Device Mismatch Effects in Diff. Amplifiers

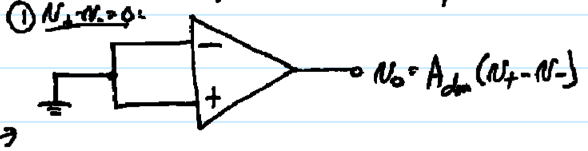
⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched

⇒ in actual ckt., get device mismatches due to processing variations

The Result:

→ Output not zero when input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!

① $v_+ - v_- = 0$:



Ideal Case: $v_o = 0$

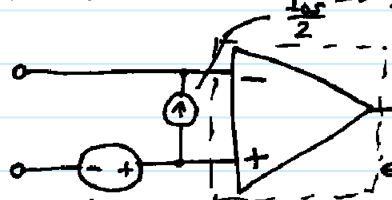
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model these effects, introduce:

① Input Offset Voltage, V_{OS} → Typ. 1-5 mV for BJT

② Input Offset Current, I_{OS} → Typ. I_{OS} = 10 nA for BJT



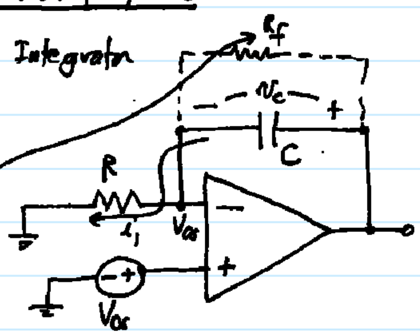
$I_{OS} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{OS} → voltage needed to set $v_o = 0$ in this ckt. (negating the effect of mismatches)

Effect of V_{OS} on Op Amp Ckts. –

Example. Integrator



$$v_o = V_{OS} + \frac{1}{C}\int_0^t i_1 dt$$

$$= V_{OS} + \frac{1}{C}\int_0^t \frac{V_{OS}}{R} dt$$

$$= V_{OS}\left(1 + \frac{t}{RC}\right) + v_c\big|_{t=0}$$

will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C

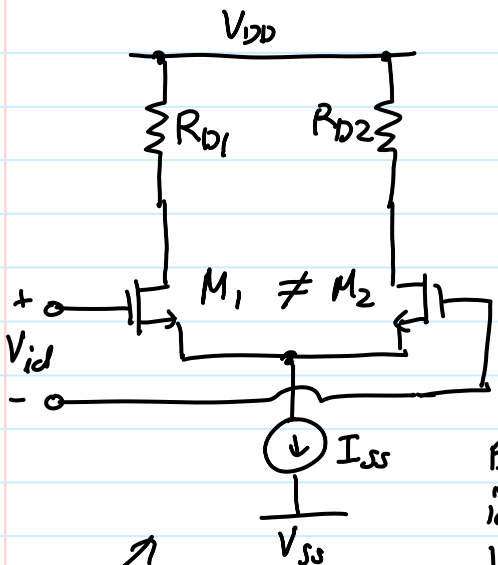
→ then $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen

→ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{OS} = small

V_{OS} is even more important in setting the resolution of A/D converters and other precision ckts.

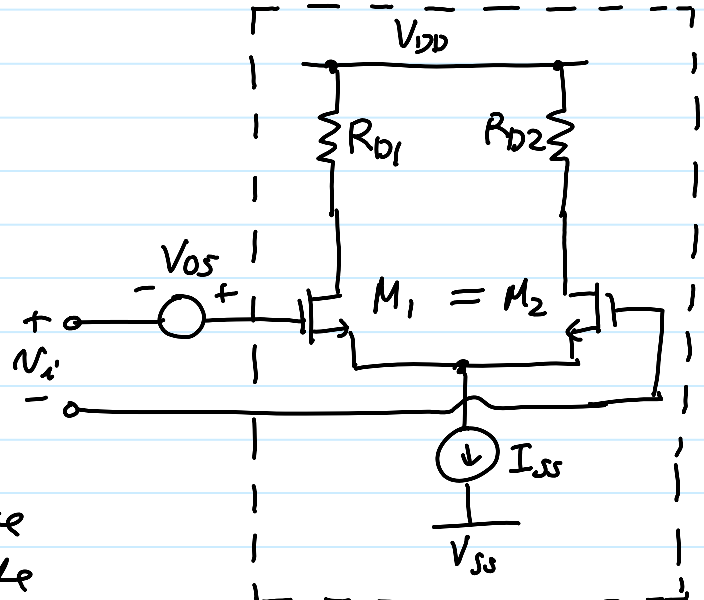
V_{OS} of a Mismatched SCP

Objective: Derive an expression for V_{OS}.



Actual SCP w/ Mismatched Xsistors & R's

Equate to an ideal SCP & use V_{OS} to model the effect of mismatches

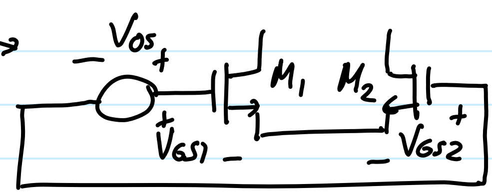


Ideal SCP w/ Matched M_1 & M_2 and $R_{D1} = R_{D2}$

Input offset voltage V_{OS} arises due to variations in:

1. Xsistors, M_1 & M_2 $\longrightarrow$ $\frac{W}{L}$ and V_t vary
2. $R_{D1} \neq R_{D2}$ $\longrightarrow$ causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \qquad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$	$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$	$V_t = \frac{V_{t1} + V_{t2}}{2}$	$R_D = \frac{R_{D1} + R_{D2}}{2}$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{os} = \Delta V_t + \sqrt{\frac{\overbrace{2(I_D + \Delta I_D/2)}^{2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1+\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}} - \sqrt{\frac{\overbrace{2(I_D - \Delta I_D/2)}^{2I_D\left(1-\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1-\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\underbrace{\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}}_{\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore\ V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0 \ \therefore\ I_{D1}R_{D1} = I_{D2}R_{D2} \longrightarrow$ mismatch in I_D must be opposite that of R_D

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

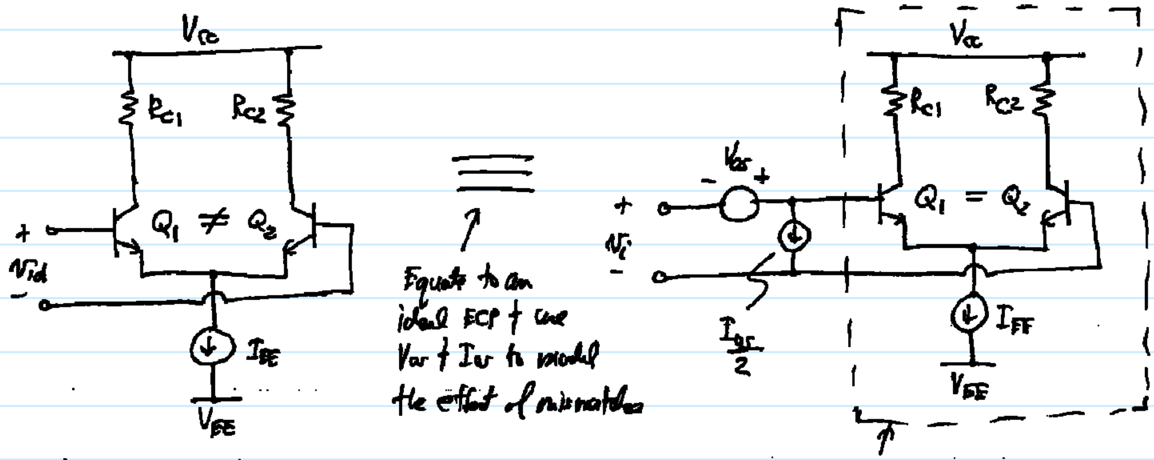
Threshold Mismatch ← bias independent

Geometric (i.e., Layout) Variation

↳ scale w/ overdrive

V_{OS} in a Mismatched ECP

Objective: Derive an expression for V_{OS}.



Actual ECP w/ Mismatched Xistors + R's — Ideal ECP w/ Matched Q_1 + Q_2 and $R_{C1} = R_{C2}$

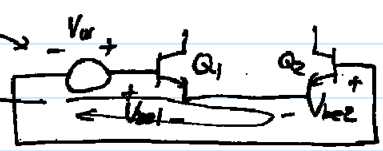
Input Offset Voltage V_{OS} arises due to variations in:

① Xistors, Q_1 + Q_2 $\longrightarrow$ I_s + β vary: $I_S = \dfrac{q n_i^2 \overline{D}_n A}{N_A W_B(V_{CB})}$ ("function of")

$\therefore$ $I_{S1} \neq I_{S2}$ can be caused by:
- (i) $A_1 \neq A_2$ (etching tolerance limits)
- (ii) $N_{A1} \neq N_{A2}$ (doping variations of base)
- (iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)

② $R_{C1} \neq R_{C2}$ $\rightarrow$ causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$, ~~which occurs when~~ in this ckt.:



KVL: $V_{OS} - V_{be1} + V_{be2} = 0$

$$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = V_T \ln\left(\frac{I_{C1}}{I_{C2}} \frac{I_{S2}}{I_{S1}}\right)$$

Find $\frac{I_{C1}}{I_{C2}}$ in terms of design elements:

[When $V_{id} = V_{OS} \rightarrow V_{od} = 0V$] $\longrightarrow$ $V_{od} = (\cancel{V_{CC}} - I_{C1}R_{C1}) - (\cancel{V_{CC}} - I_{C2}R_{C2}) = 0$

$$I_{C1}R_{C1} = I_{C2}R_{C2} \longrightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}}$$

$$\boxed{V_{OS} = V_T \ln\left(\frac{R_{C2}}{R_{C1}} \frac{I_{S2}}{I_{S1}}\right)}$$

This is an exact equation for V_{OS}. It's often more useful + intuitive to express this in terms of percent variations (and eventually standard deviations).

Convert to Percent Variation Form –

Define. $R_c = \frac{R_{c1}+R_{c2}}{2}$, $\Delta R_c = R_{c1}-R_{c2}$; $I_s = \frac{I_{s1}+I_{s2}}{2}$, $\Delta I_s = I_{s1}-I_{s2}$ } Objective: Express V_{os} in terms of percent variations $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$.

$\Downarrow$

In general: $\Delta X = X_1 - X_2$, $X = \frac{X_1+X_2}{2}$ } $X_1 = X + \frac{\Delta X}{2}$, $X_2 = X - \frac{\Delta X}{2}$ $\Rightarrow$ Thus: $R_{c1} = R_c + \frac{\Delta R_c}{2}$, $R_{c2} = R_c - \frac{\Delta R_c}{2}$; $I_{s1} = I_s + \frac{\Delta I_s}{2}$, $I_{s2} = I_s - \frac{\Delta I_s}{2}$

With these formulations:

$$V_{OS} = V_T \ln\left[\frac{R_{c2}}{R_{c1}}\frac{I_{s2}}{I_{s1}}\right] = V_T \ln\left\{\frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \cdot \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}}\right\} = V_T \ln\left\{\frac{1-\frac{\Delta R_c}{2R_c}}{1+\frac{\Delta R_c}{2R_c}} \cdot \frac{1-\frac{\Delta I_s}{2I_s}}{1+\frac{\Delta I_s}{2I_s}}\right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right] \Rightarrow V_{os} \cong V_T\left\{-\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s}\right\}$$

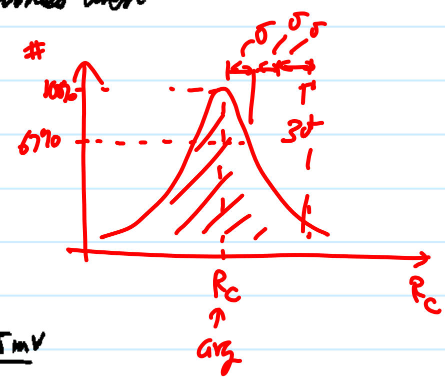
taking the first term assuming $\Delta R \ll R_c$ & $\Delta I_s \ll I_s$

$$\boxed{V_{OS} = V_T\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically varying parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V_{os}:

→ since $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{os}} = V_T\sqrt{\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta I_s/I_s}}$$



Ex: Typ. $\sigma_{\Delta R_c/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$\therefore\ \sigma_{V_{os}} = (26m)\sqrt{(0.01)^2 + (0.05)^2} = \underline{1.3mV}$ Typ. V_{os} for BJT ~ $\underline{1\text{-}5mV}$

$\boxed{V_{os}\text{ Drift w/ Temperature}}$

$$\frac{dV_{os}}{dT} = \underbrace{\frac{kT}{q}\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}_{\text{indep. of } T}\frac{1}{T} = \frac{V_{os}}{T}$$

[In kelvin]

Ex. $\frac{dV_{os}}{dT} = \frac{1.3m}{300K} = 4.3\,\mu V/°C$ drift around $T = 300K$.

I_{OS} in a Mismatched ECP

By Definition: $I_{OS} = I_{B1} - I_{B2} = \boxed{\dfrac{I_{C1}}{\beta_1} - \dfrac{I_{C2}}{\beta_2} = I_{OS}}$

To express in percent variations:

$$\begin{cases} I_{C1} = I_C + \dfrac{\Delta I_C}{2} \\ I_{C2} = I_C - \dfrac{\Delta I_C}{2} \end{cases} \qquad \begin{cases} \beta_1 = \beta + \dfrac{\Delta\beta}{2} \\ \beta_2 = \beta - \dfrac{\Delta\beta}{2} \end{cases}$$

$$\therefore I_{OS} = \frac{I_C + \frac{\Delta I_C}{2}}{\beta + \frac{\Delta\beta}{2}} - \frac{I_C - \frac{\Delta I_C}{2}}{\beta - \frac{\Delta\beta}{2}} = \frac{I_C}{\beta}\left\{\frac{1+\frac{\Delta I_C}{2I_C}}{1+\frac{\Delta\beta}{2\beta}} - \frac{1-\frac{\Delta I_C}{2I_C}}{1-\frac{\Delta\beta}{2\beta}}\right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \cdots\right] \rightarrow \; = \frac{I_C}{\beta}\left\{\left(1+\frac{\Delta I_C}{2I_C}\right)\left(1-\frac{\Delta\beta}{2\beta}\right) - \left(1-\frac{\Delta I_C}{2I_C}\right)\left(1+\frac{\Delta\beta}{2\beta}\right)\right\}$$

$$= \frac{I_C}{\beta}\left\{\cancel{1} + \frac{\Delta I_C}{2I_C} - \frac{\Delta\beta}{2\beta} - \cancel{\frac{\Delta I_C}{2I_C}\frac{\Delta\beta}{2\beta}} - \cancel{1} + \frac{\Delta I_C}{2I_C} - \frac{\Delta\beta}{2\beta} + \cancel{\frac{\Delta I_C}{2I_C}\frac{\Delta\beta}{2\beta}}\right\}$$

$$\boxed{I_{OS} = \frac{I_C}{\beta}\left\{\frac{\Delta I_C}{I_C} - \frac{\Delta\beta}{\beta}\right\}}$$

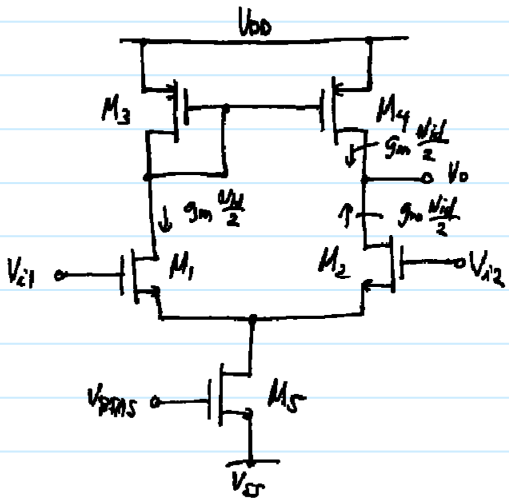
But for $V_{id} = 0V \Rightarrow \dfrac{I_{C1}}{I_{C2}} = \dfrac{R_{C2}}{R_{C1}} \longrightarrow \dfrac{\Delta I_C}{I_C} = -\dfrac{\Delta R_C}{R_C}$

$$\therefore \boxed{I_{OS} = -\frac{I_C}{\beta}\left(\frac{\Delta R_C}{R_C} + \frac{\Delta\beta}{\beta}\right)}$$

Ex. Typ: $\sigma_{\Delta\beta/\beta} = 0.1$, $\sigma_{\Delta R_C/R_C} = 0.01$

$$\longrightarrow I_{OS} = -\frac{I_C}{\beta}\left[\sigma^2_{\Delta R_C/R_C} + \sigma^2_{\Delta\beta/\beta}\right]^{1/2} \cong -0.1\frac{I_C}{\beta} \approx -0.1 I_B = I_{OS}$$

MOS Differential Stage w/ Current Mirror Load



Small Signal Gain: (similar to BJT) ↗ all the same FB effects, etc...

$$\frac{v_o}{v_{id}} = g_{m2}(r_{o2} \| r_{o4}) = \frac{g_{m2}}{g_{ds2} + g_{ds4}} = \frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{SS}}}{\frac{I_{SS}}{2}(\lambda_2 + \lambda_4)} \Rightarrow \boxed{\frac{v_o}{v_{id}} = \frac{2}{\lambda_2 + \lambda_4}\sqrt{\frac{\mu_n C_{ox}(W/L)_2}{I_{SS}}}}$$

$$\left[\frac{\Delta(W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}}\right]$$

Offset Voltage - $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{od} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{t3,4}\left(\frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{(V_{GS} - V_t)_{1,2}}{2}\left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}}\right]$$

Via similar derivation to what we just did

For small V_{OS}: ① small $(V_{GS} - V_t)$

② $g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2}$ & $\left(\frac{W}{L}\right)_{3,4} < \left(\frac{W}{L}\right)_{1,2}$