EE 140/240A Linear Integrated Circuits
Fall 2019 Homework 3

1. Model Mania

(a) For a quadratic model MOSFET, sketch the following curves. No units needed, but try to get the shape right with any significant points labeled, and label with something like "goes as sqrt(X)" or "linear in X". Assume $\lambda = 0$.

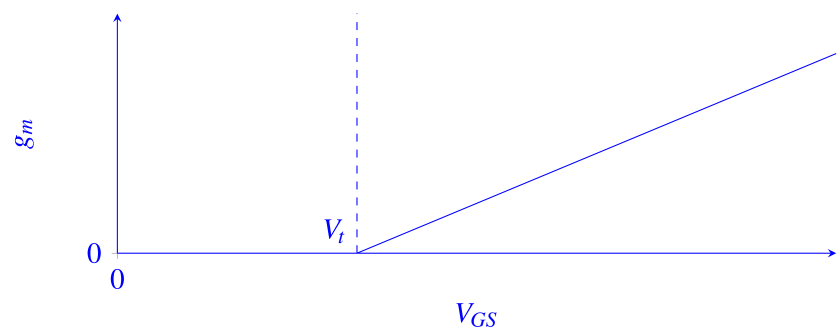
Solution: We'll start with

$$I_D = \frac{1}{2}\mu C_{ox}(V_{GS} - V_t)$$

i. g_m vs. V_{GS} with constant W/L

Solution:

$$g_m = \mu C_{ox}\frac{W}{L}(V_{GS} - V_t)$$



Linear in V_{GS}

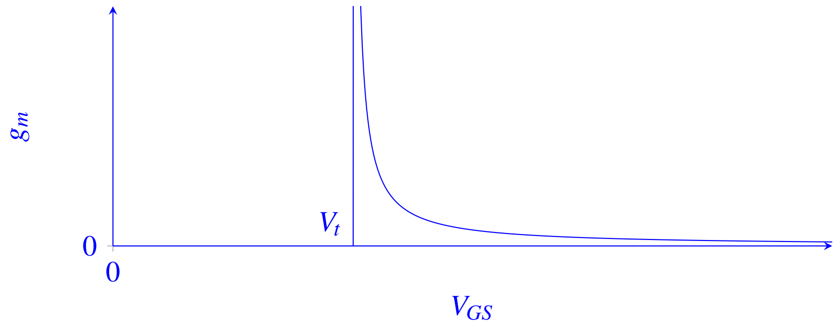
Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

ii. g_m vs. V_{GS} with constant I_D

Solution:

$$g_m = \frac{2I_D}{V_{GS} - V_t}$$



Inverse in V_{GS}

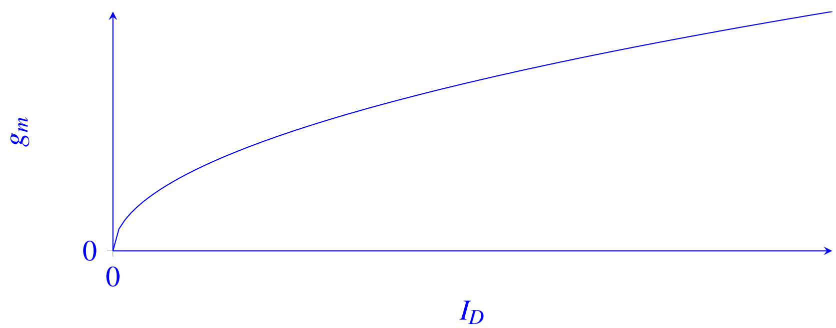
Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

iii. g_m vs. I_D with constant W/L

Solution:

$$g_m = \sqrt{2\mu C_{ox}\frac{W}{L}I_D}$$



Proportional to $\sqrt{I_D}$

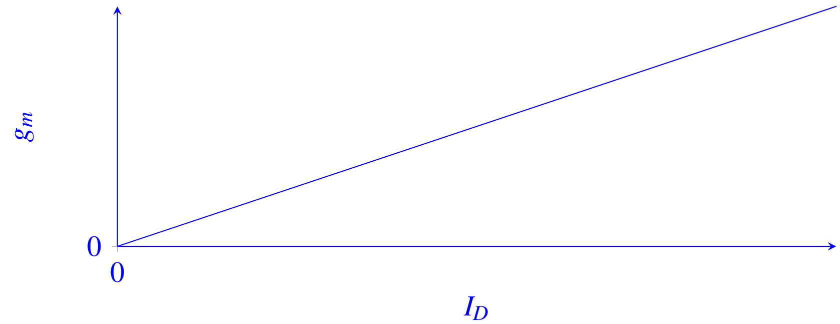
Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

iv. g_m vs. I_D with constant V_{GS}

Solution:

$$g_m = \frac{2I_D}{V_{GS} - V_t}$$



Linear in I_D

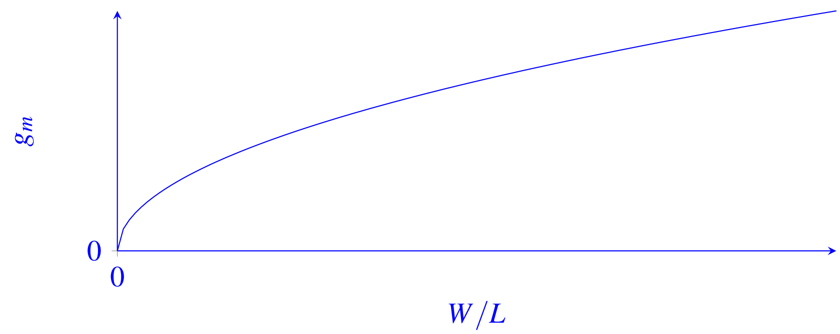
Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

v. g_m vs. W/L with constant I_D

Solution:

$$g_m = \sqrt{2\mu C_{ox}\frac{W}{L}I_D}$$



Proportional to $\sqrt{W/L}$

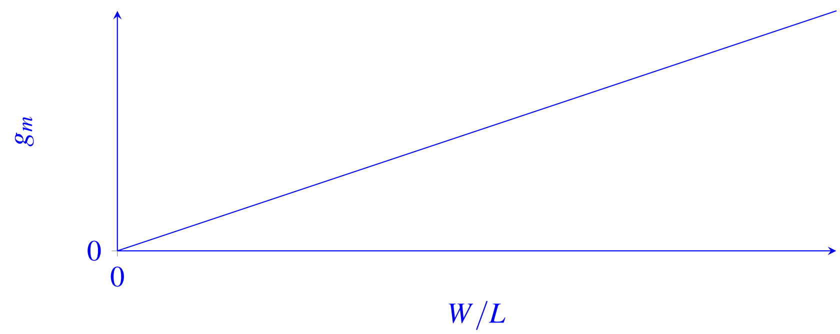
Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

vi. g_m vs. W/L with constant V_{GS}

Solution:

$$g_m = \mu C_{ox}\frac{W}{L}(V_{GS} - V_t)$$



Linear in W/L

Rubric: (3 Points)

- +1: Identifying equation
- +2: Reasonable sketch

(b) **(EE240A)** Repeat questions (a)i., (a)ii., (a)iv., and (a)vi. for subthreshold. You do not need to sketch the plot. You may assume I_S is a constant in all but $\frac{W}{L}$. Do not assume n is necessarily a constant.

Solution: The equation we'll start with is

$$I_D = I_S e^{\frac{V_{GS}}{n} \cdot \frac{q}{k_B T}}$$

where $\frac{W}{L}$ is contained within I_S (linear relationship). Finding g_m:

$$\begin{aligned} g_m &= \frac{\partial i_d}{\partial v_{gs}} \\ &= I_S e^{\frac{V_{GS}}{n} \cdot \frac{q}{k_B T}} \frac{q}{n k_B T} \\ &= I_D \cdot \frac{q}{n k_B T} \\ &= I_D \cdot \frac{\ln\left(\frac{I_D}{I_S}\right)}{V_{GS}} \end{aligned}$$

i. g_m vs. V_{GS} with constant W/L

$$g_m = I_S e^{\frac{V_{GS}}{n} \cdot \frac{q}{k_B T}} \frac{q}{n k_B T}$$

Exponentially increasing in V_{GS}

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

ii. g_m vs. V_{GS} with constant I_D

$$g_m = I_D \cdot \frac{\ln\left(\frac{I_D}{I_S}\right)}{V_{GS}}$$

Inversely proportional to V_{GS}. Note that we want this form for g_m because none of the other elements depend on V_{GS}.

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

iii. g_m vs. I_D with constant V_{GS}

$$g_m = I_D \cdot \frac{\ln\left(\frac{I_D}{I_S}\right)}{V_{GS}}$$

This is not a clean relationship.

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

iv. g_m vs. W/L with constant V_{GS}

$$g_m = I_S e^{\frac{V_{GS}}{n} \cdot \frac{q}{k_B T}} \frac{q}{n k_B T}$$

Roughly linear in W/L, since it's contained within I_S

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

(c) **(EE240A)** Repeat part (a), but for the linear region of operation. You do not need to plot for this subpart.

Solution:

$$\begin{aligned} g_m &= \frac{di_d}{dv_{gs}} \\ &= \mu C_{ox} \frac{W}{L} \\ &= \frac{I_D}{V_{GS} - V_t - \frac{1}{2} V_{DS}} \end{aligned}$$

i. g_m vs. V_{GS} with constant W/L

$$g_m = \mu C_{ox} \frac{W}{L}$$

Constant! How fascinating; this means if for some reason you're in the linear region of operation, changing V_{GS} isn't going to significantly help your g_m.

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

ii. g_m vs. V_{GS} with constant I_D

$$g_m = \frac{I_D}{V_{GS} - V_t - \frac{1}{2} V_{DS}}$$

Approximately inversely proportional.

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

iii. g_m vs. I_D with constant W/L

$$g_m = \mu C_{ox} \frac{W}{L}$$

Constant. The takeaway here is that if you want g_m and you're in the linear region of operation, your devices will likely have to get larger.

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

iv. g_m vs. I_D with constant V_{GS}

$$g_m = \frac{I_D}{V_{GS} - V_t - \frac{1}{2} V_{DS}}$$

Roughly linear

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

v. g_m vs. W/L with constant I_D

$$g_m = \mu C_{ox} \frac{W}{L}$$

Linear

Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

vi. g_m vs. W/L with constant V_{GS}

$$g_m = \mu C_{ox} \frac{W}{L}$$

Linear again!

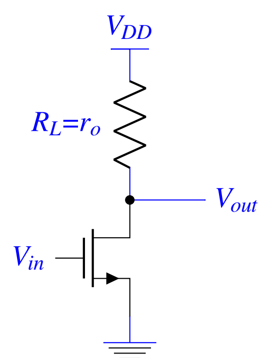
Rubric: (3 Points)

- +1: Identifying equation
- +2: Correct relationship

2. Resistive Load CS Amp Biasing

A common source FET amplifier has a resistive load and $\lambda = \frac{1}{50\text{V}}$. The output is biased at 5V, the transistor is in saturation, and the resistive load has the same impedance as the output resistance of the transistor. What is the supply voltage?

Solution:



Given $R_L = r_o$:

$$\begin{aligned} V_{DD} &= V_{OUT} + I_D R_L \\ &= V_{OUT} + I_D r_o \\ &= V_{OUT} + I_D \frac{1}{\lambda I_D} \\ &= V_{OUT} + \frac{1}{\lambda} \\ &= 5\text{V} + 50\text{V} \\ &= 55\text{V} \end{aligned}$$

$$V_{DD} = 55\text{V}$$

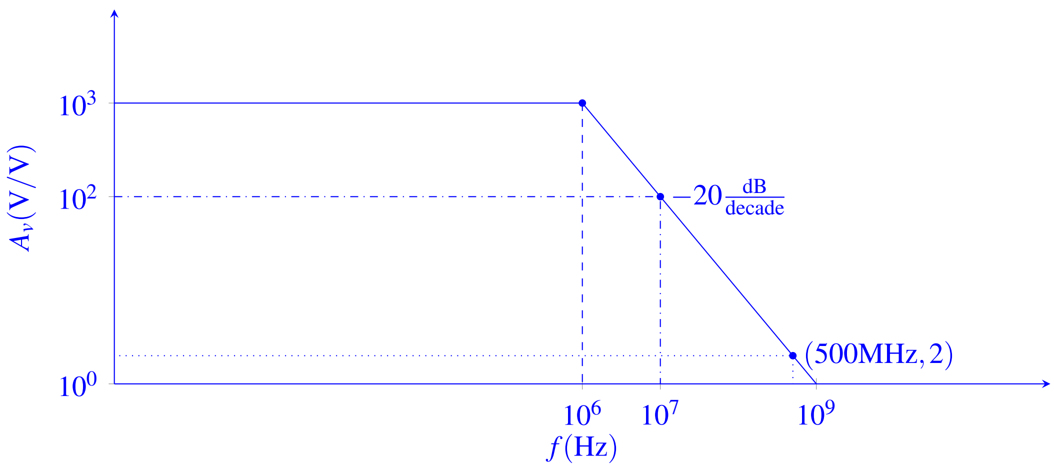
Rubric: (5 Points)

- +1: Correct choice of circuit topology
- +2: V_{DD} Equation
- +2: Correct value of V_{DD}

3. Single-Pole Amplifier

A single-pole amplifier has a low frequency gain of 1000, and a gain of 2 at 500MHz. What are the pole frequency and unity gain frequency in Hz? What is the gain at 10MHz?

Solution: It helps to draw this one out:



Given that this is a single-pole amplifier and the gain at 500MHz $= 2$, we know the unity gain frequency is roughly a third of a decade higher than 500MHz, i.e. a factor of 2, so

$$f_u = 2 \times 500\text{MHz} = 1.0\text{GHz}$$

Since gain $\times$ bandwidth is a constant for the single-pole system,

$$\begin{aligned} f_u &= A_{v0} \times f_p \\ f_p &= \frac{f_u}{A_{v0}} \\ &= \frac{1\text{GHz}}{1000} \\ &= 1\text{MHz} \end{aligned}$$

And now we want the gain at 10MHz, i.e. one decade above the pole frequency

$$A_V(10\text{MHz}) = \frac{1000}{10} = 100\text{V/V}$$

$$f_u = 1.0\text{GHz}$$
$$f_p = 1\text{MHz}$$
$$A_V(10\text{MHz}) = 100\text{V/V}$$

Rubric: (5 Points)

- +1: Correct unity gain frequency equation
- +1: Correct unity gain frequency with correct units
- +1: Correct pole frequency with correct units
- +2: Correct gain with correct units

4. Settling Time

Write down a table of settling error vs. time for $\frac{t}{\tau} = \{1,3,5,7\}$. Use a calculator for this one. Memorize the table to one significant digit.

Solution:

t/τ	$v(t) = v_0\left(1 - e^{-t/\tau}\right)$	% Error
1	0.632	36.8%
3	0.950	5.0%
5	0.993	0.7%
7	0.9991	0.09%

Rubric: (4 Points)

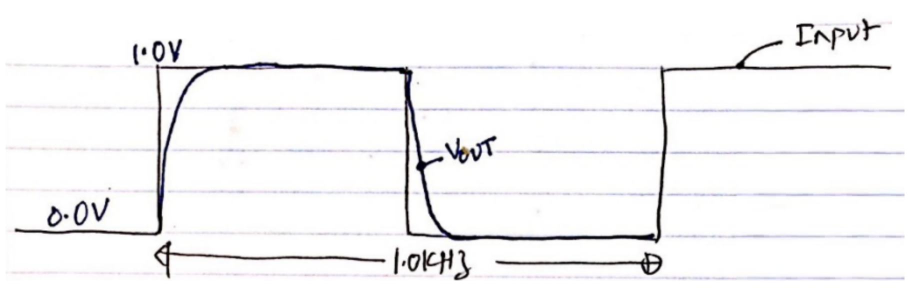
- +1: Correct error value (×4)

5. RC Low-Pass Filter Transient

An RC low pass filter with a time constant of 1μs is driven with a $0-1$V square wave. Sketch the first full cycle of the input and output when the square wave is first turned on, and the frequency of the square wave is

(a) 1kHz

Solution:

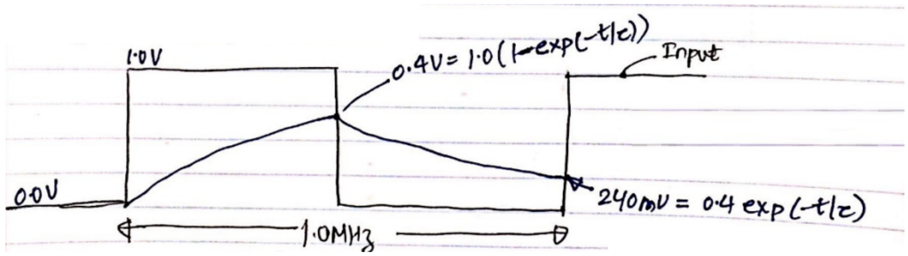


Rubric: (3 Points)

- +3: Correct waveform

(b) 1MHz

Solution:

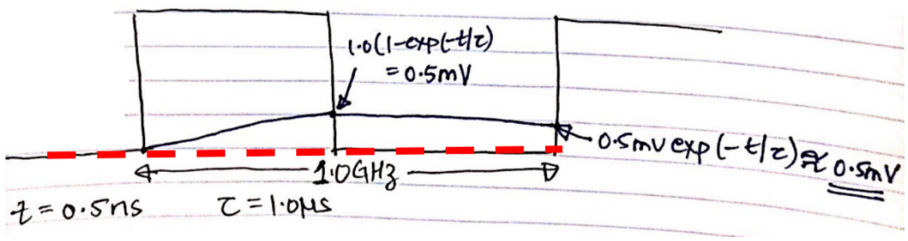


Rubric: (5 Points)

- +2: Correct waveform
- +3: Correct values of magnitude

(c) 1GHz

Solution:



Rubric: (2 Points)

- +2: Correct waveform. The plot doesn't have to be to scale.

6. Relationships for Your Cheat Sheet

Fill in the following table for a single-transistor common-source amplifier:

A_{v0} (V/V)	ω_p (rad/s)	ω_u (rad/s)	g_m ($1/\Omega$)	r_o (Ω)	C_L (F)
1000	1M				1p
	1M	0.1G		100k	
		10G		1M	20f
	10	10M			10p

Solution: The often-used equations of interest (that are useful for your cheat sheet):

$$A_{v0} = |g_m r_o|$$

$$\omega_p = \frac{1}{r_o C_L}$$

$$\omega_u = A_{v0}\omega_p = \frac{g_m}{C_L}$$

A_{v0} (V/V)	ω_p (rad/s)	ω_u (rad/s)	g_m ($1/\Omega$)	r_o (Ω)	C_L (F)
1000	1M	**1G**	**1m**	**1M**	1p
100	1M	0.1G	**1m**	100k	**10p**
200	**50M**	10G	**0.2m**	1M	20f
1M	10	10M	**0.1m**	**10G**	10p

Rubric: (12 Points)

- +1: Each correct value (×12)