Single pole response

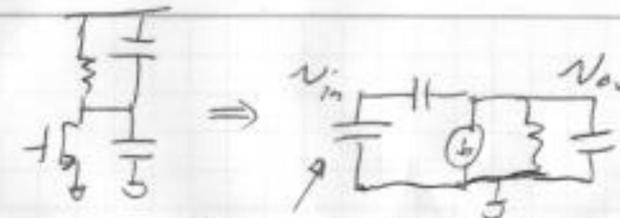
very common
effect of g_m, R_o, C_o

poles & zeros, ODEs

time domain
sinusoids, settling time

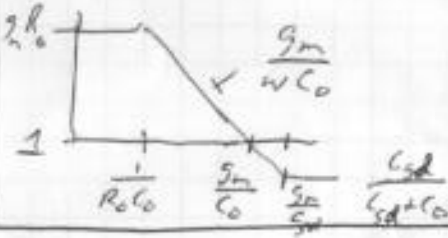
CS amp design

Last time



is more if driving w/ voltage source (why?)

diagram
same for PMOS input
current source load
active load



Poles? Zeros?

$u(t)$ — [LTI] — $y(t)$

$\downarrow \mathcal{L}$

$U(s)$ [$H(s)$] $Y(s)$

circuit designers are lazy, and use some small signal variables for time and freq.

$$y(t) + a\frac{dy}{dt} = u(t)$$

$$\downarrow$$

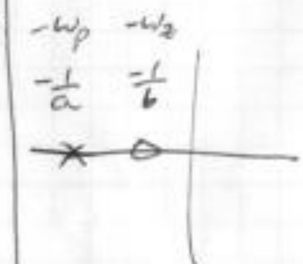
$$Y + asY = U$$

$$H(s) = \frac{Y}{U} = \frac{1}{1+as} = \frac{1}{1+s/\omega_p} \qquad \omega_p = \frac{1}{a}$$

$$y(t) + a\frac{dy}{dt} = u(t) + b\frac{du}{dt}$$

$$(1+as)Y = (1+bs)U \qquad \omega_z = \frac{1}{b}$$

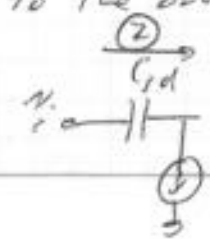
$$H(s) = \frac{Y}{U} = \frac{1+bs}{1+as} = \frac{1+s/\omega_z}{1+s/\omega_p} \qquad \omega_p = \frac{1}{a}$$



complex plain

poles & zeros

a zero means ≥2 paths from the input to the output



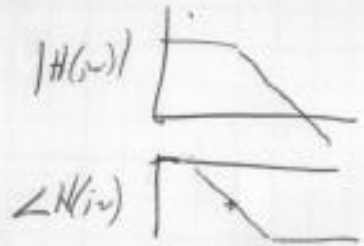
time domain

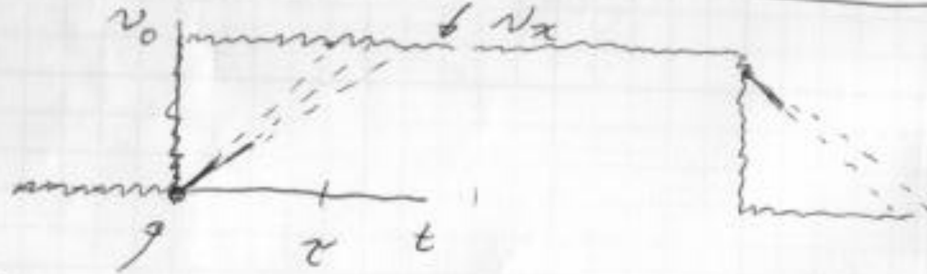
$$A\sin(\omega t) \longrightarrow \boxed{H(s)} \longrightarrow |H(j\omega)|\,A\sin(\omega t + \angle H(j\omega))$$

$$\sum A_i \sin(\omega_i t + \phi_i) \longrightarrow \boxed{H(s)} \longrightarrow \sum |H(j\omega_i)|\,A_i \sin(\omega_i t + \phi_i + \angle H(j\omega_i))$$

So knowing $|H(j\omega)|$ and $\angle H(j\omega)$ is important

$\Rightarrow$ Bode Plot





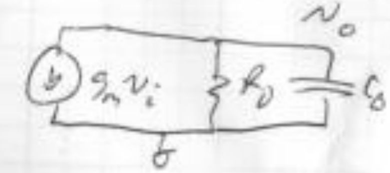
$$\frac{dv_o}{dt} = \frac{v_x - v_o}{\tau} = \frac{v_x}{\tau}$$

$$\cancel{\frac{dv_o}{dt}}\, e^{-t/\tau}$$

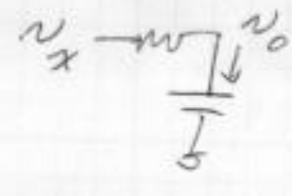
$$v_x(1 - e^{-t/\tau})$$

$$\text{error} = v_x e^{-t/\tau} \qquad v_{initial}\, e^{-t/\tau}$$

~~time~~ step response



thevenin



$$\frac{dv_o}{dt} = \frac{i_c}{C_o} \rightarrow \frac{v_x - v_o}{RC_o}$$

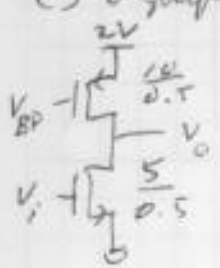
$$i_c = \frac{v_x - v_o}{R}$$

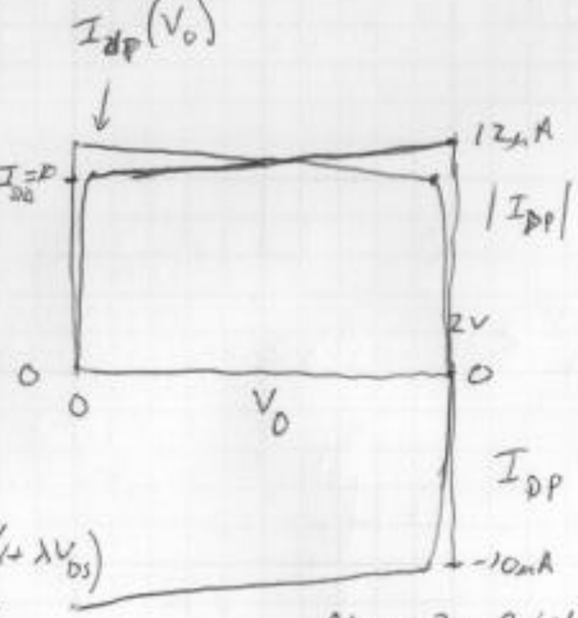
$$\frac{dv_o}{dt} = \frac{v_x - v_o}{\tau}$$

Settling time

t	$\text{error} = e^{-t/\tau}$
τ	$e^{-1} = 37\%$
3τ	$\frac{1}{e^3} = 5\%$
5τ	1%
7τ	0.1%
9τ	0.01%
14τ	1 ppm

CS amplifier





$V_I = 0.6V$

$V_{tn} = |V_{tp}| = 0.5V$

$I_{dn} = \left(\frac{100\mu A}{V^2}\right)(60)(0.1)^2(1+\lambda V_{DS})$

$= 10\mu A\left(1+\frac{V_{DS}}{10V}\right)$

$V_{BP} = 2 - 0.6V$

Find g_m, R_O, $(V_{o,min}, V_{o,max})$ = output swing.

A_v

$g_m = \frac{2I_D}{V_{OV}} = \frac{2(10\mu A)}{0.1V} = 200\frac{\mu A}{V^2}$

$r_o = \frac{1}{\lambda I_D} = \frac{10V}{10\mu A} = 1M\Omega$

$R_o = \frac{1}{2}r_o = 500k\Omega$

$A_v = \left(200\frac{\mu A}{V}\right)\left(500k\frac{V}{A}\right) = 100$

$v_i = 10mV \Rightarrow i_d = \left(200\frac{\mu A}{V}\right)(10mV) = 2\mu A$, $V_o < 0.1V$

$v_i = -10mV \Rightarrow i_d = -2\mu A$ $V_o > 1.9V$

output swing $\approx (0.1, 1.9V)$

what if $V_I = 0.82V$ $V_{ov} = 0.32V$

$V_{ov}^2 = 0.1V^2$

$I_D = \left(\frac{200\mu A}{2V^2}\right)(10)(0.1) = 100\mu A$

$g_m = \frac{2I_D}{V_{ov}} = \frac{200\mu A}{0.32V} = 600\frac{\mu A}{V}$

$r_o = \frac{1}{\lambda I_D} = \frac{10V}{100\mu A} = 100K$

$R_o = 50K$

$V_{BP} = 2 - 0.82V = 1.18$

$A_v = g_m R_o = 30$

What if $V_I = 0.5V$? $V_{ov} \approx 0 < 10mV$

Sub-V_t Say equal at 10mV Say n=2

$I_s e^{\frac{10mV}{n\,26mV}} = 100\frac{\mu A}{V^2}(10)(10^{-2}V)^2 = 0.1\mu A$

$I_s \approx 0.05\mu A$ to $0.08\mu A$

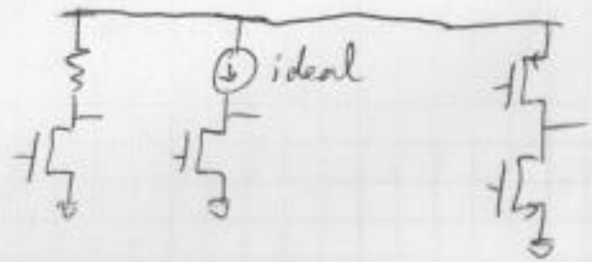
$r_o = \frac{1}{\lambda I_D} = \frac{10V}{10^{-7}A} = 10^8\Omega$

$g_m = \frac{I_D}{nV_{TH}} = \frac{80nA}{50mV} = 1.6\times10^{-6}$ S

$A_v = 160 = 80$ $A_v = \frac{I_D}{nV_{TH}}\frac{1}{\lambda I_D} = \frac{V_A}{nV_{TH}} = \frac{10V}{50mV}$

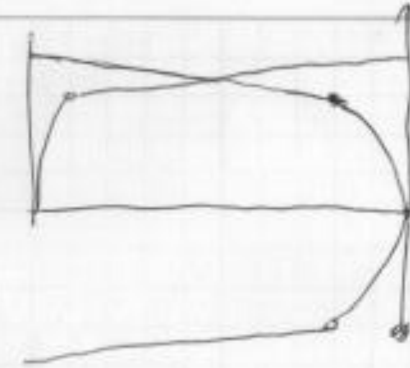
$= 200$

Back to circuits



$\leftarrow R_L\ \infty,\ A_v = g_m r_o$

$\leftarrow R_L \ll r_o,\ A_v \approx g_m R_L$



$r_{on} = r_{op} \qquad R_o = \frac{r_{on}}{2} \qquad A_v = \frac{g_m r_o}{2}$

active load usually gives better gain than Resistive load (when $|V_A| > V_{DD}$)