EE 140/240A Linear Integrated Circuits
Fall 2019 Homework 1

This homework is due September 4, 2019, at 23:00.

Submission Format
Your homework submission should consist of **one** file submitted via bCourses.

- `hw1.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Rubric: (All the Points)

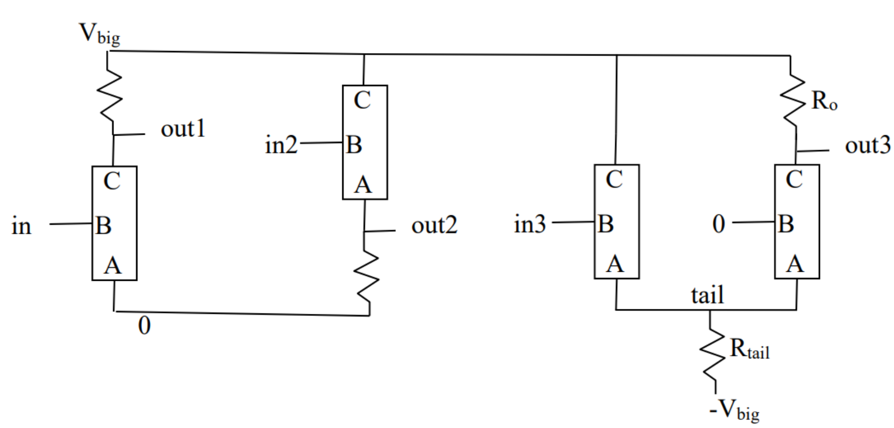
- Grade a correct approach at 60%. It is important to get the approach correct.
- Grade a correct approach + a correct numerical answer with full marks.
- A single numerical error should cost you 20%.
- Multiple numerical errors should cost you no more than 40%

1. Three-Terminal Devices

In the figure below, there are four identical three-terminal devices. These devices all have the properties:

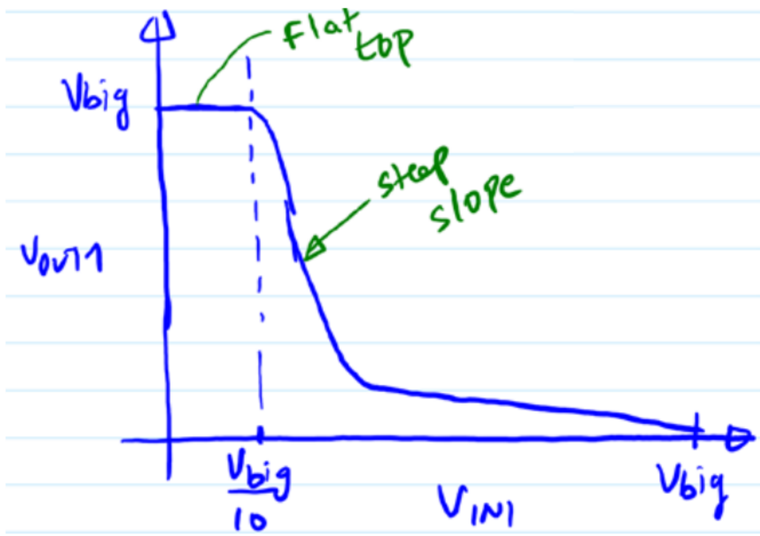
- The current from C to A is a strong function of the voltage from B to A and a weak function of the voltage from C to A.
- There is negligible current into node B.
- If the voltage from B to A is less than V_X, that the current is zero.
- If the voltage from B to A is more than $V_X = \frac{V_{big}}{10}$, the current goes up really fast.
- The voltage from C to A doesn't have much effect, as long as it is greater than 0, but if it is 0 or less, the current is 0.

(Note: Seen from far away, this describes JFETs, BJTs, Darlingtons, MOSFETs, MESFETs, vacuum tubes, IGBTs, HEMTs, and virtually every other three terminal electronic device ever made)



(a) Sketch V_{out1} as V_{in} varies from 0 to V_{big}

Solution:



Rubric: (5 Points)

- +2: Flat top for $V_{\text{in}} < \frac{V_{\text{big}}}{10}$
- +2: Steep slope
- +1: Curve never goes to 0

(b) When V_{out1} is $\frac{V_{\text{big}}}{2}$, write an expression for the V_{out1} using the derivative of the current with respect to V_{in} and the resistor R.

Solution:

$$V_{\text{out1}} = V_{\text{big}} - I_{CA1}R$$
$$V_{\text{out1}} = V_{\text{big}} - \frac{\partial I_{CA1}}{\partial V_{\text{in}}} V_{\text{in}} R$$

$$\boxed{V_{\text{out1}} = V_{\text{big}} - \frac{\partial I_{CA1}}{\partial V_{\text{in}}} V_{\text{in}} R}$$

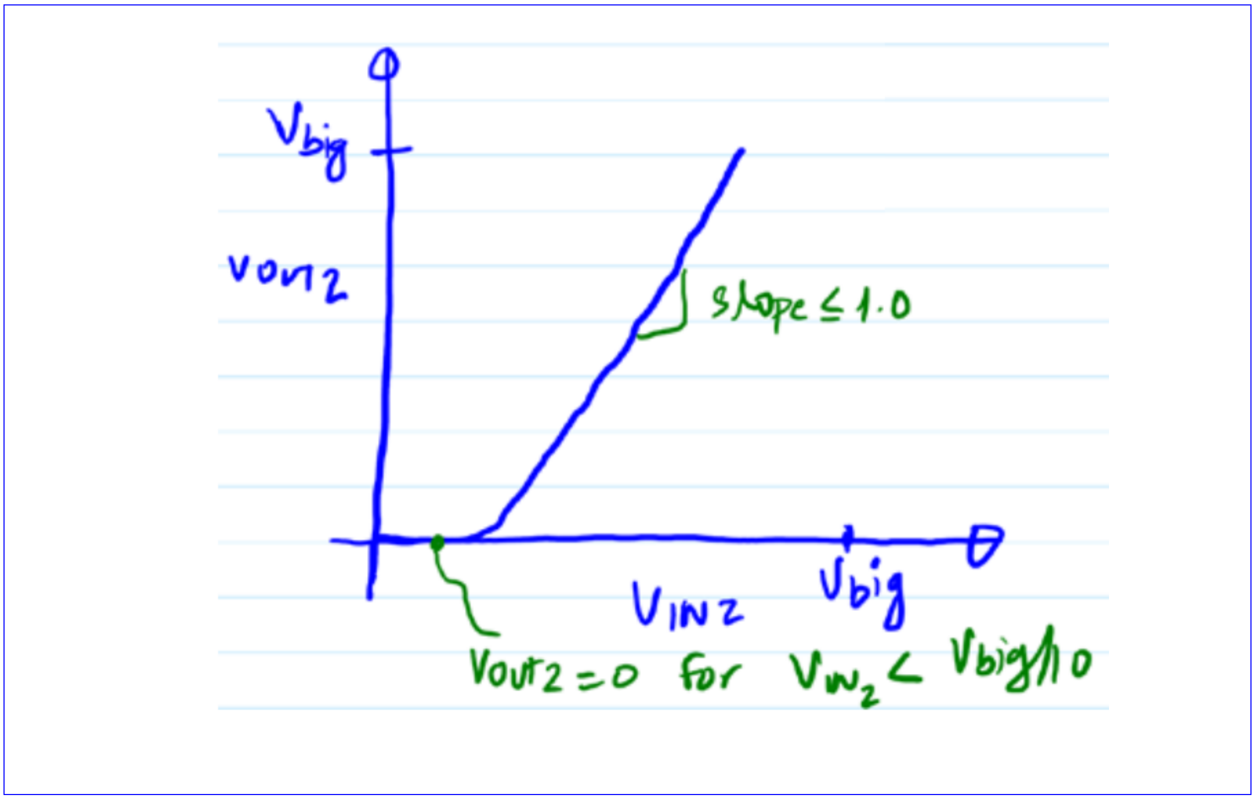
Rubric: (5 Points)

- +2: Correct derivative of I_C with V_{in}
- +2: Product of this derivative with R
- +1: Correct sign

No need to get hte full expression correct, but these aspects must be present.

(c) Sketch V_{out2} as the V_{in2} varies from 0 to V_{big}.

Solution:

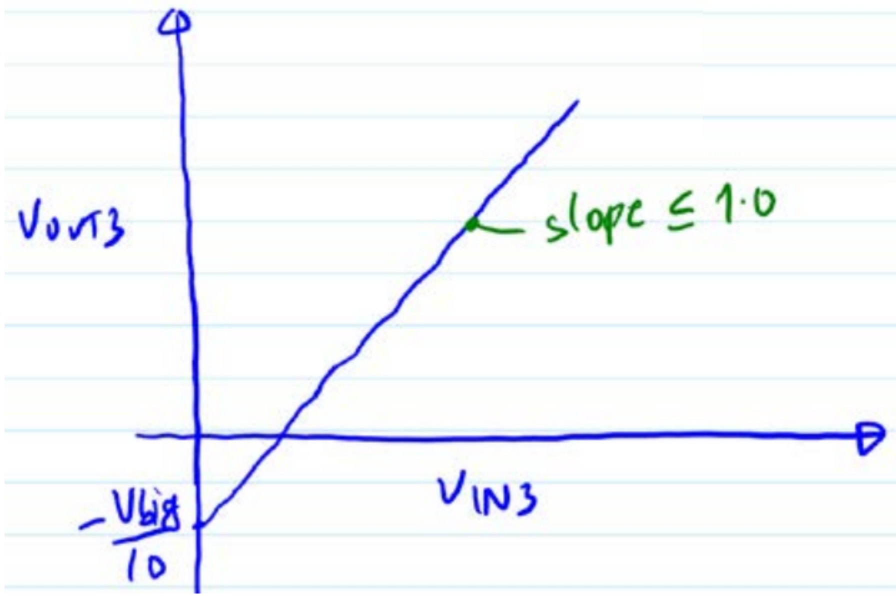


Rubric: (5 Points)

- +3: $V_{\text{out2}} = 0$ for $V_{\text{in}} < \frac{V_{\text{big}}}{10}$
- +2: Identify slope ≤ 1

(d) Sketch V_{tail} as V_{in3} varies from 0 to V_{big}.

Solution:



Errata: The y-axis should read V_{tail}

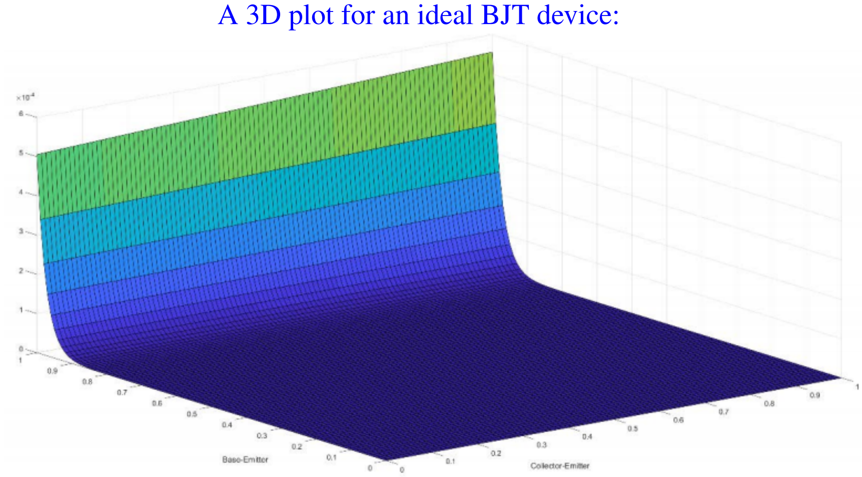
Rubric: (5 Points)

- +3: y-intercept $= -\frac{V_{\text{big}}}{10}$
- +2: Identify slope ≤ 1

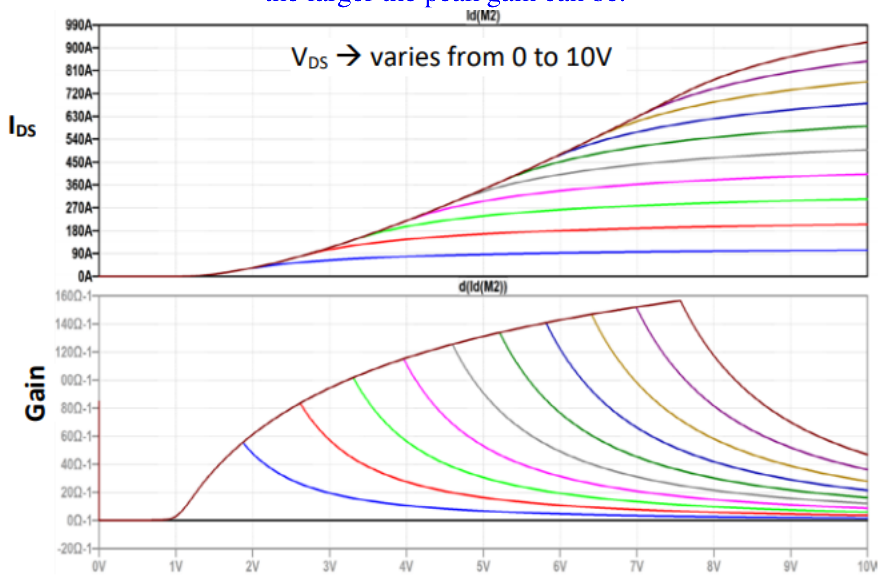
(e) **(EE240A)** Use MATLAB or some equivalent to plot 3D surfaces of current vs. control and output voltages for several of the types of 3-terminal devices listed above, preferably showing constant-current projections onto the XY plane. Comment on the gain vs. bias point.

Solution:

A 3D plot for an ideal BJT device:



We choose MOSFET AO6408 for our simulation. In real life, gain depends on both the control voltage and output voltage. The control voltage changes the bias point, which strongly affects gain. With increasing control voltage, the gain reaches a peak and drops for further increase. This peak can be further influenced by controlling the output voltage. The higher the output voltage, the larger the peak gain can be.



Rubric: (5 Points)

- +5: Comment on the gain vs. bias point

2. Berkeley Hills

You're standing on a big smooth hillside. Directly North the hill climbs up quite steeply, rising 10cm for every step you take. Directly East the hill climbs gently, rising only 1mm for every step you take. You put a stake in the ground where you are standing and call it $(0,0)$. You measure the elevation to be E. If you want to be 10 steps further north than you currently are, but don't want to change your altitude (current source!)

(a) If you walk 10 steps east and put a stake in the ground labeled $(10,0)$, what is your elevation?

Solution: Going east, we find the elevation change:

$$.1\frac{\text{cm}}{\text{step}} \times 10\text{steps} = 1\text{cm}$$

and account for the fact that our initial elevation was E to get

$$E + 1\text{cm}$$

Rubric: (5 Points)

- +3: Formulate correct expression
- +2: Correct numerical value

(b) If you walk 10 steps north from $(0,0)$ and put a stake in the ground labeled $(0,10)$, what is your elevation?

Solution: Going north, we have the elevation change:

$$10\frac{\text{cm}}{\text{step}} \times 10\text{steps} = 100\text{cm}$$

$$\boxed{E + 100\text{cm}}$$

Rubric: (5 Points)

- +3: Formulate correct expression
- +2: Correct numerical value

(c) After walking 10 steps north, how far east do you have to walk in order to go back to your original altitude? (You can have a negative answer!)

Solution: Walking 10 steps north gives us a change in elevation of $+100$cm. To neutralize this, we need a change in elevation of -100cm, which can be gained by walking west (the opposite of east) for

$$\frac{100\text{cm}}{0.1\frac{\text{cm}}{\text{step}}} = 1000 \text{ steps}$$

$$\boxed{1000 \text{ steps west (or } -1000 \text{ steps east)}}$$

Rubric: (5 Points)

- +2: Formulate correct expression
- +2: Correct direction
- +1: Correct number of required steps

(d) (Note: altitude is current. East/west is drain/source voltage. North/south is gate/source voltage. The first stake is the DC operating point, the origin of the local coordinate systems. Step counts are small signal voltage changes. What is the intrinsic gain?

Solution: Using our definition of intrinsic gain and the analogy,

$$\begin{aligned}
a_0 &= -g_m r_o \\
&= -\frac{di_d}{dv_{gs}}\frac{dv_{ds}}{di_d} \\
&= -\frac{\text{change in altitude climbing north}}{\text{change in altitude climbing east}} \\
&= -\frac{10}{0.1} \\
&= -100
\end{aligned}$$

Note that generally, gain has no units.

$$\boxed{-100}$$

Rubric: (5 Points)

- +3: Formulate correct expression
- +2: Correct numerical value

(e) Do any of the previous answers depend on E, or the GPS location of the stake that you labeled (the specific values of the DC operating point)?

Solution:

Simply put, yes! This is analogous to the notion that g_m and r_o depend on the biasing condition of your amplifier.

Rubric: (5 Points)

- +5: Correct answer. No explanation necessary.

3. Linearizing Gold Mining

You live in an area with a lot of gold mines. Everyone knows that the amount of gold G that has been extracted from a mine as a function of the time the mine has been open H is given by

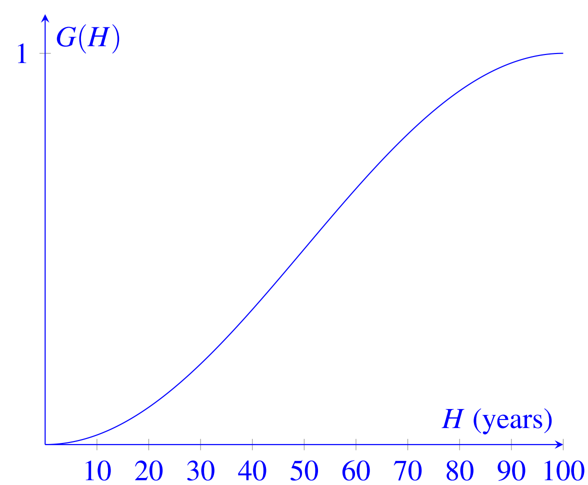
$$G(H) = \frac{G_{\text{total}}}{2}\left(1 - \cos\left(\frac{H}{T}\pi\right)\right)$$

from the time that people first start excavating the mine (when $H = 0$) until the time that all of the gold is gone, $H = T$. In other words, G is strictly increasing over since no one's putting gold back into the mine.

(This is about linearization. You get more bang for your buck (more g_m per µA) with transistors that are biased just right. There's a low bias where you get nothing, a high bias where everything is maxed out, and a sweet spot somewhere in the middle.)

(a) If you only get to work for one hour in the mine, does it matter if you work at the beginning, vs. the middle or the end? Why or why not?

Solution: Let's have $G_{\text{total}} = 1$. Plotting $G(H)$:



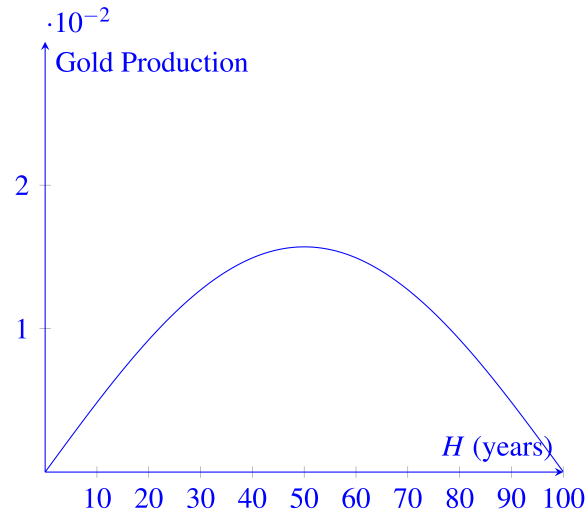
As shown below, the slope of G changes. Thus, depending on when one starts mining, the return will surely vary.

Rubric: (10 Points)

- +10: Correct answer with some reasonable justification

(b) When should you work to mine the most gold, and how much will you get?

Solution: The plot of $G(H)$ is the total gold coming out of the mine, but that's the integral of what we're interested in! If we want to maximize the amount of gold we get, we want to find a time when the gold production is highest:



Sometime about the midway around 50 years. If one wants to work for one year from the midway, $1.8 \cdot 10^{-6}$ fraction of the total gold can be mined.

Rubric: (10 Points)

- +10: Correct estimate of time when gold extraction maximizes

(c) Let $T = 100$ years. If you start working at $H = \frac{T}{4}$ and work for a year, does your gold mining rate change much from month to month?

Solution:

As the maximum is flat and the x-axis is in years, the rate of gold mining will not change significantly from month to month. For all practical purposes, it would stay fixed.

Rubric: (15 Points)

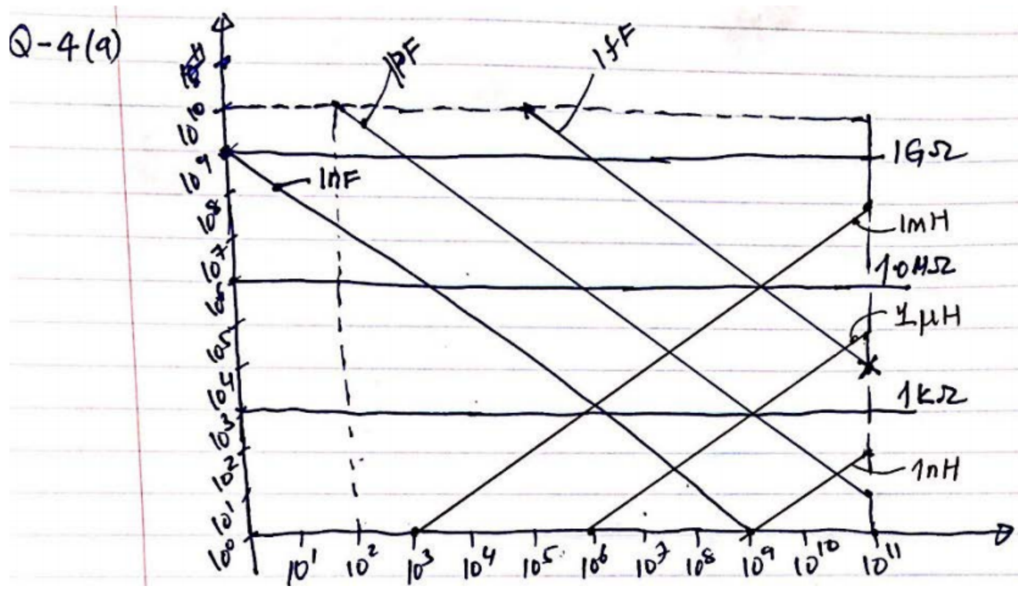
- +15: Correct answer with reasonable explanation

4. Impedance Plotting

Graph the magnitude of the impedance of the following elements and circuits by hand. Use a log/log scale, with the frequency axis varying from 1 to $10^{11}\,\frac{\text{rad}}{\text{s}}$, and the impedance axis varying from 1Ω to $10G\Omega$.

(a) Resistors of magnitude $1k\Omega$, $1M\Omega$, $1G\Omega$ and capacitors of 1nF, 1pF, and 1fF; and inductors of magnitude 1mH, 1μH, 1nH (all 9 of these components should be on the same plot)

Solution:



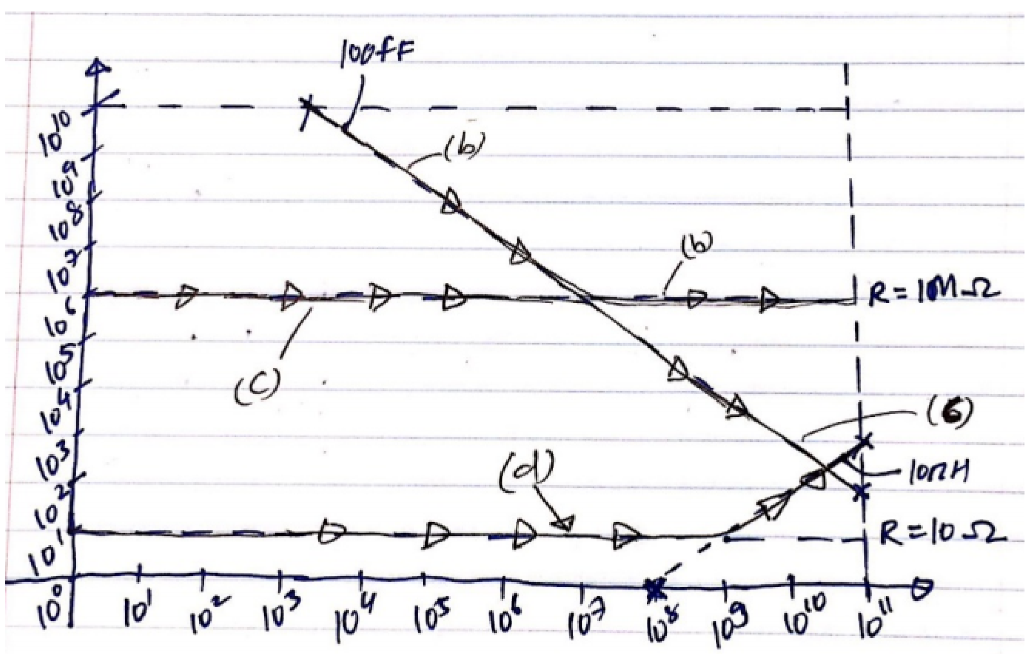
Rubric: (5 Points)

- +2: Correct shape for all plots
- +3: Correct values for all plots

(b) The following three impedances should be on a single plot:

- The series combination of 1MΩ and 100fF
- The parallel combination of 1MΩ and 100fF
- The series combination of 10Ω and 10nH (real inductors always have series resistance)

Solution:



Rubric: (15 Points)

- +2: Correct shape for single plot ($\times 3$)
- +3: Correct values for single plot ($\times 3$)