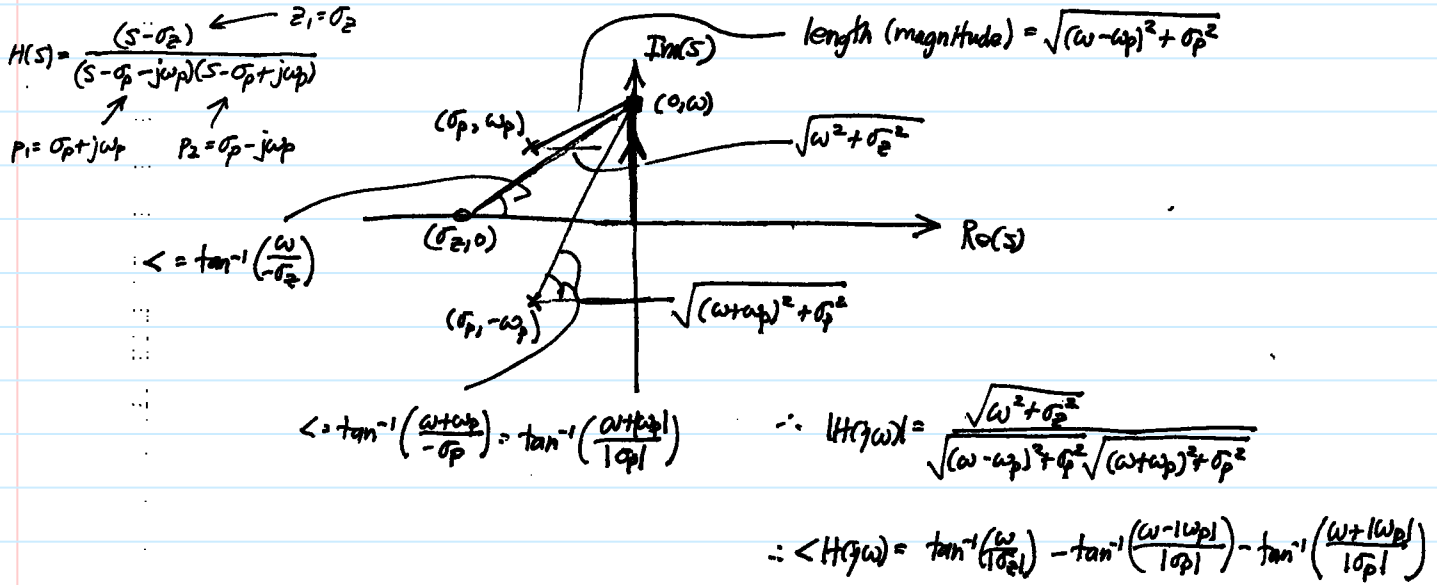


The frequency response of a given system is completely characterized by knowledge of the poles, zeros, and dc gain factor ( $H_0$ ) of the system in question. In fact, the magnitude & phase of the network can be determined graphically from a pole/zero diagram.



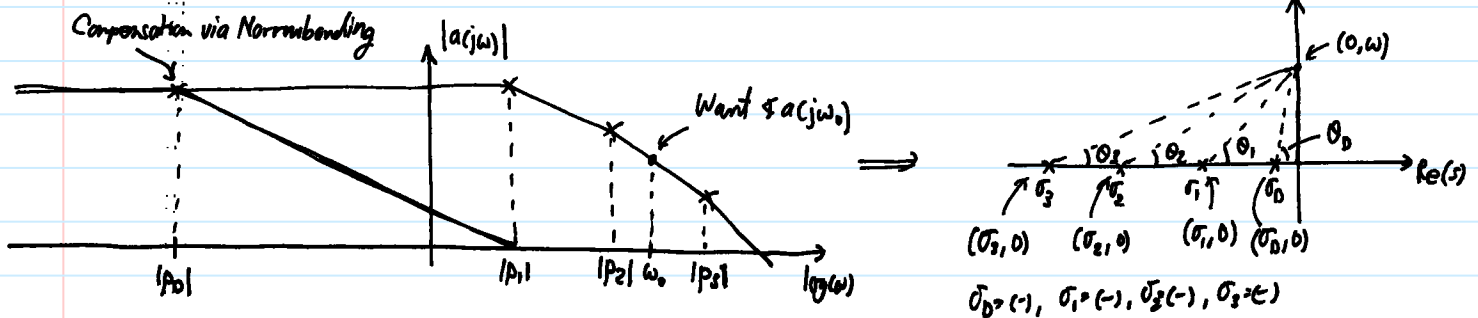
In general:

$$|H(j\omega)| = H_0 \frac{\prod_{j=1}^m (\text{mag. of vectors from zeros to } j\omega)}{\prod_{i=1}^n (\text{mag. of vectors from poles to } j\omega)} = H_0 \frac{\prod_{j=1}^m |j(\omega - \omega_{zj}) - \sigma_{zj}|}{\prod_{i=1}^n |j(\omega - \omega_{pi}) - \sigma_{pi}|}$$

$$\angle H(j\omega) = \sum \text{angles from zeros} - \sum \text{angles from poles} + \angle H_0$$

$z_j = \sigma_{zj} + j\omega_{zj}$   
 $p_i = \sigma_{pi} + j\omega_{pi}$   
 could be  $0^\circ$  or  $180^\circ$   
 (-)    (+)

Precise Determination of Phase



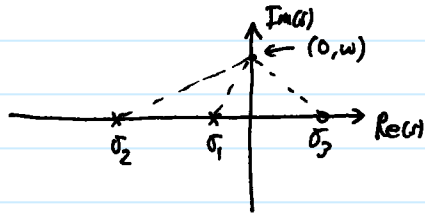
$$\angle a(j\omega) = -\theta_0 - \theta_1 - \theta_2 - \theta_3$$

$$= -\tan^{-1}\left(\frac{\omega}{-\sigma_0}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)$$

$$\therefore \angle a(j\omega_0) = -90^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_0}{|P_2|}\right) - \tan^{-1}\left(\frac{\omega_0}{|P_3|}\right)$$

since  $\omega_0 \gg |P_0| \ll |P_1|$

Ex: System w/ 2 poles & 1 RHP zero



$\sigma_1 = (-)$      $\sigma_2 = (-)$  since it's a zero!

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) + \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{|\sigma_1|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_2|}\right) + \tan^{-1}\left(-\frac{\omega}{|\sigma_3|}\right)$$

$$\angle H(j\omega) = -90^\circ - 90^\circ + (-90^\circ) \Rightarrow \angle H(j\omega) = -270^\circ$$

$\omega \rightarrow \infty$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + \sigma_3^2}}{\sqrt{\omega^2 + \sigma_1^2} \sqrt{\omega^2 + \sigma_2^2}}$$

Note: The RHP zero contributes a (-) phase shift!

