HW6 Due Friday 10/18
HW7 Due Wed 10/23
HW8 Due Wed 10/30
Midterm Nov 1 in class on HW1-8

Feedback & stability

why "$j\omega$"?

graphical $|H(\omega)|$ and $\angle H(\omega)$

Phase margin

Poles move in feedback
1 pole
2 poles

Feedback is great
- set stable gain w/ ratio of components
- build math operations
- raise/lower input/output impedances appropriately [later]

but

there be dragons!

sometimes feedback systems oscillate

need to understand poles, Heaviside transform

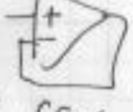
In the time domain

negative feedback looks good

if phase around loop $= \pm 180°$, positive feedback!

$A_{cl} = \frac{A}{1+Af} \approx 1/f$

positive feedback generally not

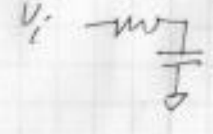


buffer

latch (permanent?)

inverter w/ hysteresis

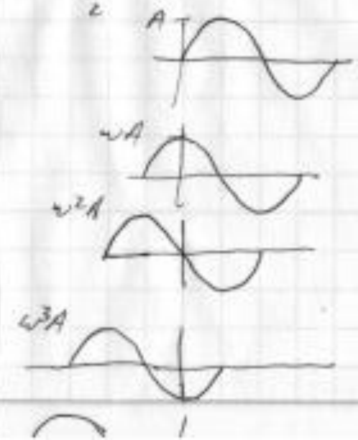
output derivatives introduce negative phase. in $H(j\omega)$



$i_o = C\frac{dv_o}{dt}$ $\quad i_o = \frac{v_i - v_o}{R}$

$RC\frac{dv_o}{dt} + v_o = v_i$

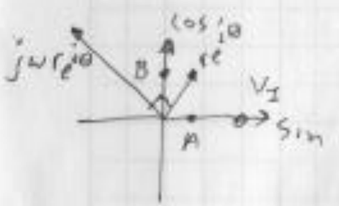
say $v_o = A\sin\omega t$	A
$\frac{dv_o}{dt} = \omega A\cos\omega t$	$j\omega A$
$\frac{d^2v_o}{dt^2} = -\omega^2 A\sin\omega t$	$(j\omega)^2 A$
$\frac{d^3v_o}{dt^3} = -\omega^3 A\cos\omega t$	$(j\omega)^3 A$
$d^4v_o - \omega^4 A\sin\omega t$	$(j\omega)^4 A$

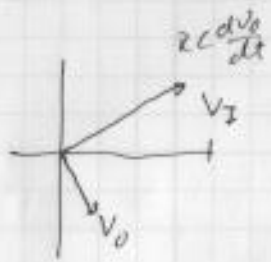


$RC\frac{dv_o}{dt} + v_o = v_i \qquad v_i = V_I \sin(\omega t)$

$v_o = A\sin\omega t + B\cos\omega t =$

$RC\frac{dv_o}{dt} = RC\omega A\cos\omega t - RC\omega B\sin\omega t$

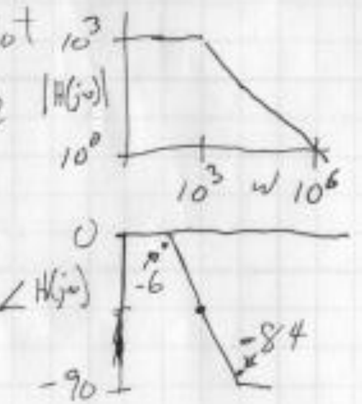




$\omega \ll \frac{1}{RC}$

Bode plot

single pole



$H(s) = \frac{1}{1+s/\omega_p}$

$H(j\omega) = \frac{1}{1+j\omega/\omega_p}$

$$H(j\omega) \approx \begin{cases} 1 & \omega \ll \omega_p \\ \frac{1}{1+j} & \omega = \omega_p \\ \frac{\omega_p}{j\omega} & \omega \gg \omega_p \end{cases}$$

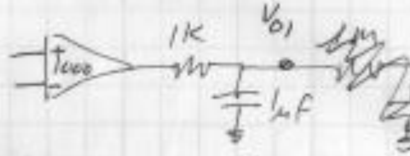
$j\omega$ just book keeps derivatives effect on sin, cos

at high freq, derivative of output dominates, w/

90° positive phase shift relative to output

so that derivative term is approx V_i

so output must lag V_i



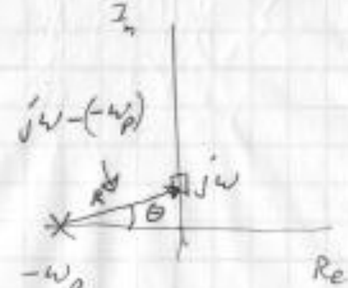
~~w~~ pole, $\omega_p = 10^3$

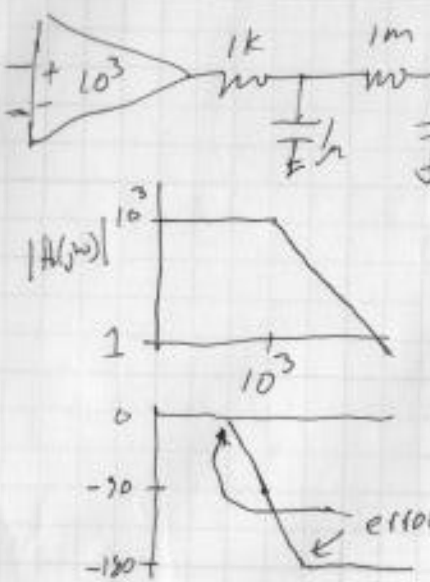
$H(s) = \frac{1}{1+s/\omega_p} = \frac{\omega_p}{s+\omega_p}$

$|H(j\omega)| = \frac{|\omega_p|}{|j\omega+\omega_p|} = \frac{|\omega_p|}{|j\omega - -\omega_p|}$

$|H(j\omega)| = \frac{|\omega_p|}{R}$

$\angle H(j\omega) = \angle\omega_p - \angle Re^{j\theta} = -\theta$





Above $10^4 \frac{rad}{s}$

$\angle H(j\omega) = -180$

looks like positive feedback!

amplifier sings!
(rings in oscillation)

~~poles~~ what if $f = 0.001$? 0.005? 0.01?

need a measure of how close we are to trouble, defined as when

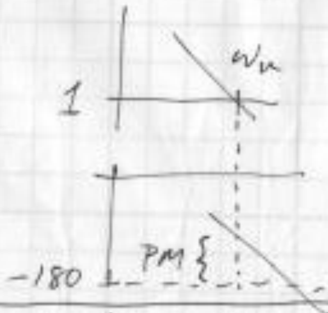
$$\angle H(j\omega_{180}) = -180$$

$$|H(j\omega_{180})| \geq 1$$

phase margin

$PM = \angle H(j\omega_1) - (-180)$

PM desired varies
30 – 80°
45–60 typ.
see EE128



Bode plot of open loop amplifier tells you about the stability of the closed loop amplifier

What about the closed loop transfer function?

$$A_{CL} = \frac{A}{1+Af} \qquad A = \frac{N(s)}{D(s)}$$

$$= \frac{N(s)}{D(s)} \frac{1}{1 + \frac{N(s)}{D(s)} f} = \frac{N(s)}{D(s) + f N(s)} \Leftarrow \text{characteristic polynomial}$$

roots of characteristic polynomial are poles of CL sys.

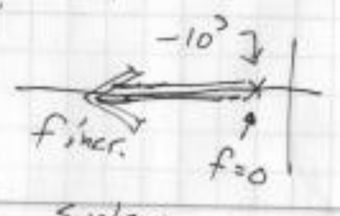
roots depend on N, D, f – everything.

1 pole, passive feedback (f real $0 < f \leq 1$)

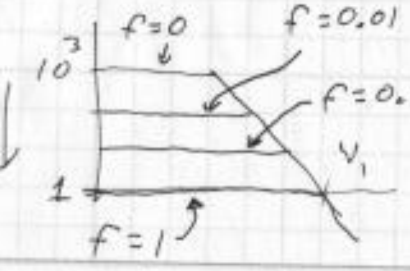
$$A = \frac{A_o}{1 + s/\omega_p} \qquad A_{CL} = \frac{A_o}{1 + s/\omega_p + A_o f}$$

$$A_{CL} = \frac{A_o}{1 + A_o f} \frac{1}{1 + \frac{s}{\omega_p(1 + A_o f)}} \approx \frac{1}{f} \frac{1}{1 + \frac{s}{\omega_{pCL}}}$$

$$\omega_{pCL} = \omega_p(1 + A_o f)$$



s-plane



ω_1 stays same