HW6 Friday (delayed)
HW7 Next Wed (normal)
HW8 Wed 10/30
midterm in class Friday 11/1

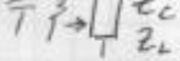
2 pages, 4 sides

HW 1-8, labs

OH today

circuit $\xrightarrow{\text{nodal analysis}}$ ODE $\sum a_i \frac{d^i}{dt^i} v_o = \sum b_j \frac{d^j}{dt_j} v_i$

$\downarrow$ H/L


Z_C Z_L $\xrightarrow{N.A.}$ $H(s) = \frac{N(s)}{D(s)} = \frac{\sum b_j s^j}{\sum a_i s^i}$

$\downarrow$ Heaviside/Laplace

N.A. $v_s = 0$ → $G_m(s)$

N.A. $v_i = 0$ → $R_o(s)$

in feedback

$$N(s) = A_{CL} = \frac{A}{1+Af}$$

$$= \frac{N(s)}{D(s) + N(s)f}$$

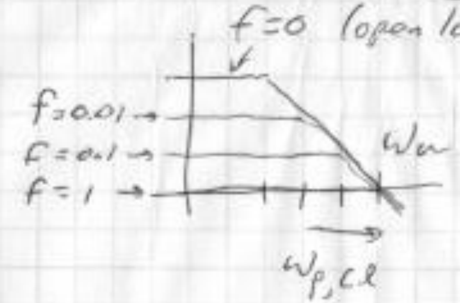
characteristic polynomial

roots are poles p_i, depend on $D(s), N(s), f$

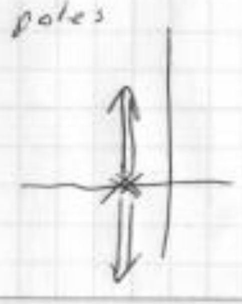
zeros don't move in feedback

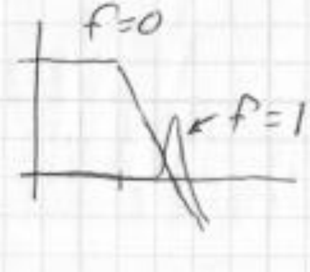
Last time

1 pole



2 poles





$Q \approx 15$

2 poles

$$A = \frac{A_0}{(1+s/\omega_p)^2} = \frac{A\omega_p^2}{(s+\omega_p)^2}$$

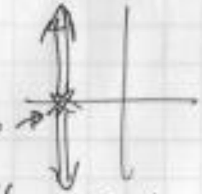
$$A_{CL} = \frac{A}{1+Af} = \frac{A\omega_p^2}{(s+\omega_p)^2 + A\omega_p^2 f}$$

$$s^2 + 2\omega_p s + \omega_p^2 + A\omega_p^2 f$$

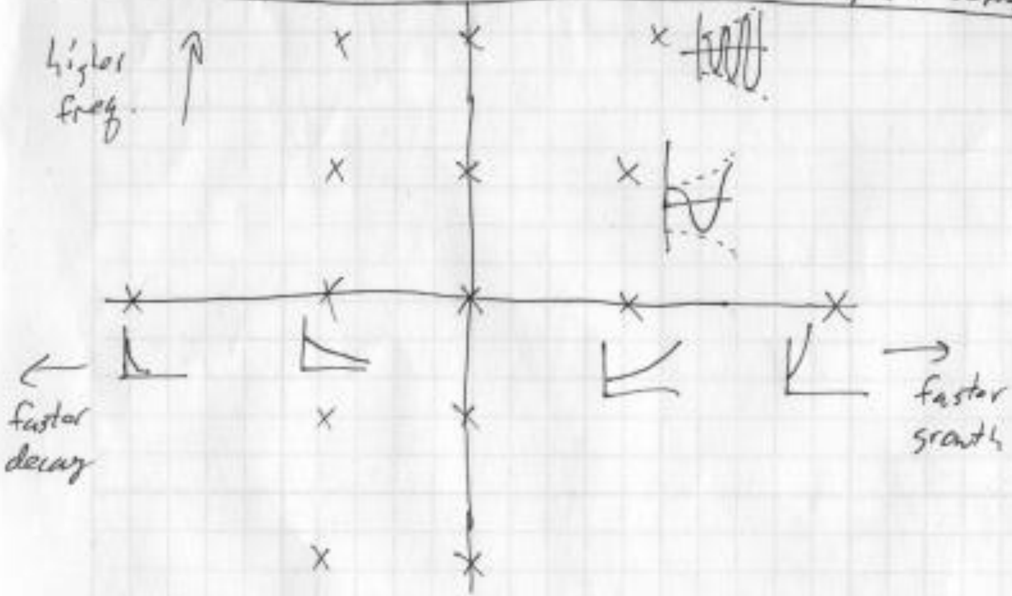
$$\omega_{p,CL} = \frac{-2\omega_p \pm \sqrt{(2\omega_p)^2 - 4(\omega_p^2)(1+A_0 f)}}{2}$$

$$= -\omega_p \pm \omega_p\sqrt{1-(1+A_0 f)}$$

$$= -\omega_p \pm j\sqrt{A_0 f}\,\omega_p$$



s-plane CL poles



σ gives envelope

ω gives osc. freq.

time response ~~of~~ vs. pole location

recall ODE $\sum_{i=0}^{n} a_i \frac{d^i}{dt^i} v_o = \sum_{i=0}^{m} b_i \frac{d^i}{dt^i} v_i$

Heaviside (Laplace) $\Sigma a_i s^i V_o = \Sigma b_i s^i V_I$

$$H(s) = \frac{V_o}{V_I} = \frac{\Sigma b_i s^i}{\Sigma a_i s^i}$$

roots of $\Sigma a_i s^i = 0$ are poles p_i

impulse response of pole p_i proportional to $e^{p_i t}$

if $p_i = \sigma$ real then exponential

if $p_i = \sigma \pm j\omega$ then sinusoidal $e^{\sigma t}\cos(\omega t + \phi)$

3 poles

$$A(s) = \frac{A_0}{(1+s/\omega_p)^3} = \frac{A_0\omega_p^3}{(s+\omega_p)^3}$$

$$A_{CL} = \frac{A}{1+Af} = \frac{A_0\omega_p^3}{(s+\omega_p)^3 + A_0 f \omega_p^3}$$

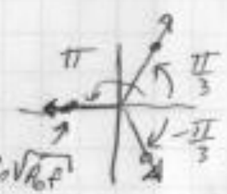
to find poles:

$$(s+\omega_p)^3 + A_0 f \omega_p^3 = 0$$

easy if we sub $v = s+\omega_p$

$$s+\omega_p = \sqrt[3]{-A_0 f \omega_p^3}$$

$$= \omega_p \sqrt[3]{A_0 f}\,\sqrt[3]{-1}$$



$$e^{j(\pi + n2\pi)}$$

$$-1 = e^{j\pi} = \left(e^{j\theta_p}\right)^3 = e^{j3\theta_p}$$

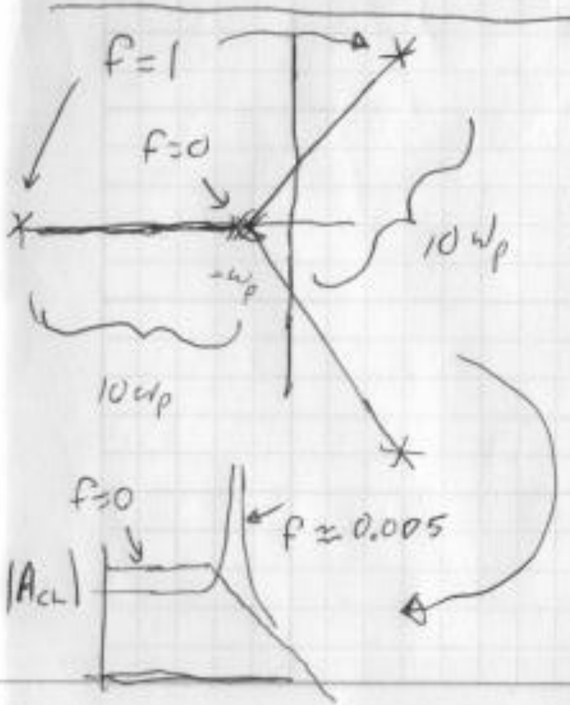
$$\pi(1+2n) = 3\theta_p \qquad n = 0,1,2$$

$$\theta_p = \frac{\pi}{3}(1+2n)$$

$$= \frac{\pi}{3},\ \frac{3\pi}{3},\ \frac{5\pi}{3}$$

$$= \frac{\pi}{3},\ \pi,\ -\frac{\pi}{3}$$

poles are $p_i = -\omega_p + \omega_p\sqrt[3]{A_0 f}\, e^{j\{\pi, \pm\frac{\pi}{3}\}}$



unstable poles!