EE 140/240A Linear Integrated Circuits
Fall 2019 Homework 6

This homework is due October 18, 2019 (late October 19, 2019 09:00).

Submission Format
Your homework submission should consist of **one** file.

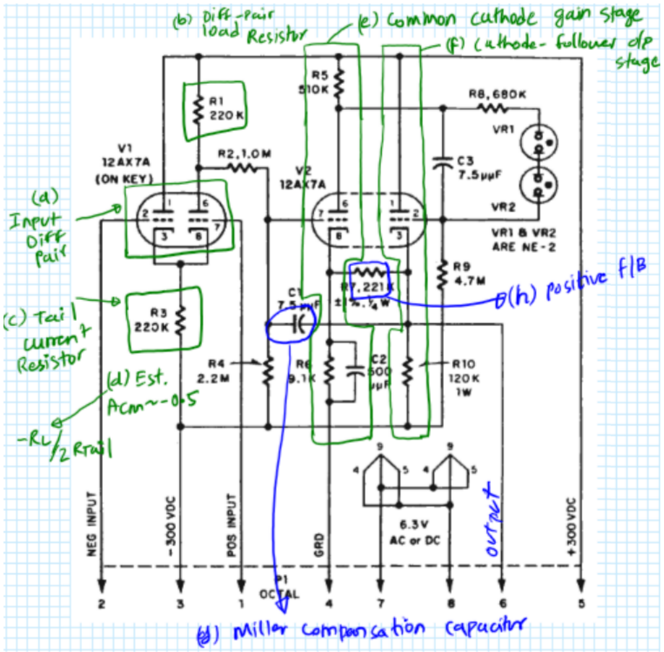
- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

1. K2-W

Check out the datasheet for the K2-W tube op-amp: http://philbrickarchive.org/k2-w_operational_amplifier_later.htm. This op-amp, released in 1952, was the first production op-amp. It runs from a ±300V supply, and has a bandwidth of 300kHz (or k-cycles/s, as they said back then—the unit Hertz not having been established yet). There's a schematic on page 2. Pins 1, 2, and 6 on the bottom of the figure are V_+, V_-, and V_{out}. V_{R1} and V_{R2} are neon bulbs that provide a low impedance level shift of roughly 100V to center the output between the rails. Identify (circle and label):

(a) input differential pair
(b) diff-pair load resistor
(c) tail current resistor
(d) Estimate the common mode gain and write it near the tail resistor
(e) Common-cathode gain stage (like CS or CE)
(f) Cathode-follower output stage (like source-follower or emitter follower, CD, CC)
(g) Miller-multiplied compensation capacitor from the output back to the input of the gain stage
(h) **(BONUS)** positive feedback in this amplifier, designed to increase the low frequency gain (which ended up at about 20,000)

Solution:



You can find more info on the K2-W here: https://www.electronicdesign.com/analog/whats-all-k2-w-stuff-anyhow

Rubric: (8 Points)

- +1: For each correct marking (with no extra devices)

2. More Single-Pole Amplifiers

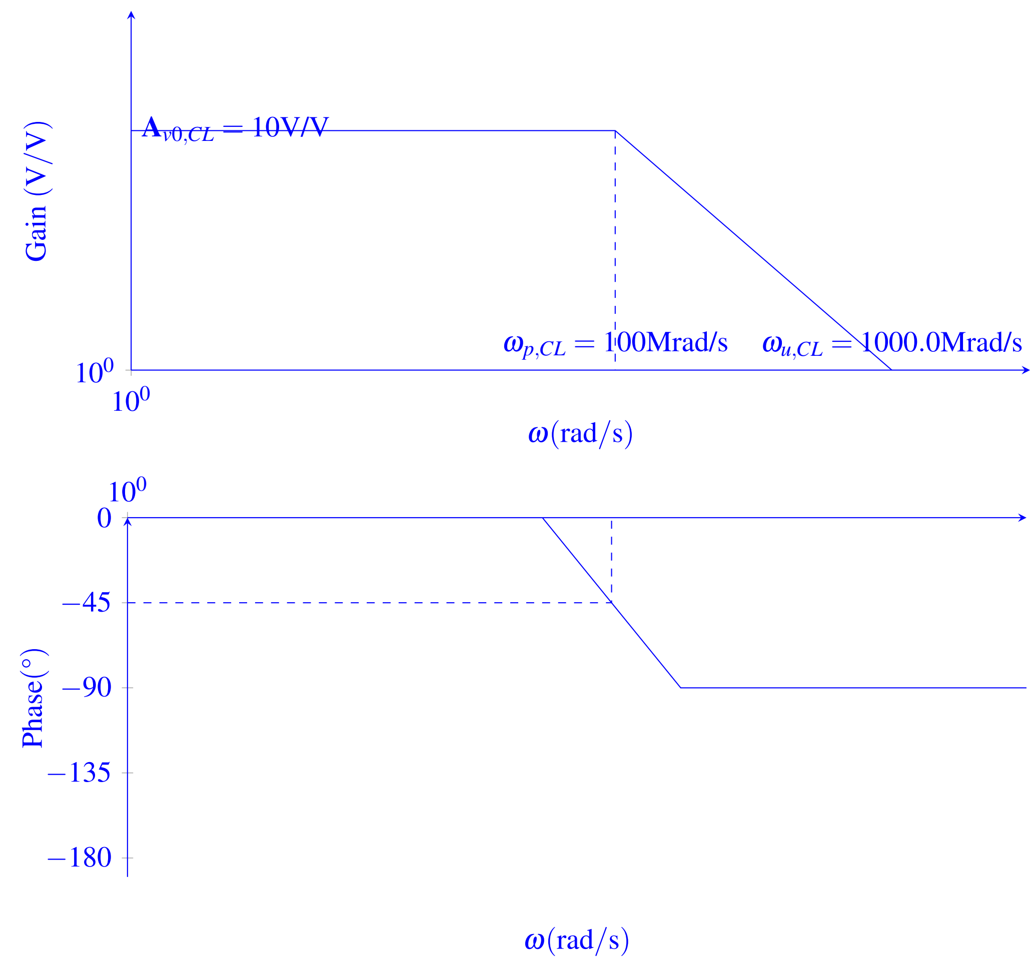
You have an opamp with a low-frequency gain of 1000 and a single pole at 1Mrad/s. Plot the location of the pole as a function of the feedback factor f from $f = [0, 1]$.

Solution:

Rubric: (3 Points)

- +1: Pole location with $f = 0$
- +1: Pole location with $f = 1$
- +1: Trajectory of pole location correct

(a) **From now on, assume f = 0.1**. Sketch the Bode plot of the closed-loop amplifier.
Solution:



Rubric: (2 Points)

- +1: Correct 3dB frequency
- +1: Correct closed-loop gain

(b) What is the fractional gain error?

Solution: Following the equation for fractional gain error:

$$-\frac{1}{Af}$$

$$-1 \cdot 10^{-2}$$

Rubric: (2 Points)

- +1: Correct equation
- +1: Correct numerical answer

(c) What is the time constant of the step response? How does it compare to the open-loop time constant?

Solution:

$$\tau_{\text{OL}} = \frac{1}{\omega_{p,\text{OL}}} \qquad \tau_{\text{CL}} = \frac{1}{\omega_{p,\text{OL}} A_0 f}$$

$$\tau_{\text{OL}} = 1\mu\text{s}$$

$$\tau_{\text{CL}} = 1 \cdot 10^{-2}\mu\text{s}$$

The closed loop time constant is faster than the open-loop time constant by a factor of the loop gain $A_0 f$

Rubric: (2 Points)

- +1: Correct closed-loop time constant
- +1: Correct comparison to open-loop time constant

(d) What is the unity gain frequency? How does it compare to the open-loop unity gain frequency?
Solution:

The unity gain frequency stays constant

$$\omega_u = 1000.0\frac{\text{Mrad}}{\text{s}}$$

Rubric: (2 Points)

- +1: Correct ω_u
- +1: Correct comparison between closed-loop and open-loop unity gain frequency

3. Now With Three Poles!

You have an opamp with a low-frequency gain of 1000 and **three** poles at 1Mrad/s.

(a) Plot the location of the three poles as a function of the feedback factor f.
Solution: First, define ω_p as the open loop pole frequency.

The open-loop transfer function:

$$A_{\text{OL}}(s) = \frac{A_0\omega_p^3}{(s+\omega_p)^3}$$

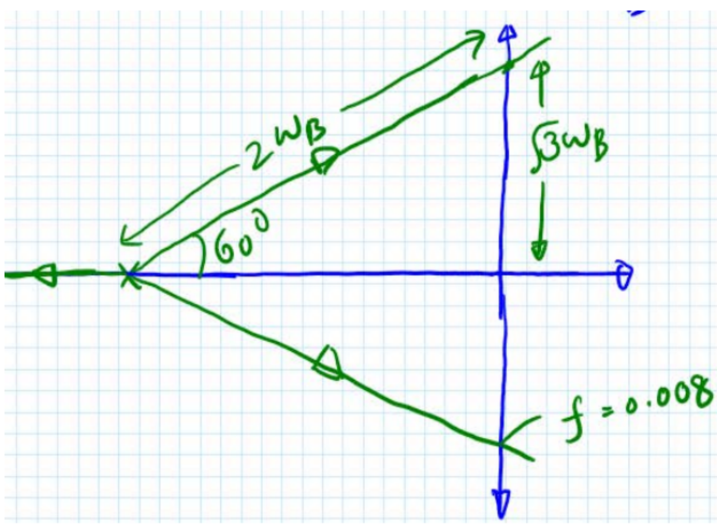
And plug this into the closed loop transfer function equation:

$$A_{\text{CL}}(s) = \frac{A_{\text{OL}}(s)}{1+A_{\text{OL}}(s)f} = \frac{A_0\omega_p^3}{(s+\omega_p)^3 + A_0 f\omega_p^3}$$

The poles of the characteristic equation can be found by setting the denominator to 0:

$$(s+\omega_p)^3 + A_0 f \omega_p^3 = 0$$
$$s+\omega_p = \sqrt[3]{A_0 f}\,\omega_p \cdot \exp\left(j\phi_i\right), \phi_i = 180 \pm 60^\circ$$
$$s = -\omega_p\left(1+\sqrt[3]{A_0 f}e^{\pm j\frac{\pi}{3}}\right)$$

where the portion of the expression above with the complex exponential gives the angle and magnitude of the vector which progresses from the initial pole location.



Rubric: (9 Points)

- +1: Calculated correct pole location in terms of f ($\times 3$)
- +1: Correct starting point when $f = 0$ ($\times 3$)
- +1: Correct angle of trajectory for each pole as f changes ($\times 3$)

(b) At the point where the poles cross the $j\omega$ axis, annotate the plot with the value of f that gives this pole location.

Solution:

Using our answer to the previous part and with some triangle geometry, at the $j\omega$ axis, the real part is 0, so

$$2\omega_p = \sqrt[3]{A_0 f}\,\omega_p$$
$$f = \frac{8}{A_0}$$
$$= 0.008$$

See the plot above for the annotation.

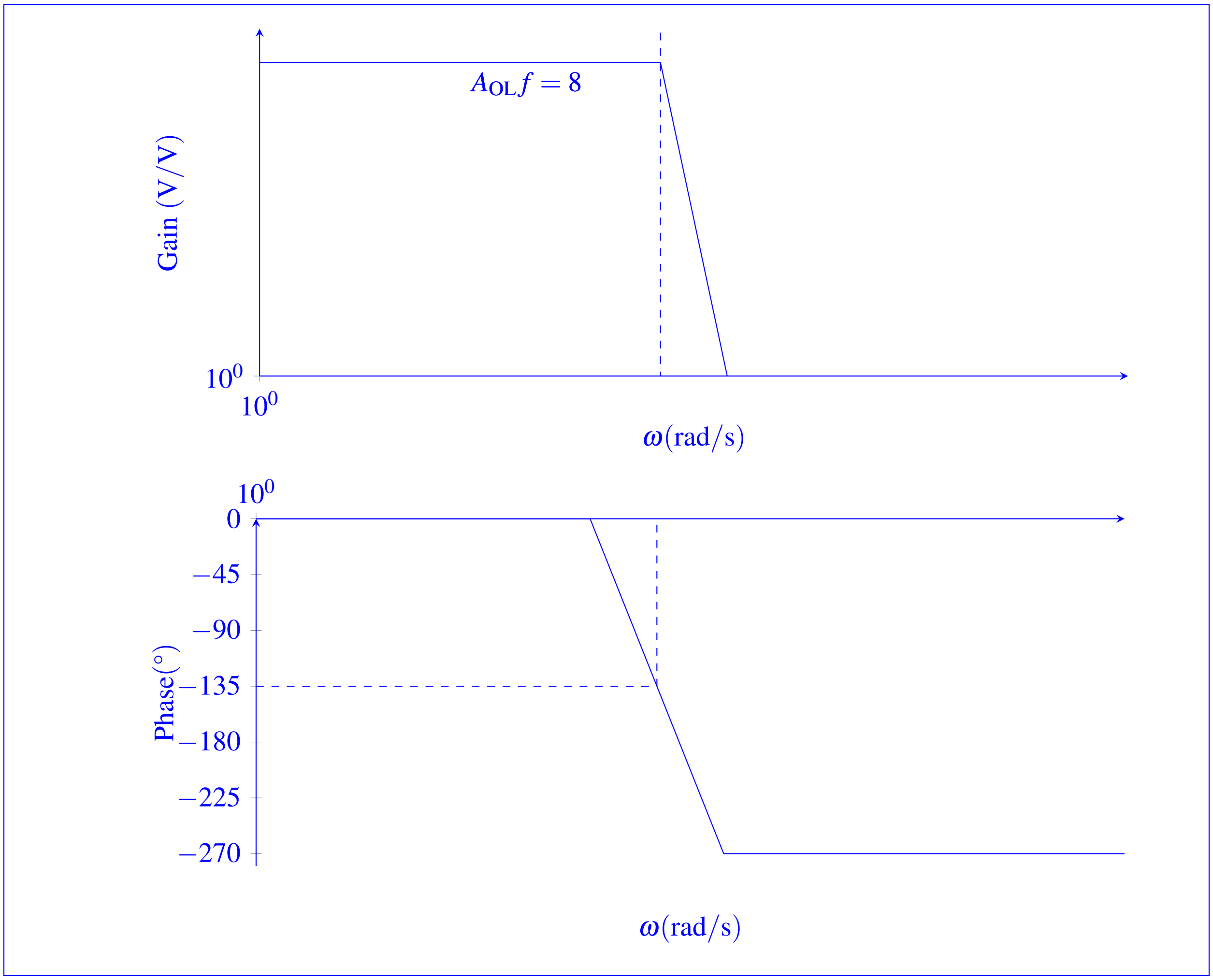
Rubric: (2 Points)

- +1: Set real portion to 0 in expression for poles in previous part (don't double-penalize yourself—even if your expression for the pole location earlier was incorrect, you should still give yourself credit if you went through the correct process here)
- +1: Correctly calculated f given the expression from the previous part

(c) Using this value for f, draw a Bode plot of the loop gain Af

Solution:

$$A_{\text{OL}} f = \frac{8}{\left(1+\frac{s}{\omega_p}\right)^3}$$

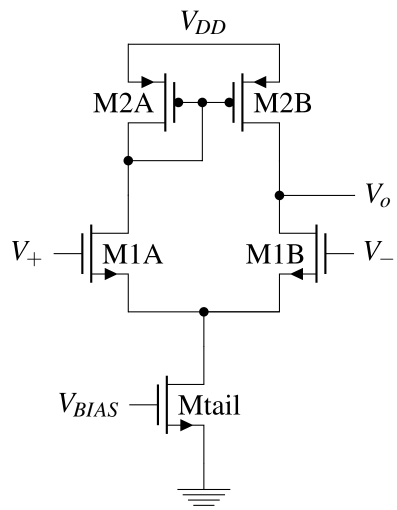


Rubric: (3 Points)

- +1: Correct DC Af
- +1: Correct pole location *and* −60dB/decade slope
- +1: ω_u of $Af < 10^7 \, \frac{\text{rad}}{\text{s}}$

4. Virtual Ground Doesn't Fly

Estimate the output resistance of a CMOS differential amplifier with current mirror load.



You may assume that $g_m r_o \gg 1$ for all combinations of g_m and r_o. The following steps may help:

(a) Estimate the impedance seen looking into the source of M1A

Solution:

$$R_a = \frac{\frac{1}{g_{m2a}} + r_o}{1 + g_{m1a} r_o}$$
$$\approx \frac{r_o}{g_{m1a} r_o}$$

$$\boxed{R_a \approx \frac{1}{g_{m1a}}}$$

Rubric: (1 Points)

- +1: Correct impedance estimate

(b) Estimate the impedance seen looking down from the source of M1B

Solution:

$$R_b = \frac{1}{g_{m1a}} || r_{o3}$$
$$\approx \frac{1}{g_{m1a}}$$

$$\boxed{R_b \approx \frac{1}{g_{m1a}}}$$

Rubric: (1 Points)

- +1: Correct impedance estimate

(c) Estimate the impedance seen looking into the drain of M1B

Solution:

$$\begin{aligned} R_c &= R_b + r_o(1 + g_m R_b) \\ &\approx r_o(1 + g_m R_b) \\ &\approx 2r_o \end{aligned}$$

$$\boxed{R_c \approx 2r_o}$$

Rubric: (1 Points)

- +1: Correct impedance estimate

(d) For the R_o calculation, estimate i_{d1B} as a function of v_o.

Solution:

$$\begin{aligned} i_{d1B} &= \frac{v_o}{R_{\text{out}}} \\ &= \frac{v_o}{2r_{o1B}} \end{aligned}$$

$$\boxed{i_{d1B} = \frac{v_o}{2r_{o1B}}}$$

Rubric: (1 Points)

- +1: Correct relationship between i_{d1B} and v_o

(e) The current i_{d2B} is due to both the output resistance and the mirrored current. Estimate both parts.

Solution:

$$\begin{aligned} i_{d2B} &= (\text{mirrored current}) + (\text{current due to output resistance}) \\ &\approx i_{d1B} + \frac{v_o}{r_{o2B}} \\ &= \frac{3v_o}{2r_o} \end{aligned}$$

$$\boxed{i_{d2B} = \frac{3v_o}{2r_o}, r_o = r_{o1B} = r_{o2B}}$$

Rubric: (2 Points)

- +1: Correct mirrored current
- +1: Correct current due to output resistance

(f) Estimate the total output current $i_o = i_{d1B} + i_{d2B}$.

Solution:

$$\begin{aligned} i_o &= i_{d1B} + i_{d2B} \\ &\approx \frac{v_o}{2r_{o1B}} + \frac{v_o}{2r_{o1B}} + \frac{v_o}{r_{o2B}} \\ &= v_o\left(\frac{1}{r_{o1B} + r_{o2B}}\right) \end{aligned}$$

$$\boxed{i_o \approx v_o\left(\frac{1}{r_{o1B} + r_{o2B}}\right)}$$

Rubric: (1 Points)

- +1: Correct total output current given previous answers (don't double-penalize)

(g) Show that $R_o \approx r_{o1B}||r_{o2B}$. Magic!

Solution:

$$\begin{aligned} R_o &= \frac{v_o}{i_o} \\ &\approx \frac{1}{\left(\frac{1}{r_{o1B}} + \frac{1}{r_{o2B}}\right)} \\ &= r_{o1B}||r_{o2B} \end{aligned}$$

$$\boxed{R_o \approx r_{o1B}||r_{o2B}}$$

Rubric: (1 Points)

- +1: Correct final calculation

5. (EE240A) More Poles

A single-stage op-amp has a low frequency gain of 200 and a dominant pole at 10Mrad/s.

(a) Draw the s-plane with the real axis from -10^7 to 0, and the imaginary axis from 0 to 10^7. Mark the pole location and draw a dot at $10^7 j$.

Solution: See the solution for part (c)

Rubric: (2 Points)

- +1: Correct imaginary and real axes
- +1: Correct pole location

(b) Draw the vector from the pole to $10^7 j$. Calculate the magnitude and phase of this vector.

Solution:

$$\text{magnitude} = \sqrt{2} \cdot 10^7 \qquad \text{phase} = -\arctan\left(\frac{10^7}{-10^7}\right) = 45^\circ$$

See the solution for part (c) for the plot.

$$\text{magnitude} = \sqrt{2} \cdot 10^7$$

$$\text{phase} = 45^\circ$$

Rubric: (3 Points)

- +1: Drew vector
- +1: Correct magnitude
- +1: Correct phase

(c) Draw a dot at $10^6 j$. Draw the vector from the pole to $10^6 j$. Calculate the magnitude and phase of this vector.

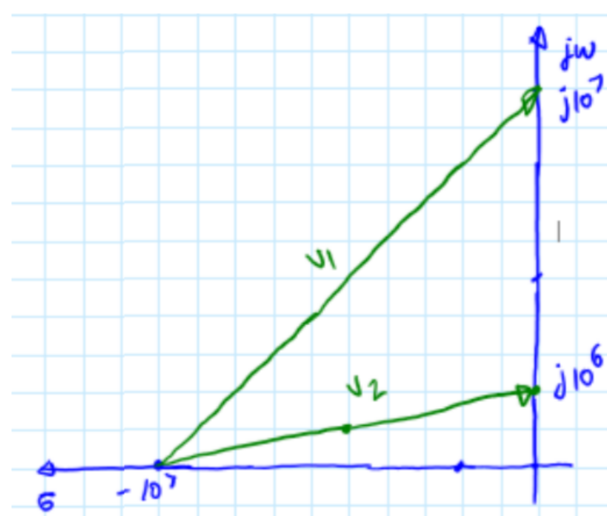
Solution:

$$\text{magnitude} = \sqrt{(10^7)^2 + (10^6)^2} \approx 10^7$$

$$\text{phase} = -\arctan\left(\frac{10^6}{10^7}\right) \approx 0.1\text{rad} \longleftarrow \text{small angle approximation} \approx 5.73^\circ$$

$$\text{magnitude} \approx 10^7$$

$$\text{phase} \approx 5.73^\circ$$



Rubric: (3 Points)

- +1: Drew vector and dot
- +1: Correct magnitude
- +1: Correct phase

(d) Repeat parts (a) and (b), but with the imaginary axis from 0 to 10^8 and the dot at $10^8 j$. Keep the pole in the same location.

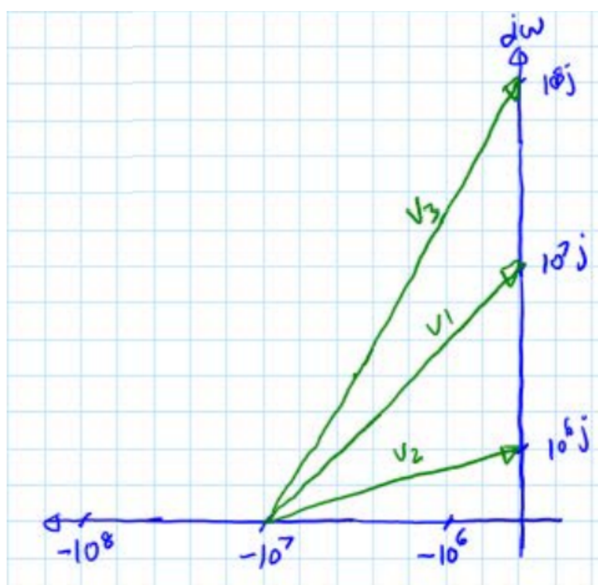
Solution:

$$\text{magnitude} = \sqrt{(10^7)^2 + (10^8)^2} \approx 10^8$$

$$\text{phase} = -\arctan\left(\frac{10^8}{10^7}\right) \approx 1.47\text{rad} \approx 84.3°$$

$$\text{magnitude} \approx 10^8$$

$$\text{phase} \approx 84.3°$$

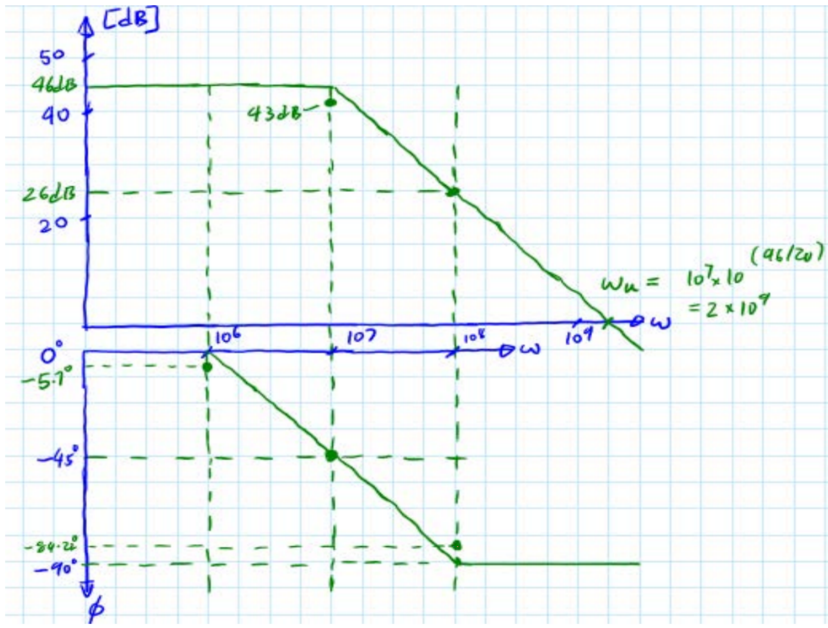


Rubric: (3 Points)

- +1: Drew vector
- +1: Correct magnitude
- +1: Correct phase

(e) Draw a Bode plot of the gain of your amplifier, with frequency running from 10^5 to 10^9rad/s. Use the straight-line approximations for the Bode plot, and then add dots showing the results of parts (b), (c), and (d).

Solution:



Rubric: (4 Points)

- +1: Correct DC magnitude
- +1: Correct pole frequency
- +1: Correct magnitude slope
- +1: Correct phase start and end values

6. Compensating for Something

A two-stage CMOS op-amp running at a particular bias point has the following parameters:

- $G_{m1} = 1\text{mS}$
- $R_{o1} = 1\text{M}\Omega$
- $C_1 = 0.1\text{pF}$
- $C_C = 0\text{pF}$
- $G_{m2} = 1\text{mS}$
- $R_{o2} = 100\text{k}\Omega$
- $C_2 = 10\text{pF}$

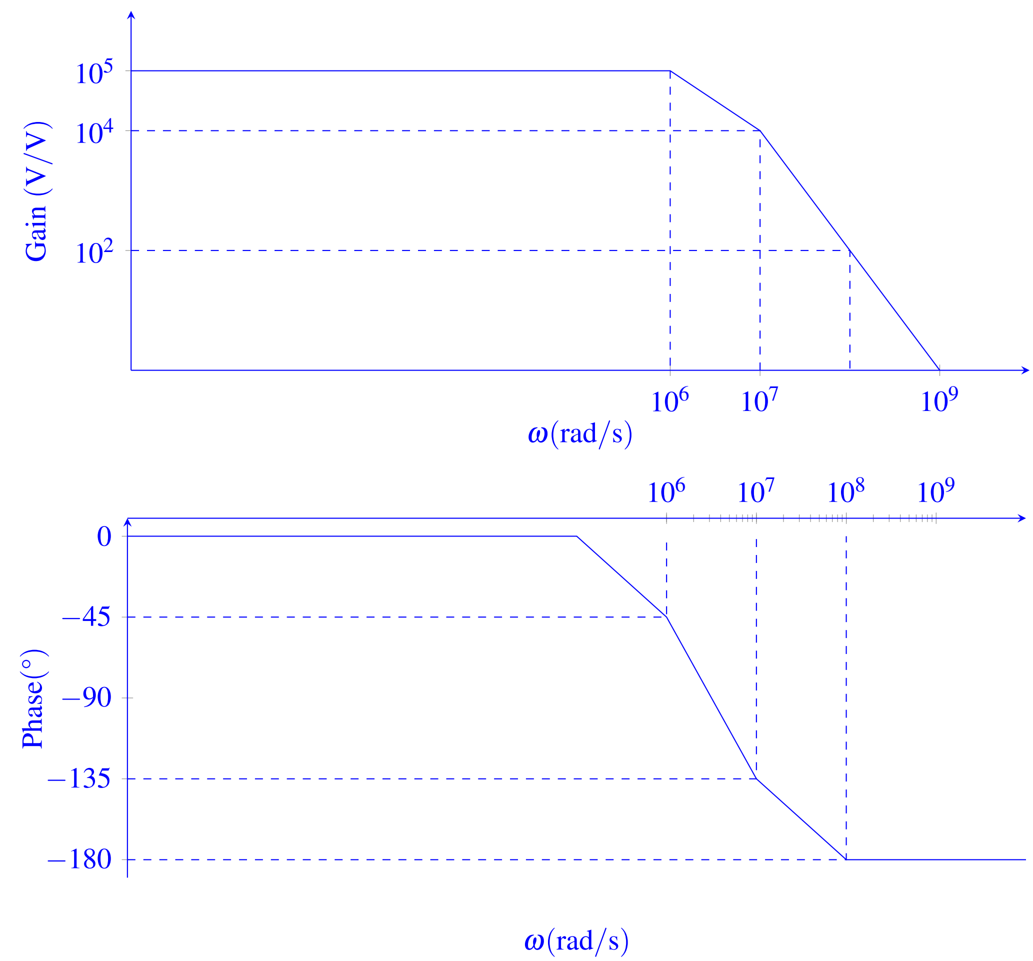
(a) Plot the magnitude and phase of the overall gain of this uncompensated amplifier.

Solution:

$$\omega_{p1} = \frac{1}{R_{o1}C_1} = 10^7\,\frac{\text{rad}}{\text{s}}$$

$$\omega_{p2} = \frac{1}{R_{o2}C_2} = 10^6\,\frac{\text{rad}}{\text{s}}$$

$$A_{v0} = G_{m1}R_{o1}G_{m2}R_{o1} = 10^5\,\frac{\text{V}}{\text{V}}$$



Rubric: (6 Points)

- +1: Correct pole frequency (2×)
- +1: Correct DC gain
- +1: Correct phase relationship about pole frequencies (2×)
- +1: Correct unity gain frequency

(b) Where are the poles of the uncompensated amplifier? Is it unity-gain stable?

Solution: See the plot in part (a)

$$\omega_{p,1} = 10^7 \frac{\text{rad}}{\text{s}}, \omega_{p,2} = 10^6 \frac{\text{rad}}{\text{s}}$$

No, the amplifier is not unity gain stable.

Rubric: (1 Points)

- +1: Correct answer of if the amplifier is unity gain stable

7. Continuing... For the same amplifier above, we now add $\mathbf{C_C = 1}$pF. You may ignore the RHP zero that this introduces. On the figures provided below,

(a) Plot the magnitude of the second stage gain vs. frequency.

Rubric: (3 Points)

- +1: Correct DC gain
- +1: Correct pole frequency
- +1: 20dB/decade drop-off

(b) Plot the magnitude of the input *capacitance* of the second stage (including Cc) vs. frequency.

Rubric: (4 Points)

- +1: Correct DC capacitance (if you didn't include C_1 that's fine)
- +1: Correct high-frequency capacitance (fine if you didn't include C_1)
- +1: Correct pole location for gain dropping the capacitance
- +1: Correct ω_{u2} location where C_C no longer Millerizes

(c) Plot the magnitude of the input *impedance* of the second stage vs. frequency. Add a line for the output impedance of the first stage.

Rubric: (4 Points)

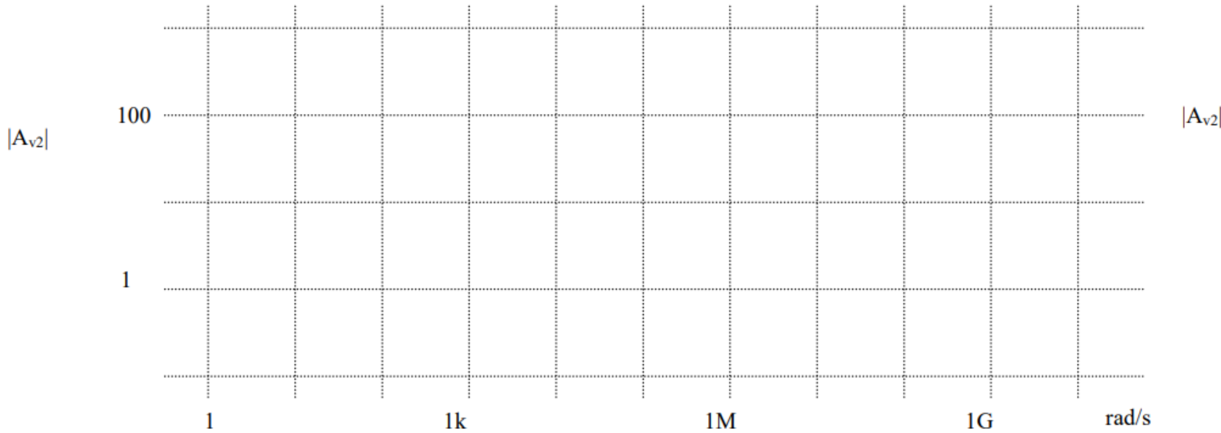
- +1: Correct low frequency R_{o1}
- +1: Correct impedance line for Millerized C_C
- +1: Correct zero location in the impedance when Miller effect begins to decrease
- +1: Correct impedance line for non-Millerized C_C

(d) Now plot the magnitude of the gain of the first stage on the top plot, and the magnitude of the overall gain of the amplifier.

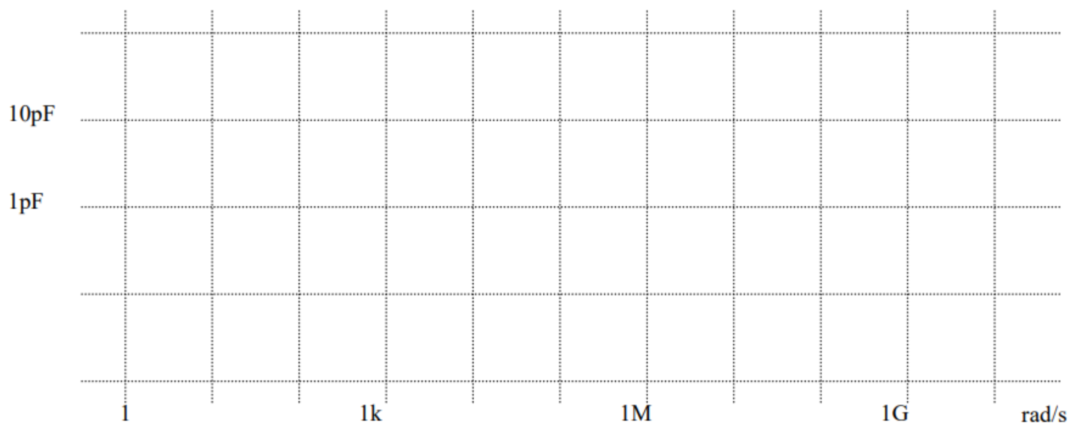
Rubric: (9 Points)

- +1: Correct DC gain of first stage
- +1: Correct pole and zero locations of the first stage gain (3×)
- +1: Correct DC gain of combined stages
- +1: Correct pole locations and slope of combined stages (4×)

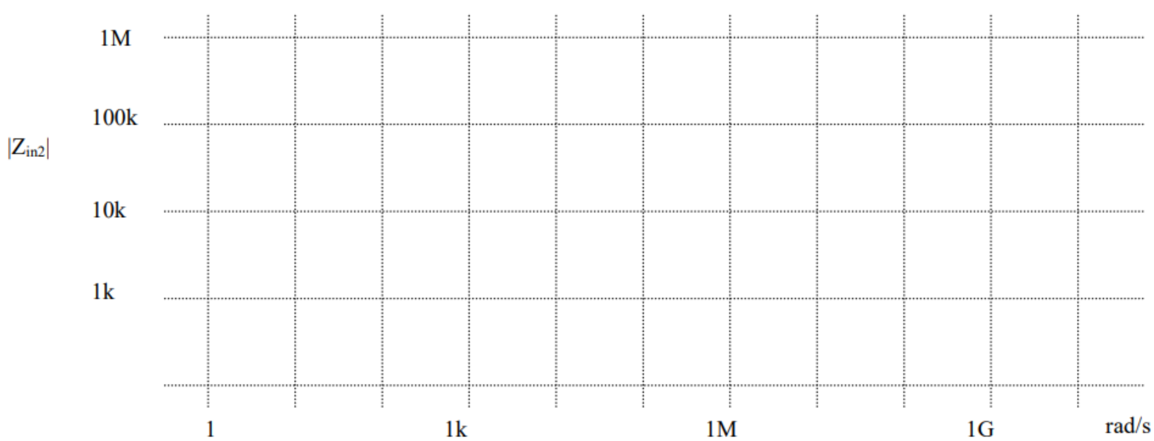
Second stage gain – $|A_{v2,0}|$, and first stage and overall gains



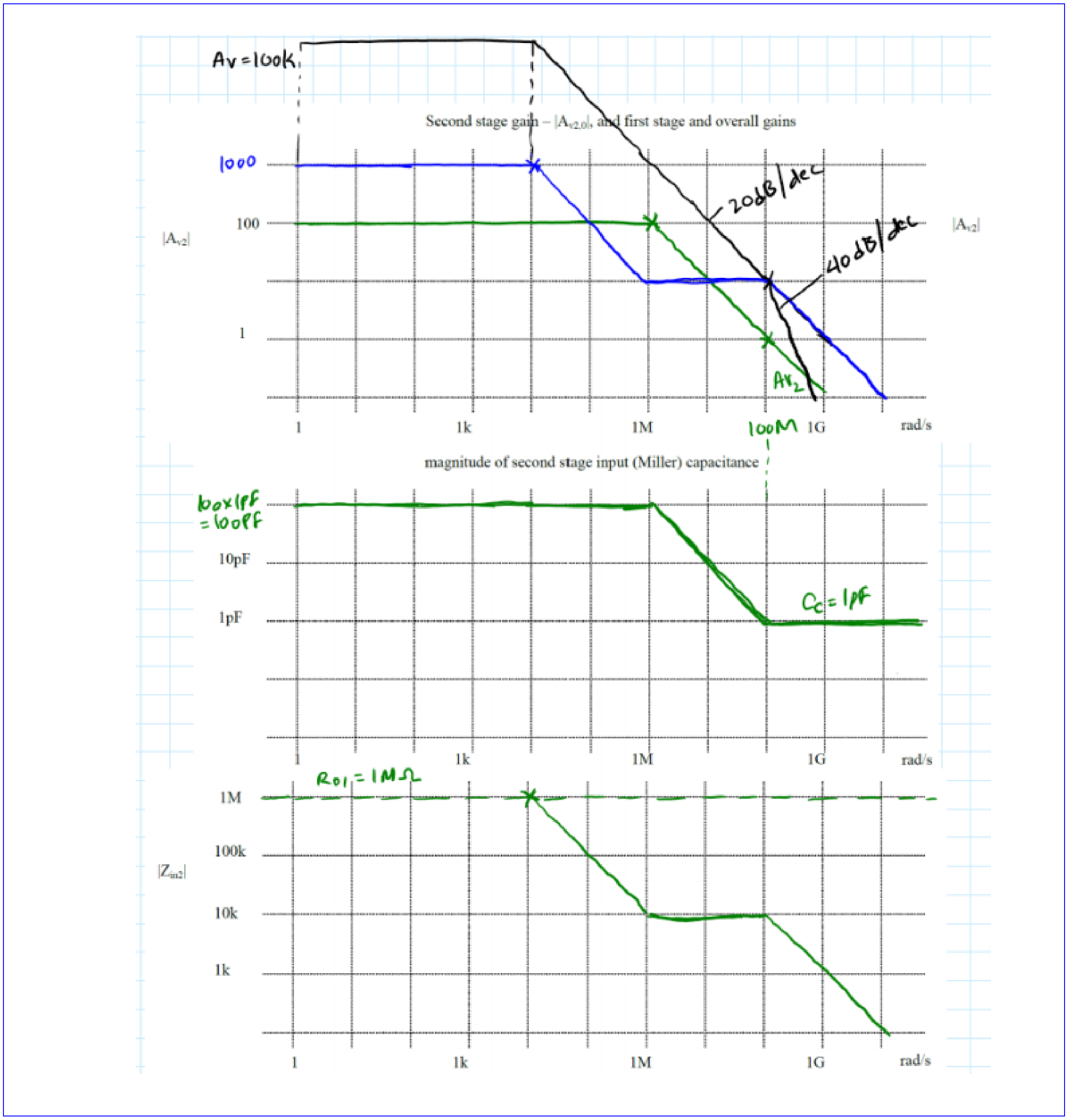
magnitude of second stage input (Miller) capacitance



second stage input impedance, and R_{o1}



Solution:



(e) What are the compensated poles of the amplifier? If C_c were 0pF, where would the poles of the amplifier be?

Solution:

	ω_{p1} (rad/s)	ω_{p2} (rad/s)
Uncompensated	10^7	10^6
Compensated	10^4	10^8

Rubric: (4 Points)

- +1: Correct compensated poles of the amplifier (2×)
- +1: Correct uncompensated poles of the amplifier (2×)

8. Virtual Ground Is A Lie

(EE240A) For a standard 5 transistor CMOS differential amplifier show that the gain from a differential input to the (so called virtual ground!) tail voltage is $\frac{1}{4}$. You can assume that $g_m r_o \gg 1$ for all combinations of g_m and r_o. You can win bets with experienced IC designers with this knowledge!

Solution:

Estimating G_m:

$$\begin{aligned} v_{mirr} &\approx -\frac{v_i}{2} \\ v_d &\approx g_m v_i \left(\frac{r_o}{2}\right) \\ i_o &\approx -\frac{v_x}{r_o} - \frac{v_d}{r_o} \\ &\approx \frac{v_i}{2r_o} - \frac{g_m}{2} v_i \\ G_m &\approx -\frac{g_m}{2} \end{aligned}$$

Estimating R_o

$$\begin{aligned} R_o &\approx r_o || \frac{r_o + \frac{1}{g_m}}{1 + g_m r_o} || \frac{r_o + r_o}{1 + g_m r_o} \\ &\approx r_o || \frac{1}{g_m} || \frac{2}{g_m} \\ &\approx \frac{2}{3g_m} \\ &\approx \frac{1}{2g_m} \text{ if you ignore the additional } r_o \text{ on the non-diode connected branch} \end{aligned}$$

Rubric: (4 Points)

- +1: Correct G_m estimate with correct sign
- +1: Correct R_o estimate (using $\frac{2}{3g_m}$ is acceptable)
- +2: Correct gain with correct sign

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