_{ce2} > V_{ce1}$

① Case: $V_{ce} = V_{ce1} \rightarrow x_1 \rightarrow I_{c1} \propto$ slope of this line

② Now, increase $V_{ce1} \rightarrow V_{ce2} \rightarrow V_{cb}\uparrow \rightarrow x\uparrow$ to $x_2 \rightarrow I_{c2} \propto$ slope of this line

$\therefore I_{c2} > I_{c1}$

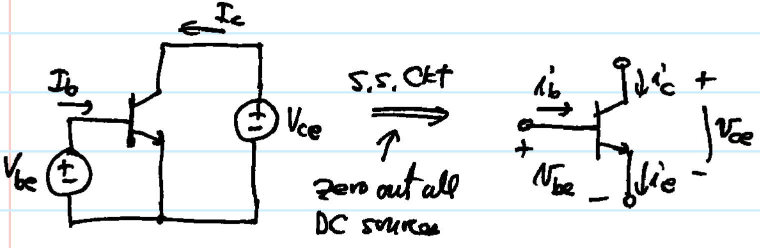
Thus, $V_{ce}\uparrow \rightarrow I_c\uparrow$ due to $x_{depl.}\uparrow$

Result: $I_c = f(I_b, V_{ce})$ in forward-active!

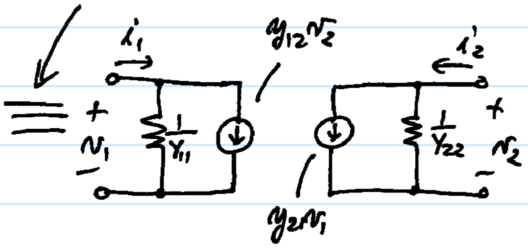
$$\boxed{I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1 + \frac{V_{ce}}{V_A}\right]}$$

← This, then, is a more accurate I_c equation.

Small-Signal Models for Forward-Active Bipolar Xsistors



Y-Parameter Model

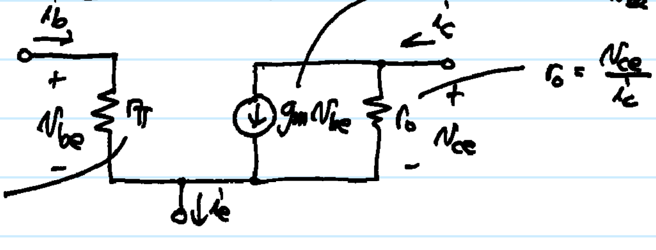


$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$ $y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$

$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$ $y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$

If only interested in the forward direction

Hybrid-π Model



$g_m = \frac{i_c}{v_{be}}$

$r_o = \frac{v_{ce}}{i_c}$

$r_\pi = \frac{v_{be}}{i_b}$

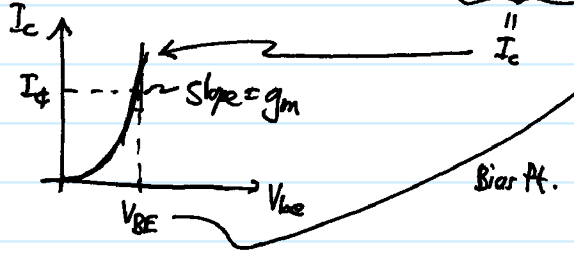
} Specified by the bias point.

Determine the S.S. elements:

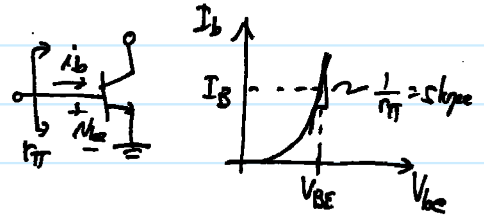
$$g_m = \frac{i_c}{v_{be}} = \frac{\partial I_c}{\partial v_{be}}\Big|_{Q\text{-pt.}} = \frac{\partial}{\partial v_{be}}\Big[\underbrace{I_s \exp\left(\frac{v_{be}}{V_T}\right)}_{= I_c}\Big]\Big|_{v_{be}=V_{BE}} = \frac{I_s}{V_T}\exp\left(\frac{V_{BE}}{V_T}\right) \Longrightarrow \boxed{g_m = \frac{I_C}{V_T}}$$

$I_s \exp\left(\frac{V_{BE}}{V_T}\right) = I_C$

Note: function of the DC operating pt.



$r_\pi = \frac{v_{be}}{i_b}$:

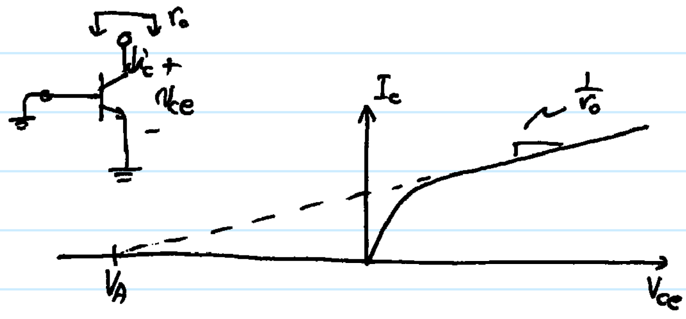


$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{g_m} = \frac{\beta}{\frac{I_C}{V_T}} = \frac{V_T}{I_B}$$

$$\therefore \boxed{r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

← Again, function of the DC operating pt.

$r_o = \frac{v_{ce}}{i_c}$:



$$r_o = \frac{\partial V_{ce}}{\partial I_c}\Big|_Q = \left[\frac{\partial I_c}{\partial V_{ce}}\Big|_{Q\text{-pt.}}\right]^{-1}$$

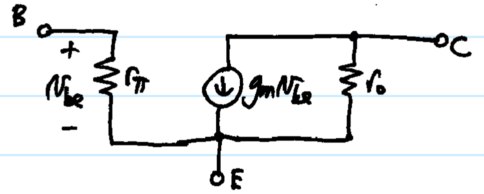
$$= \left[\frac{\partial}{\partial V_{ce}}\left(\left[I_s\exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1+\frac{V_{ce}}{V_A}\right]\right)\Big|_{V_{be}=V_{BE}}\right]^{-1}$$

$$= \left[\frac{I_s\exp\left(\frac{V_{BE}}{V_T}\right)}{V_A}\right]^{-1} = \left[\frac{I_C}{V_A+V_{CE}}\right]^{-1} = \frac{V_A+V_{CE}}{I_C}$$

$I_s\exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_C}{1+\frac{V_{CE}}{V_A}}$

$$\therefore \boxed{r_o = \frac{V_A+V_{CE}}{I_C} \underset{[V_A \gg V_{CE}]}{\approx} \frac{V_A}{I_C}}$$

... and thus, we have the hybrid-π model:



SPICE: BF

$$\boxed{\begin{aligned} r_\pi &= \frac{\beta}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{V_A + V_{CE}}{I_C} \approx \frac{I_C}{V_A} \end{aligned}}$$

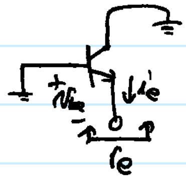
SPICE: VAF

Remarks:

① g_m is independent of device specifics (i.e., β, I_s); depends only on temperature (thru V_T) and biasing I_C → quite different from MOS, as we'll see

② small-signal model valid for $v_{be} \ll V_T$ ← $\approx$ 26mV @ 300K

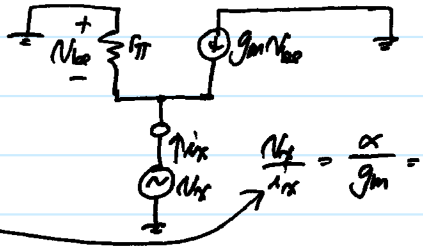
What about emitter resistance?



$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\Rightarrow \boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

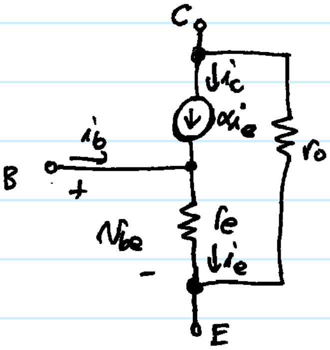
Note that although it's not explicitly shown in the hybrid-π model, r_e is present.
⇒ i.e., if you analyze this, you find that



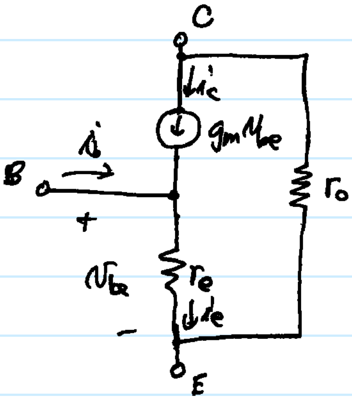
$$\frac{v_x}{i_x} = \frac{\alpha}{g_m} = r_e \checkmark$$

To explicitly show emitter resistance, use the T-model:

T-Model: (Common Base Model)



or



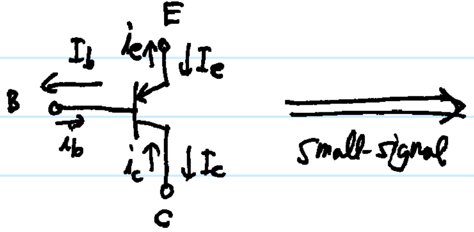
where as before:

$$g_m = \frac{I_C}{V_T}$$

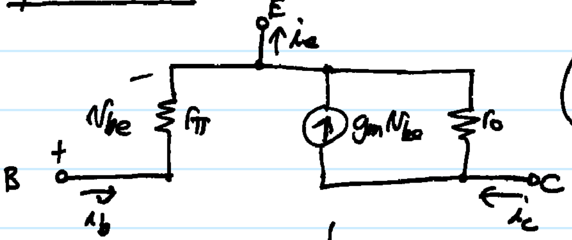
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

$\boxed{\text{Small-Signal Models for forward-active pnp Transistors}}$

For pnp transistors, use the same small-signal models as npn with no change in relative polarities!



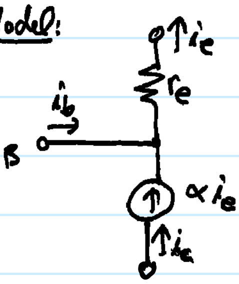
Hybrid-π Model:



(same as npn model … just upside-down)

Note that in these S.S. models, the same current directions as used for npn are used too
⇒ i.e., no change in S.S. polarities
(large-signal directions, however, can be as before)

T-Model:



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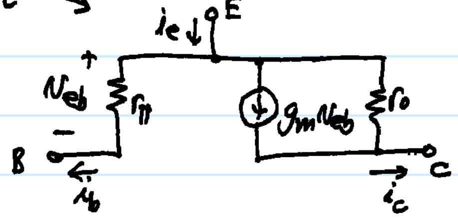
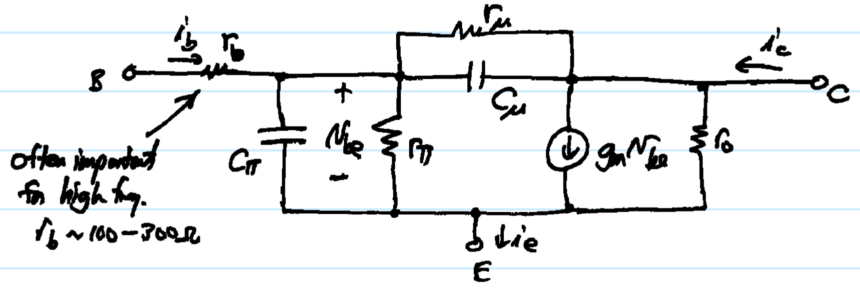
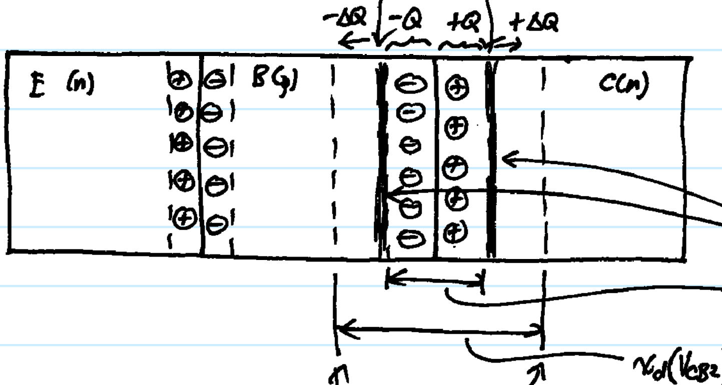


More Complete Hybrid-Π Model: (adding frequency effects & 2nd order effects)



C_μ – Base-to-Collector Capacitance

charge modulates here ∴ these are the plates of a capacitor → C_μ



S.S.

if have $V_{cb1} = V_{CB1} + v_{cb1}$, then ∂q gets modulated over these small regions ⇒ thus, these are effectively the plates of a capacitor!

$x_d(V_{CB1})$

$x_d(V_{CB2})$, where $V_{CB2} > V_{CB1}$

if change to a different bias V_{CB2}, get new capacitor plate locations → $C_\mu \sim \frac{\epsilon_s}{x_d(V_{CB2})}$

$C_\mu = f(V_{CB})$!

$$\boxed{C_\mu = \frac{C_{\mu 0}}{\sqrt{1+\frac{V_{CB}}{\phi_j}}}}$$

$= f(V_{CB})$

where $C_{\mu 0}$ = capacitance for $V_{CB} = 0$

ϕ_j = function of the built-in potential between p and n-type semiconductors $\Big\} = \frac{kT}{q} \ln\left(\frac{N_{aB} N_{dC}}{n_i^2}\right)$

n_i: $1.45 \times 10^{10}\,\text{cm}^{-3}$

In general:

In SPICE:
CJC
VJC
MJC

$$C_\mu = \frac{C_{\mu 0}}{\left(1+\frac{V_{CB}}{\phi_j}\right)^m}$$, where $m = \frac{1}{2}$ or $\frac{1}{3}$ depending upon how abrupt the junction is

Detailed Derivation: [FYI]

$$x_d \cong x_a = \left[\frac{2\epsilon(\psi_0+V_{CB})}{qN_A\left(1+\frac{N_A}{N_D}\right)}\right]^{1/2} \longrightarrow Q = qN_A A x_d = A\left[\frac{2\epsilon q N_A(\psi_0+V_{CB})}{1+\frac{N_A}{N_D}}\right]^{1/2}$$

$[N_A \ll N_D]$

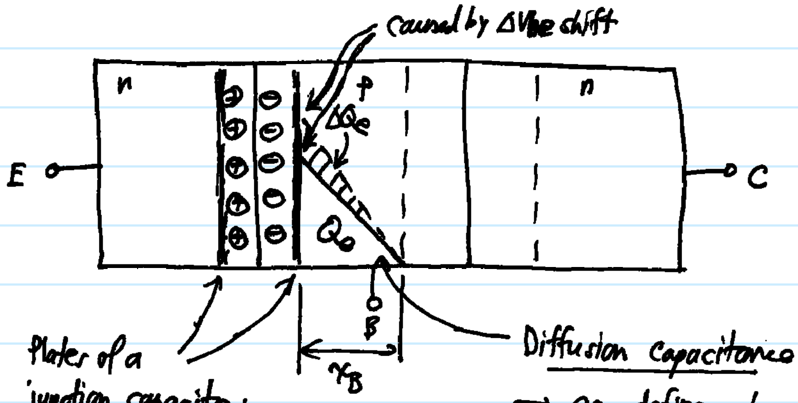
$$C_j = \frac{dQ}{dV_{CB}}\bigg|_{V_{CB}} = \left[\frac{2\epsilon q N_A}{1+\frac{N_A}{N_D}}\right]^{1/2} \frac{A}{2}(\psi_0+V_{CB})^{-1/2} = \boxed{A\left[\frac{q\epsilon N_A N_D}{2(N_A+N_D)}\right]^{1/2} \frac{1}{\sqrt{\psi_0+V_{CB}}} = C_j\big|_{V_{CB}}}$$

$$C_j = \frac{\epsilon_s A}{x_d(V_{CB})}$$

C_π – Base-to-Emitter Capacitance

Two components comprise C_π: ① Junction capacitance, C_{je}

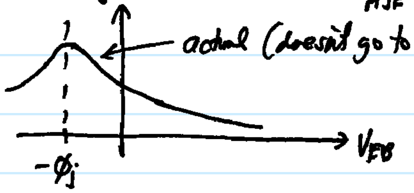
② Diffusion capacitance, C_b



Plates of a junction capacitor: → bias level determines where the plates are

$$C_{je} = \frac{C_{jeo}}{\left(1 + \frac{V_{EB}}{\varphi_0}\right)^m}$$

In SPICE: CJE, VJE, MJE



$C_\pi = C_b + C_{je}$ ($C_{je} \approx 2C_{jeo}$)

$$\boxed{C_\pi = \tau_F g_m + \frac{C_{jeo}}{\left(1 + \frac{V_{EB}}{\varphi_0}\right)^m}}$$

Diffusion capacitance: (or Base Charging Capacitance)

⇒ can define a base transit time:

$$\tau_F = \frac{Q_e}{I_c} = \frac{x_B^2}{2D_n}$$ } avg. time spent by carrier in crossing base

think of I_c as the rate of xsfer of charge through the base

$Q_e = \tau_F I_c$

$\Delta Q_e = \tau_F \Delta I_c$

Switch to S.S. parameters (variables):

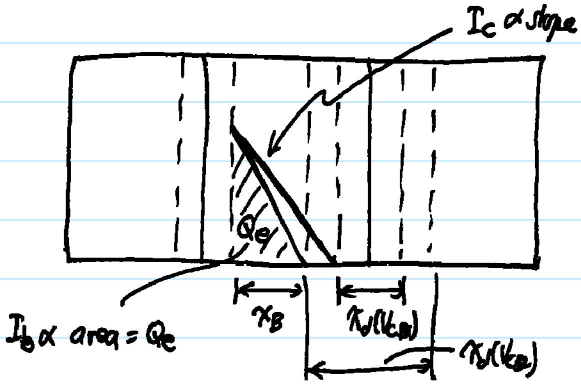
$q_e = \tau_F i_c$

$$q_e = C_b v_{be} \rightarrow C_b = \frac{q}{v_{be}} = \tau_F \frac{i_c}{v_{be}} = \boxed{\tau_F g_m = \tau_F \frac{I_C}{V_T} = C_b}$$

SPICE: TF

$\therefore \boxed{C_b \propto I_C}$

Collector-to-Base Feedback Resistor, r_μ

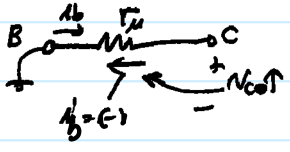


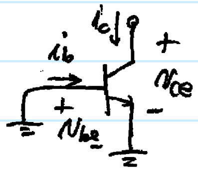
Remember, recombination base current $I_{rB} = \frac{Q_e}{\tau_b}$!

$\therefore v_{ce}\uparrow \rightarrow x_B\downarrow \rightarrow Q_e\downarrow \rightarrow i_b\downarrow$

$\searrow i_c\uparrow$ (due to Early effect)

$v_{ce}\uparrow \rightarrow i_b\downarrow$ can be modeled by an r_μ connecting C-to-B





⇒ here, $v_{be}=0 \rightarrow v_{ce}=v_{cb}$

$$\therefore \frac{i_c}{v_{ce}} = \frac{1}{r_o} = \frac{i_c}{v_{cb}} = \frac{\beta i_b}{v_{cb}} \rightarrow \frac{v_{cb}}{i_b} = \boxed{\beta r_o = r_\mu}$$

↑ assuming all of i_b is recombination current

In general, base recombination current is only part of the total base current and is the only component dependent on V_{bc} ⇒ thus, $\boxed{r_\mu > \beta_o r_o} \rightarrow r_\mu = 2\text{-}10\beta_o r_o$

↑ lateral pnp where base recomb. more significant; ↑ npn → I_b is 10% recomb.

Complete Forward-Active BJT S.S. Model (including parasitics)

⇒ Actual integrated BJT:

should draw this on the board

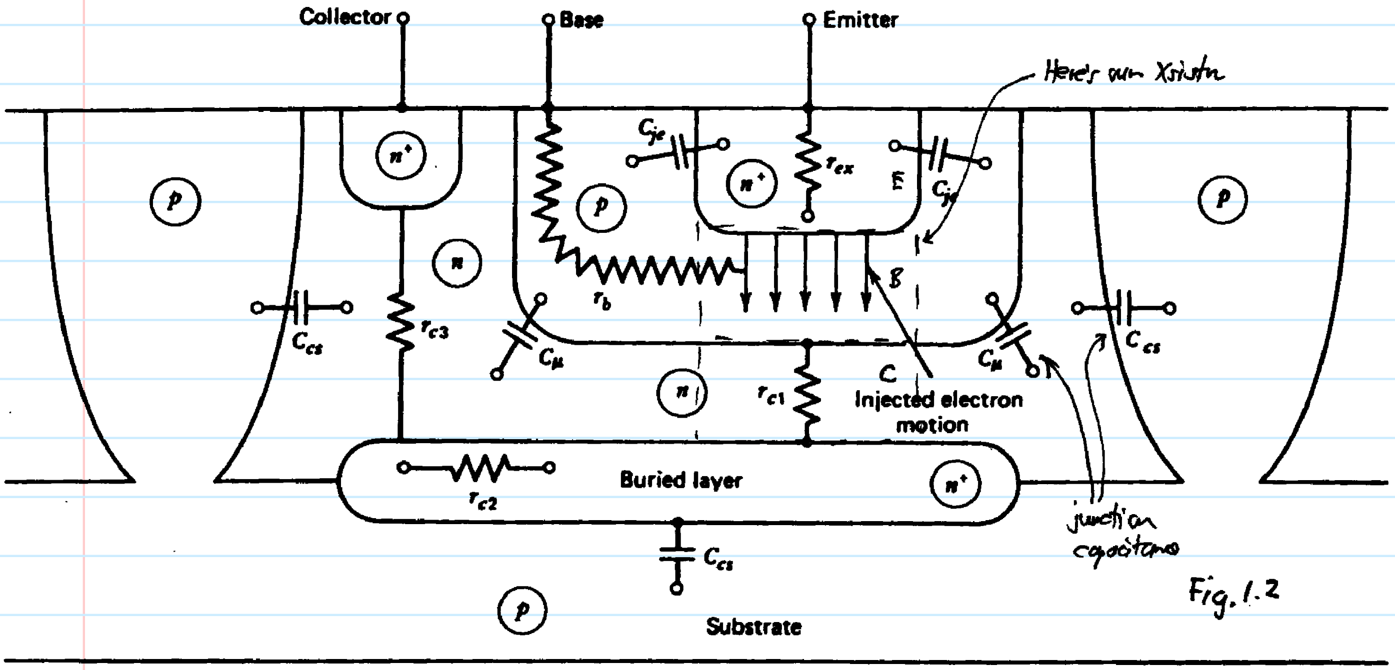
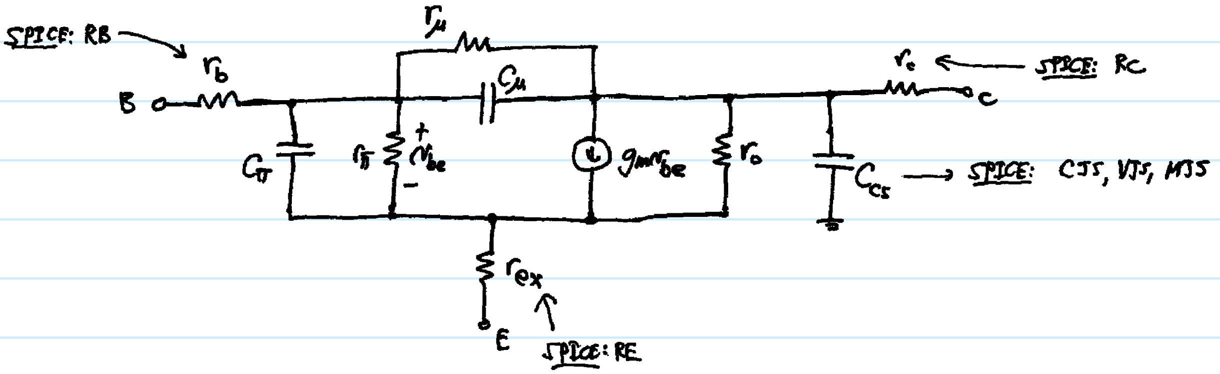
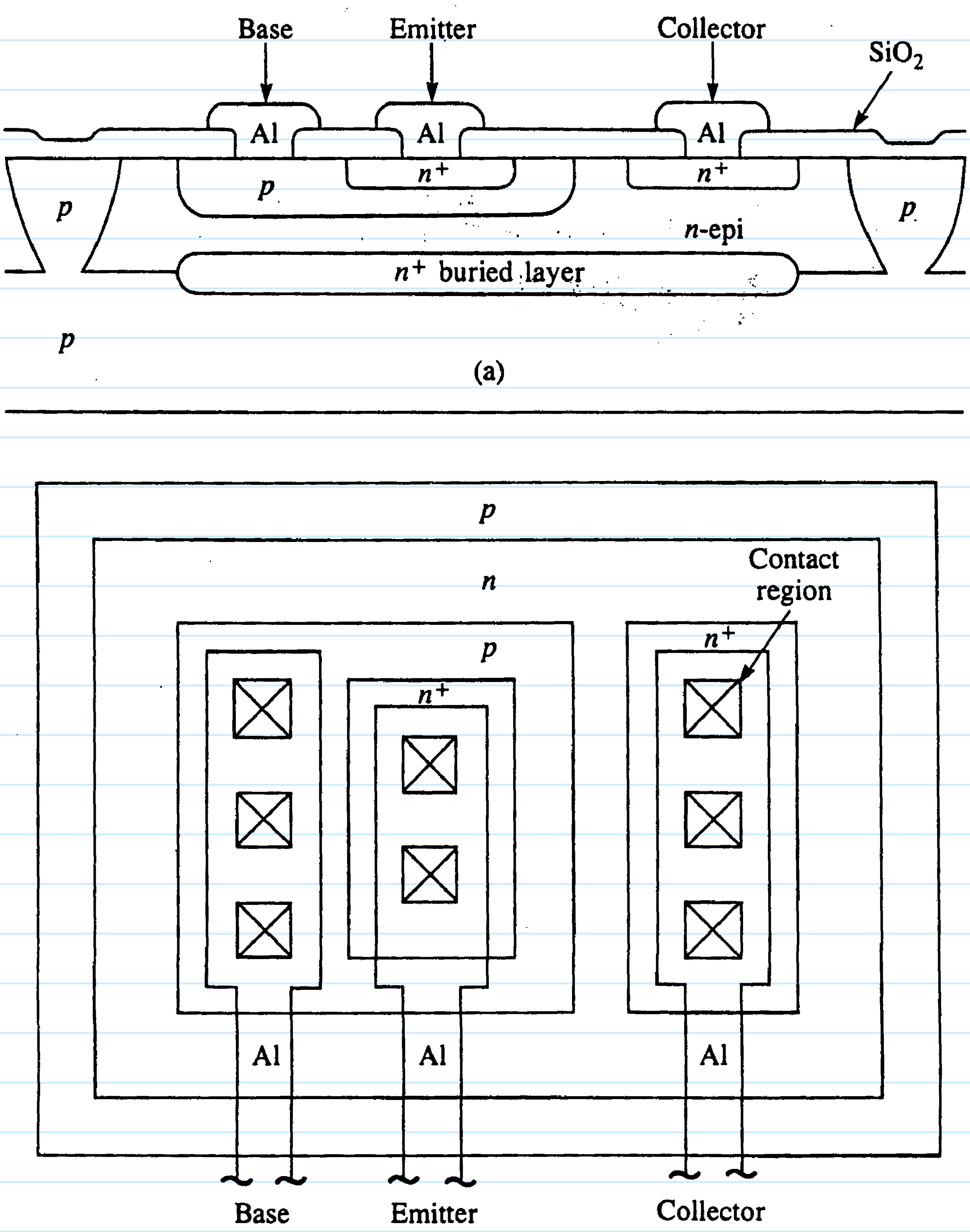


Fig. 1.2



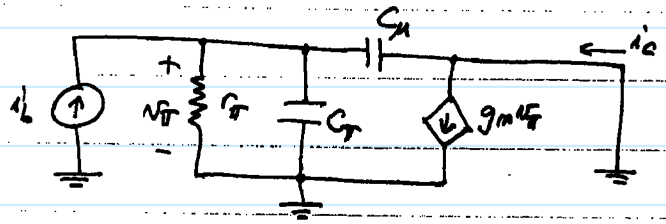

Base
Emitter
Collector
SiO$_2$
Al
Al
Al
n^+
n^+
p
p
p
n-epi
n^+ buried layer
p
p
n
p
Contact region
n^+
n^+
Al
Al
Al
Base
Emitter
Collector

(a)

Fig. 1.1

f_T (unity gain freq. for β)

Find $\beta(j\omega)$: (β as a function of freq.)



Find $\left.\frac{i_c}{i_b}\right|_{v_c=0}$:

$$v_\pi = i_b\left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right)$$

$$i_c = g_m v_\pi - sC_\mu v_\pi = (g_m - sC_\mu)v_\pi \approx g_m v_\pi$$

$[g_m >> sC_\mu]$

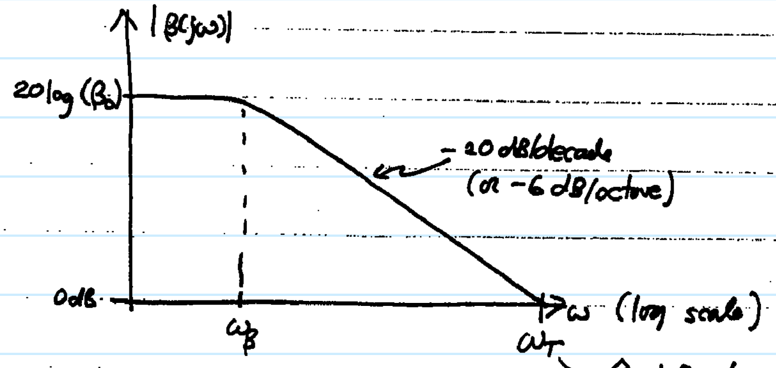
$$i_c = g_m\left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right) i_b$$

$$\frac{i_c}{i_b} = \frac{g_m}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)} = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)} = \frac{\beta_0}{1 + s r_\pi (C_\pi + C_\mu)}$$

$[\beta_0 = g_m r_\pi]$ (low freq. β)

$$\boxed{\beta(j\omega) = \frac{\beta_0}{1 + \frac{j\omega}{\omega_\beta}}} \qquad \boxed{\omega_\beta = \frac{1}{r_\pi (C_\mu + C_\pi)}}$$

Plot $|\beta(j\omega)|$: (Bode plot)



≙ defined as the radian frequency where $|\beta(j\omega)| = \beta_0$

For ω large: (e.g. ω close to ω_T)

$$|\beta(j\omega_T)| \approx \frac{\beta_0}{\omega_T r_\pi (C_\pi + C_\mu)} = 1 \longrightarrow \boxed{\omega_T = \frac{g_m}{C_\pi + C_\mu}} \Rightarrow f_T = \frac{\omega_T}{2\pi}$$

is a figure of merit for the frequency performance of a transistor.

f_T = 100 MHz → 15 GHz for bipolar Xsistors.

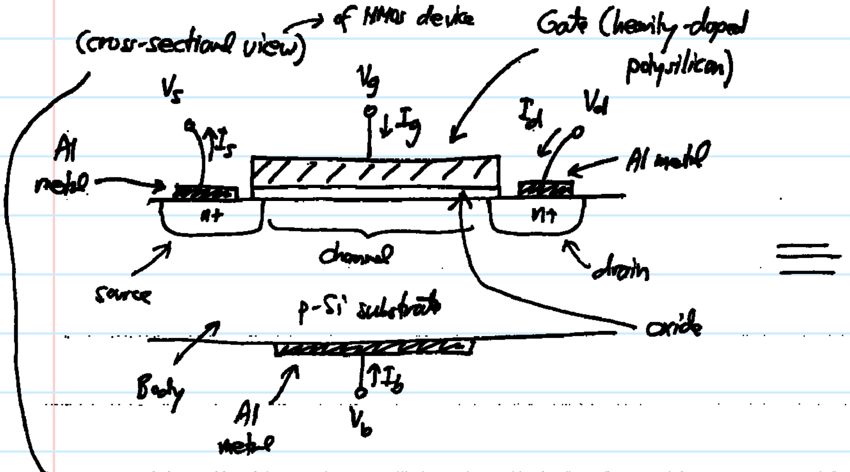
Also, note that $\boxed{\omega_T = \beta_0 \omega_\beta}$

$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$$

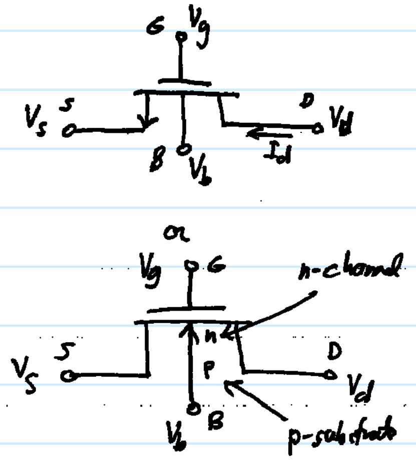
MOS Transistors

Physical Structure & Device Symbols –

(cross-sectional view) → of NMOS device

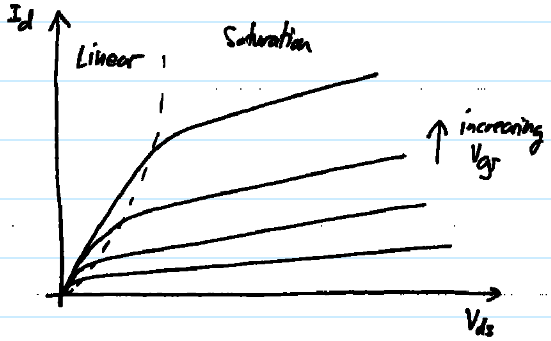


NMOS Xsistor Device Symbol

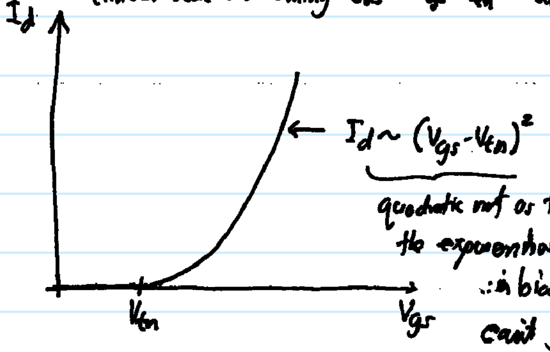


But first start w/ a perspective-view: (This also defines dimensions.)
↳ use the viewgraph on next page → pg. ⑭a

IV Characteristics (NMOS)

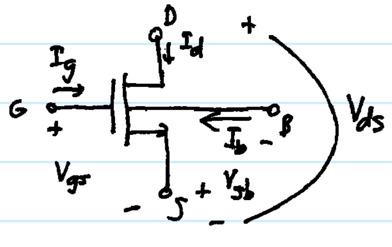


(linear scale → assuming $V_{ds} > V_{gs} - V_{tn}$ → saturation)



quadratic not as fast as the exponential of npn
∴ in biasing for MOS, can't just assume a $V_{GS(on)}$
↓
rather, must calculate!

NMOS Xsistor Mathematical Model



① Cut-Off Region: $(V_{gs} \leq V_t)$

$$I_g = I_b = 0 \ ; \ I_d = 0$$

② Linear (or Triode) Region: $(V_{gs} - V_{tn} \geq V_{ds} \geq 0)$

$$I_g = I_b = 0; \ I_d = \mu_n C_{ox} \frac{W}{L}\left(V_{gs} - V_{tn} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$= k_n\left(V_{gs} - V_{tn} - \frac{V_{ds}}{2}\right)V_{ds}$$

Body Factor → $\gamma = \frac{1}{C_{ox}}\sqrt{2q\underset{\uparrow}{\epsilon_s} N_{sub}}$ ← substrate doping conc.
permittivity in Si

③ Saturation Region: $(V_{ds} \geq V_{gs} - V_{tn} \geq 0)$

$$I_g = I_b = 0; \ I_d = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs} - V_{tn})^2(1 + \lambda V_{ds})$$

$$= \frac{1}{2}k_n(V_{gs} - V_{tn})^2(1 + \lambda V_{ds})$$

General:

$k_n = k_n' \frac{W}{L} = \mu_n C_{ox} \frac{W}{L}$

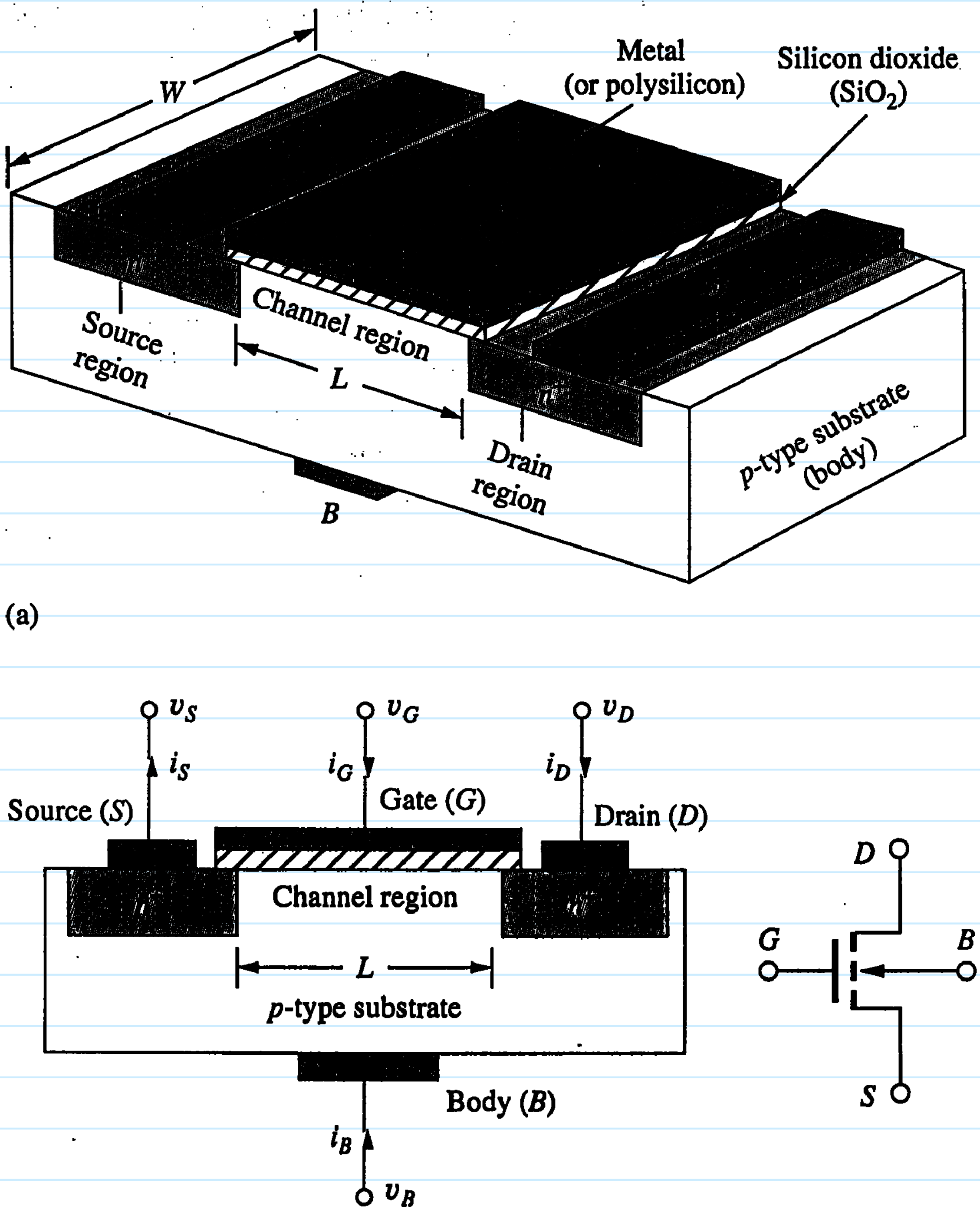
$I_g = I_b = 0$ for all regions (at least for dc)

$V_{tn} = f(V_{SB}) = V_{t0} + \gamma\left(\sqrt{V_{SB} + 2|\phi_F|} - \sqrt{2|\phi_F|}\right)$

$\mu_n \triangleq e^-$ mobility in the channel

$C_{ox} \triangleq$ gate oxide capacitance per unit area

MOS Transistor Structure



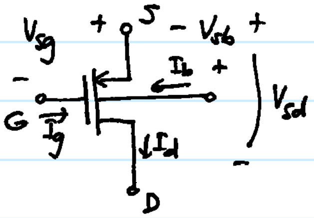
(a)

(b)

(c)

Fig. 2.1

PMOS Xistr Mathematical Model



① **Cut-Off Region:** $(V_{sg} \le -V_{tp})$ or $(|V_{gs}| \ge |V_{tp}|)$

$$I_{sd} = 0$$

② **Linear (or Triode) Region:** $(V_{sg} + V_{tp} \ge V_{sd} \ge 0$; or $|V_{gs}| - |V_{tp}| \ge |V_{ds}| \ge 0)$

$$I_{sd} = k_p\left(V_{sg} + V_{tp} - \frac{V_{sd}}{2}\right)V_{sd} = \mu_p C_{ox}\frac{W}{L}\left(V_{sg} + V_{tp} - \frac{V_{sd}}{2}\right)V_{sd}$$

$$= \mu_p C_{ox}\frac{W}{L}\left(|V_{gs}| - |V_{tp}| - \frac{|V_{ds}|}{2}\right)|V_{ds}|$$

③ **Saturation Region:** $(V_{sd} \ge V_{sg} + V_{tp} \ge 0;\ |V_{ds}| \ge |V_{gs}| - |V_{tp}| \ge 0)$

$$I_{sd} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{sg} + V_{tp})^2(1 + \lambda V_{sd}) = \frac{1}{2}k_p(V_{sg} + V_{tp})^2(1 + \lambda V_{sd})$$

$$= \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(|V_{gs}| - |V_{tp}|)^2(1 + \lambda|V_{ds}|)$$

$\mu_p \triangleq h^+$ mobility in the channel
$C_{ox} \triangleq$ gate oxide capacitance per unit area

For all regions:

$$k_p = k_p'\frac{W}{L} = \mu_p C_{ox}\frac{W}{L}$$

$I_g = 0$ and $I_b = 0$ (at dc)

$$V_{tp} = V_{to} - \gamma\left(\sqrt{V_{BS} + 2|\phi_f|} - \sqrt{2|\phi_f|}\right)$$

Threshold Voltage

$$\boxed{V_t = \phi_{ms} - \psi_s - \frac{Q_B}{C_{ox}} - \frac{Q_{SS}}{C_{ox}}}$$

, where ϕ_{ms} = work function difference [in V] between gate material and bulk Si
ψ_s = surface potential in the Si @ onset of strong inversion
$= 2\phi_f$ for uniformly doped substrate $(\phi_f \sim 0.3\,V)$
Q_{SS} = oxide charge per unit area at the oxide-Si interface [C/cm²]
Q_B = charge stored per unit area in the depletion region (at onset of inversion)

$$\Rightarrow |Q_B| = \sqrt{2q\epsilon_s \underset{\uparrow\ \text{conc. in bulk}}{N_B}\,(2|\phi_f| + \underset{\uparrow\ \text{reverse bias}}{|V_{SB}|})} \quad [C/cm^2]$$

C_{ox} = gate oxide capacitance per unit area [F/cm²]

Threshold Voltage

Case: $V_{SB}=0 \Rightarrow V_t(V_{SB}=0) = V_{t0} = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}$, where

$$Q_{B0} = \sqrt{2q\epsilon_{si}N_B(2|\phi_f|+|V_{SB}|)}$$

Then:

$$V_t = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_B}{C_{ox}}$$

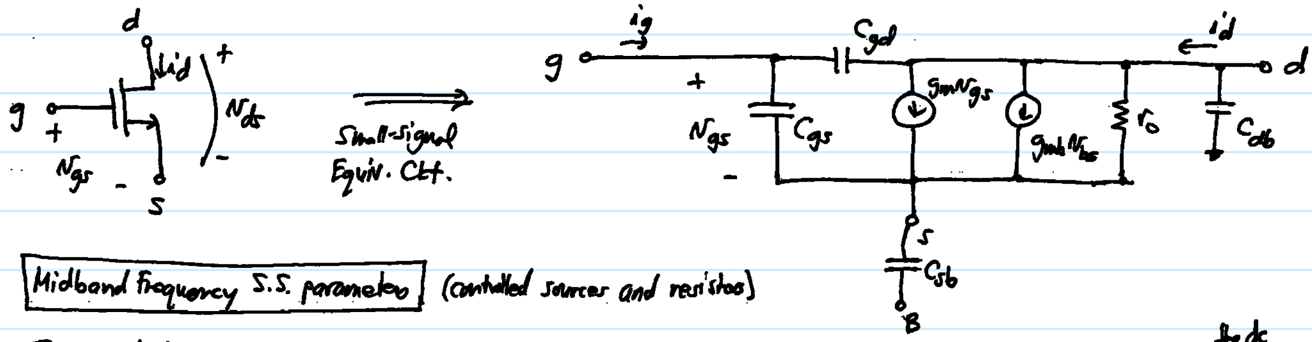
$$= \underbrace{\phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}}_{V_{t0}} - \frac{Q_B - Q_{B0}}{C_{ox}}$$

$$\boxed{V_t = V_{t0} - \gamma\left(\sqrt{2|\phi_f|+|V_{SB}|} - \sqrt{2\phi_f}\right), \quad \gamma = \frac{1}{C_{ox}}\sqrt{2q\epsilon_{si}N_B}}$$

Signs in the V_t Equation:

Parameter	NMOS	PMOS
Substrate	p-type	n-type
ϕ_{ms}: metal gate	−	−
n+ Si gate	−	−
p+ Si gate	+	+
ϕ_f	−	+
Q_{B0} (or Q_B)	−	+
Q_{ss}	+	+
γ	−	+
C_{ox}	+	+

MOS Small-Signal Model (for NMOS) → in saturation



Midband Frequency S.S. parameters (controlled sources and resistors)

Transconductance, g_m:

$$g_m = \frac{i_d}{v_{gs}} = \frac{\partial I_d}{\partial V_{gs}}\bigg|_{Q\text{-pt}} = \frac{\partial}{\partial V_{gs}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})^2\right)\bigg|_{Q\text{-pt}} = \mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})\bigg|_{V_{gs}=V_{DS}}$$

the dc V_{DS}

$$\therefore \boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{tn}) = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}}$$

$$\left[I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{tn})^2 \rightarrow (V_{GS}-V_{tn}) = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}\right]$$

$$\frac{i_d}{v_{bs}} = g_{mb} = \frac{-i_d}{v_{sb}} = -\frac{\partial I_d}{\partial V_{sb}}\bigg|_{Q\text{-pt}} = -\left(\frac{\partial I_d}{\partial V_{tn}}\cdot\frac{\partial V_{tn}}{\partial V_{sb}}\right)\bigg|_{Q\text{-pt}}$$

$$\frac{\partial I_d}{\partial V_{tn}}\bigg|_{Q\text{-pt}} = -\frac{\partial I_d}{\partial V_{gs}}\bigg|_{Q\text{-pt}} = -g_m \quad ; \quad \frac{\partial V_{tn}}{\partial V_{sb}}\bigg|_{Q\text{-pt}} = \frac{\partial}{\partial V_{sb}}\left[V_{t0} + \gamma\left(\sqrt{V_{sb}+2|\phi_F|} - \sqrt{2|\phi_F|}\right)\right]\bigg|_{Q\text{-pt}} = \boxed{\frac{\gamma}{2\sqrt{V_{SB}+2|\phi_F|}} \equiv \eta}$$

$$\therefore \boxed{g_{mb} = \eta g_m}$$

↑ often neglected!

Note $V_{SB}\uparrow \rightarrow V_T\uparrow \rightarrow \eta\downarrow \rightarrow I_d\downarrow$

↳ g_{mb} is minimized by maximizing V_{SB}!

Output Resistance, r_o: $\left(= \frac{1}{g_{ds}}\right)$

$$\Rightarrow \text{output conductance} = g_{ds} = \frac{i_d}{v_{ds}} = \frac{\partial I_d}{\partial V_{ds}}\bigg|_{Q\text{-pt}} = \frac{\partial}{\partial V_{ds}}\left(\overbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_{tn})^2}^{I_{dsat}}(1+\lambda V_{ds})\right)\bigg|_{Q\text{-pt}}$$

explain what λ is!

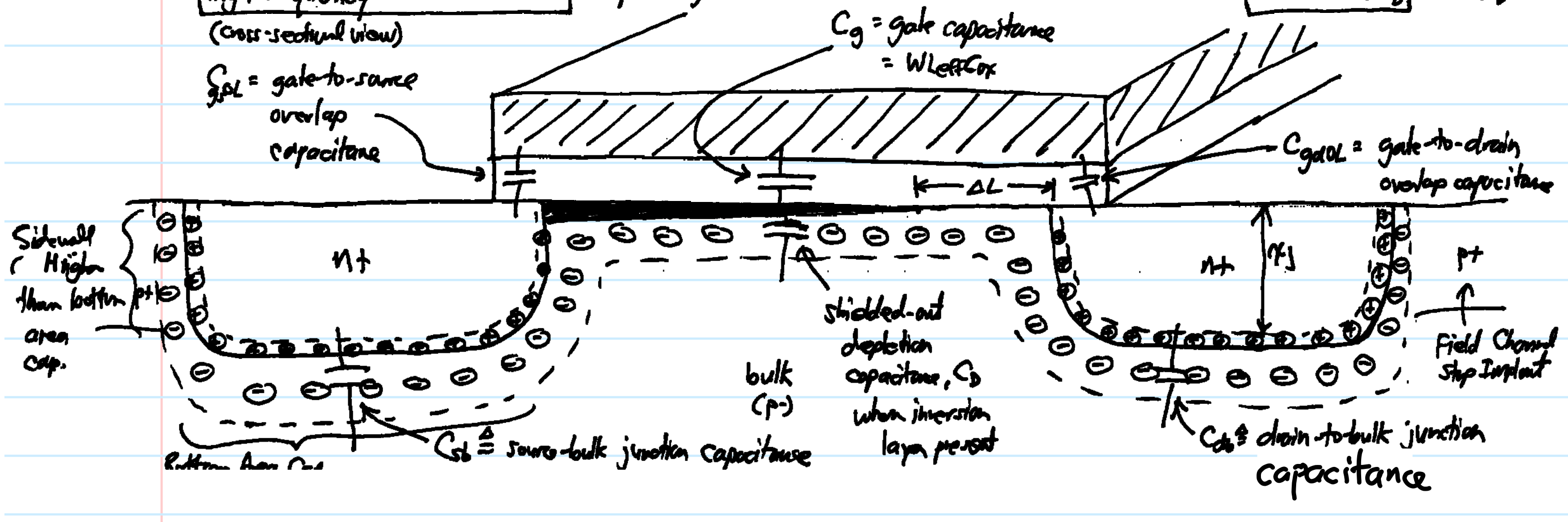
$$= \lambda I_{dsat} = \frac{\lambda I_D}{1+\lambda V_{DS}} \underset{[1 \gg \lambda V_{DS}]}{\approx} \lambda I_D = g_{ds}$$

$$\therefore \boxed{r_o = g_{ds}^{-1} = \frac{1}{\lambda I_D}} = \frac{\frac{1}{\lambda}+V_{DS}}{I_D}$$

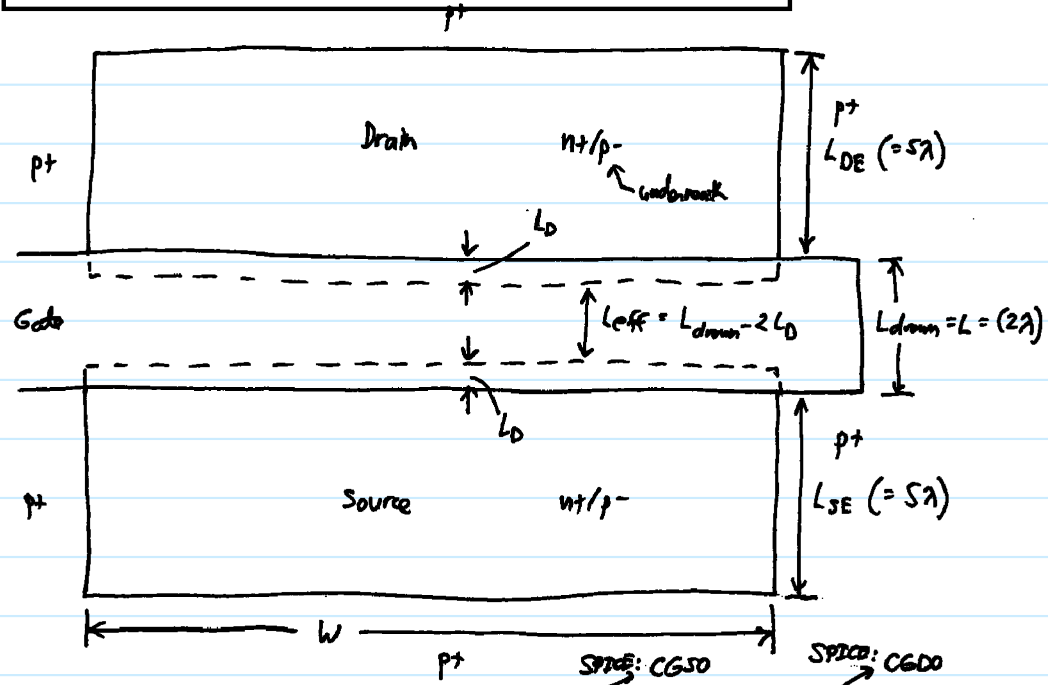
if V_{DS} is very large

High Frequency S.S. Parameters (capacitors)

(cross-sectional view)

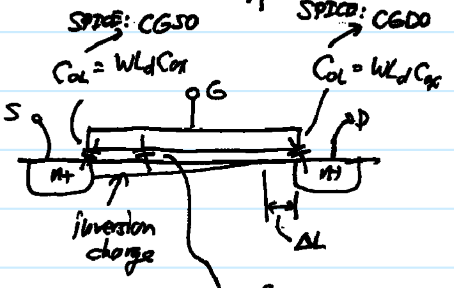


(layout view)



(still considering saturation region)

In saturation, the inversion charge is not present near the drain:



$\frac{2}{3}C_g = \frac{2}{3}WL_{eff}C_{ox}$

obtained by integrating the charge over the gate length

Gate-to-Source Capacitance, C_{gs}:

$$C_{gs} = C_{oL} + \frac{2}{3}WL_{eff}C_{ox} \quad \text{(inversion charge integrated)}$$

Gate-to-Drain Capacitance, C_{gd}:

$$C_{gd} = C_{oL} \quad \text{(no inversion charge near the drain in saturation)}$$

Source/Drain Junction Capacitance, C_{sb} & C_{db}: (must include these in SPICE simulations!)

⇒ there are depletion capacitors associated w/ the drain-to-bulk and source-to-bulk pn junctions

⇒ bottom-side capacitance per unit area is different from that at sidewalls due to higher bulk doping at the sidewalls
(there is higher doping in the field areas to prevent channels from forming under interconnect wires)

⇒ take drain capacitance as an example:

$\left(\frac{q\epsilon_s N_A N_D}{2(N_A+N_D)\psi_0}\right)^{1/2}$

SPICE: CJ

bulk doping level

$$C_{j0} = \sqrt{\frac{q\epsilon_s N_B}{2|\phi_f|}} \rightarrow \cancel{\left(\frac{q\epsilon_s N_B}{2|\phi_f|}\right)^m}$$

$\approx 2\psi_0$?

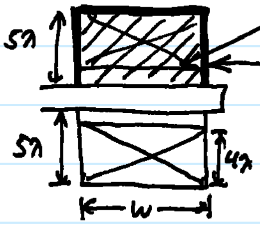
depl. cap. per unit area @ bottom-side w/ $V_{SB}=0V$

$$C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{\psi_0}}}$$

, $C_{db0} \triangleq$ depletion capacitance with $V_{DB}=0V$

$= (\text{junction bottom-side area})\,C_{j0} + (\text{junction outside perimeter})\,C_{jsw}$

$= W(5\lambda)C_{j0} + (W + 2(5\lambda))C_{jsw}$



depletion cap. along sidewalls per unit length for $V_{DB}=0V$

$$C_{jsw} = \sqrt{\frac{q\epsilon_s N_{cs}}{2|\phi_f|}} \times x_j$$

$\cancel{\left(\frac{q\epsilon_s N_{cs}}{2|\phi_f|}\right)^m}$

channel-stop implant doping level

SPICE: CJSW

s/D junction depth