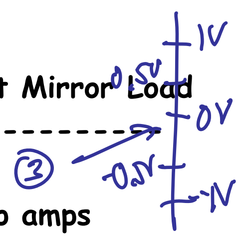
Lecture 14: Half Circuits & Current Mirror Load

- **Announcements**:
  - Problem 5 on HW#5 was not moved to HW#6, since it already had plenty of problems
  - New versions of prepared lecture notes that print better are online
  - I will miss next week's Thursday lecture
  - Will inform you how we will make it up; perhaps by special software that records directly off my computer

OH: W 10:30-11:30 → Th 3-4

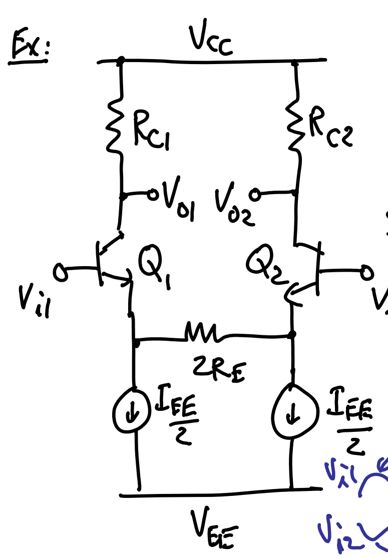
Prof Nguyen's

- **Lecture Topics**:
  - Emitter Coupled Pair (ECP)
  - Half Circuits
  - Source Coupled Pair w/ Current Mirror Load



- --
- **Last Time**:
- Going through prepared notes on op amps
- Got to ECP ... will continue with this

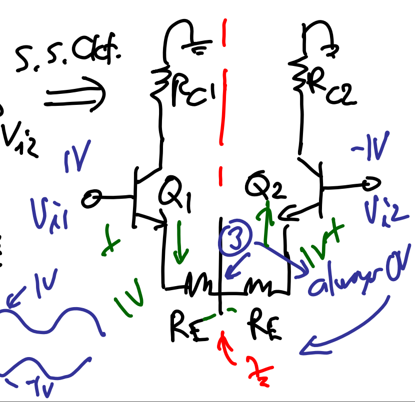
Ex:



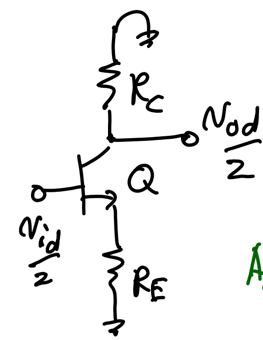
$Q_1 = Q_2 \to r_{o1} = r_{o2} = r_o$, $\to g_{m1} = g_{m2} = g_m$

$R_{C1} = R_{C2} = R_C$

S.S. Ckt. ⟹



(i) Find the differential-mode gain: (A_{dm})

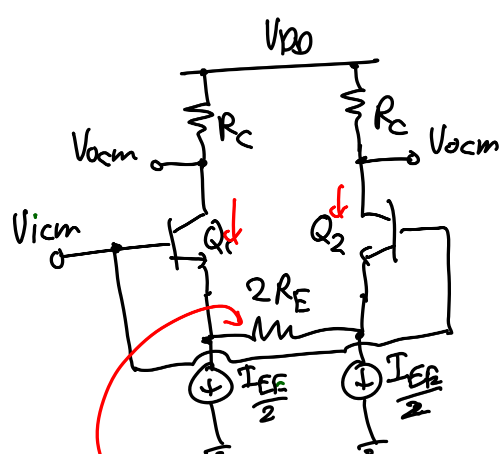


$$\frac{(v_{od}/2)}{(v_{id}/2)} = -\frac{g_m(r_o \| R_c)}{1 + g_m R_E}$$

$$A_{dm} = \frac{v_{o1} - v_{o2}}{v_{i1} - v_{i2}} = \frac{v_{od}}{v_{id}} =$$

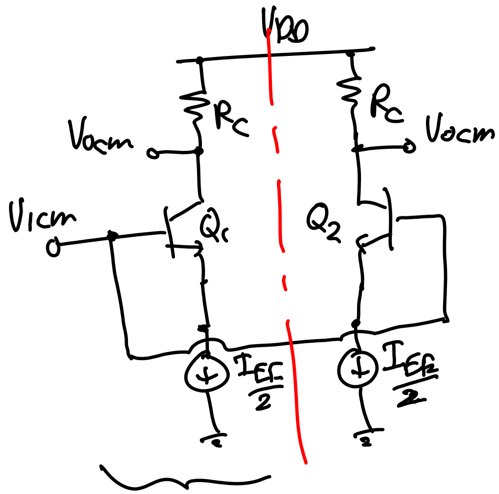
Diff. Half Ckt.

(ii) Find the common-mode gain, A_{cm}:

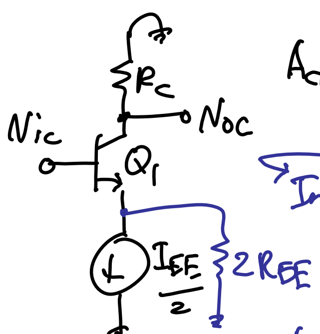


How much current goes through this resistor?

↳ $i = 0$ ∴ the resistor effectively isn't there!



Common-mode Half Ckt.



$$A_{cm} = \frac{v_{oc}}{v_{ic}} = \frac{-g_m(r_o \| R_C)}{1+g_m(\infty)} = 0$$

for the ideal case!

In reality, have

$$A_{cm} = \frac{v_{oc}}{v_{ic}} = -\frac{g_m(r_o \| R_C)}{1+g_m(2R_{EE})}$$

If there is a mismatch in the ckt., e.g., $R_{C1} \neq R_{C2}$, then we can define:

$A_{cm\text{-}dm} \triangleq$ common-mode input to differential-mode output gain

$$= \frac{v_{od}}{v_{ic}} = \frac{v_{o1}-v_{o2}}{v_{i1}} = \frac{v_{o1}-v_{o2}}{v_{i2}} \quad (\text{w/ } v_{i1}=v_{i2})$$

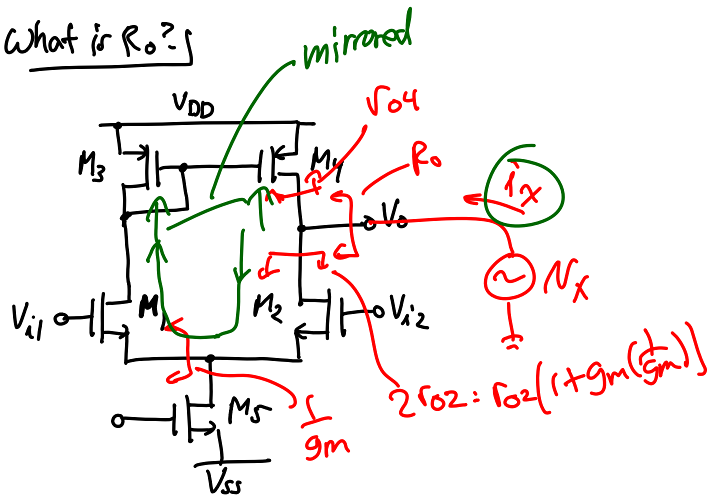
$A_{dm\text{-}cm} \triangleq$ differential-mode input to common-mode output gain

$$= \frac{v_{oc}}{v_{id}} = \frac{v_{oc}}{v_{i1}-v_{i2}} \quad \left[\text{w/ } v_{oc} = \frac{1}{2}(v_{o1}+v_{o2})\right]$$

You'll will be experiencing these in your HW.

- Now go back to the handout, to the source coupled pair (SCP) section
- Go through this until you get to "What is Ro?"
- Then …

What is R_o?



$$i_x = \frac{v_x}{r_{o4}} + \frac{v_x}{2r_{o2}} + \underbrace{\frac{v_x}{2r_{o2}}}_{\text{mirrored}}$$

$$\frac{v_x}{i_x} = R_o = \frac{1}{\frac{1}{r_{o2}}+\frac{1}{r_{o4}}} = (r_{o2} \| r_{o4})$$

- Now go back to the handout, at the current mirror loaded SCP section

Small-Signal Analysis of Diff. Pair

Consider v_{i2} & short v_{i1}:

$+\frac{1}{2}g_m v_{i2}R_C$

$v_{o1} = -\frac{1}{2}g_m v_{i1}R_C$

$v_{o2} = -\frac{1}{2}g_m v_{i2}R_C$

$+\frac{1}{2}g_m v_{i1}R_C$

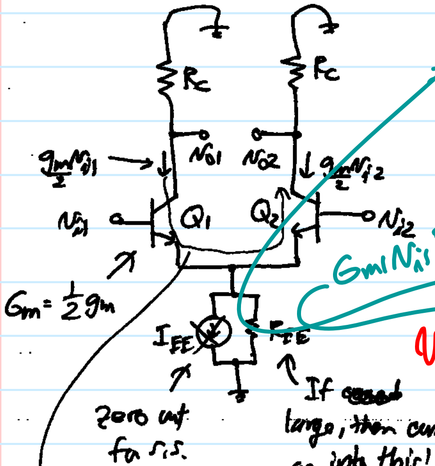
$v_{o1} + v_{o2}$

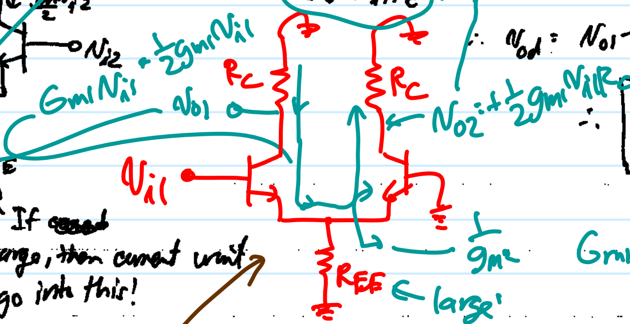
$v_{o1} = -\frac{1}{2}g_m R_C(v_{i1}-v_{i2})$

$v_{o2} = +\frac{1}{2}g_m R_C(v_{i1}-v_{i2})$

$\therefore v_{od} = v_{o1} - v_{o2} = -g_m R_C(v_{i1}-v_{i2})$

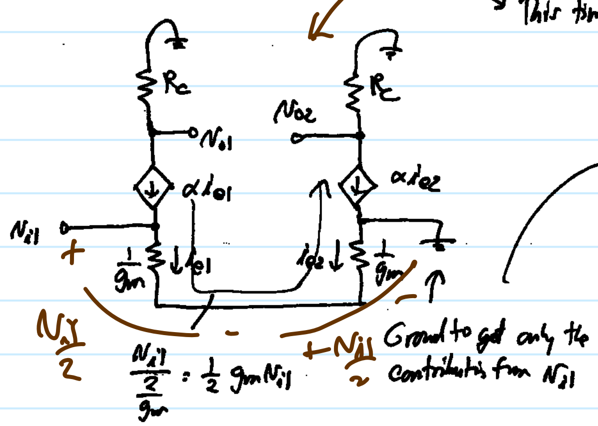
$\therefore \boxed{\frac{v_{od}}{v_{id}} = A_{dm} = -g_m R_C}$





⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt)

↳ This time also get the $\frac{v_{o1}-v_{o2}}{v_{i1}}$ gain!



$v_{o1} = -\frac{1}{2}g_m R_C v_{i1}$

$v_{o2} = +\frac{1}{2}g_m R_C v_{i1}$

$v_{o1} - v_{o2} = -g_m R_C v_{i1}$

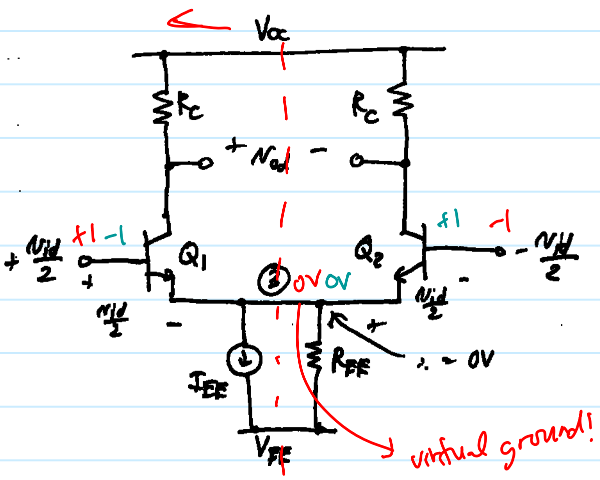
$\therefore \boxed{\frac{v_{od}}{v_{i1}} = -g_m R_C}$

$\boxed{\frac{v_{o2}}{v_{i1}} = \frac{1}{2}g_m R_C}$

$\boxed{\frac{v_{o1}}{v_{i1}} = -\frac{1}{2}g_m R_C}$

Diff. Mode Analysis

Assume a ckt. w/ only diff. input:



Total current thru I_{EE} = const.

⟶ V_E = const. as input changes

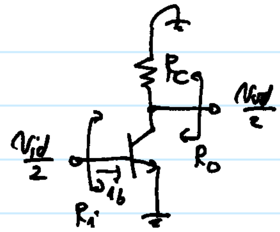
⟶ ③ acts as an incremental ground! ⟶ $v_③ = 0V$ (always!)

$\therefore$ we can ground ③, and thus, have a $\boxed{\text{Differential Half Ckt.}}$

↓

Note: Can really only make this for a purely symmetrical ckt.!

Differential Half Ckt.



By Inspection: $\dfrac{v_{od}/2}{v_{id}/2} = \dfrac{v_{od}}{v_{id}} = \boxed{A_{dm} = -g_m R_C}$

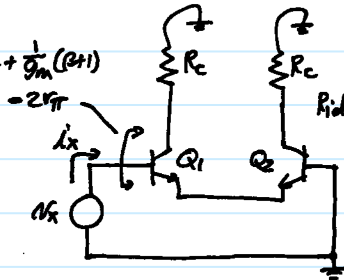
$\dfrac{v_{id}/2}{i_b} = r_\pi \rightarrow R_{id} = \dfrac{v_{id}}{i_b} = \boxed{2r_\pi = R_{id}}$

$\dfrac{v_{od}/2}{i_o} = r_o \| R_C \rightarrow R_{od} = \dfrac{v_{od}}{i_o} = 2(r_o\|R_C) \approx \boxed{2R_C = R_{od}}$

S.S. params. determined w/ $I_C = \dfrac{I_{EE}}{2}$

First define

$r_\pi + \frac{1}{g_m}(\beta+1) = 2r_\pi$

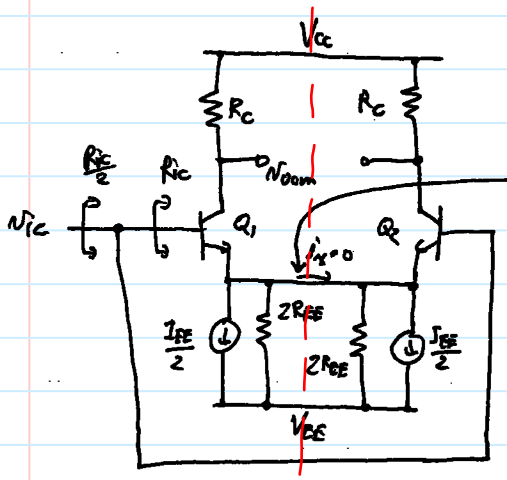


$R_{id} = \dfrac{v_x}{i_x} = 2r_\pi$

← this basically grounded, so can inspect this

Common-Mode Analysis

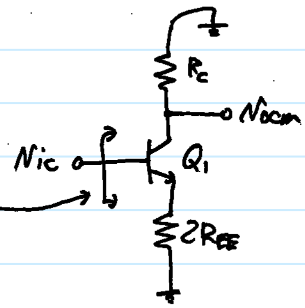
Assume a pure CM input → tie inputs together



By symmetry, $i_x = 0$. ⇒ thus, really have the equivalent of an open ckt. here

∴ ⇒ can split the ckt. into CM half-ckts.!

S.S. CM Half-Ckt.



$R_{ic} = r_\pi + (\beta+1)(2R_{EE})$
@ each input

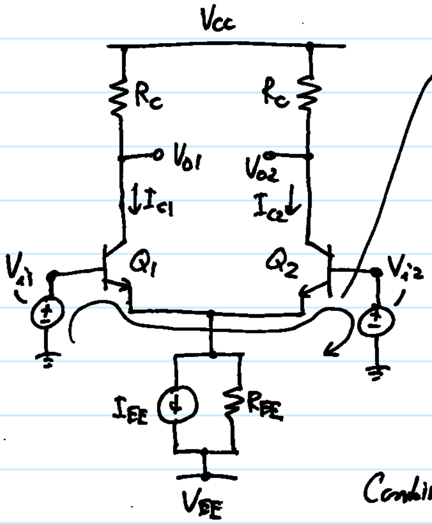
$$\boxed{A_{cm} = \frac{v_{ocm}}{v_{ic}} = -\frac{g_m R_C}{1+g_m(2R_{EE})} \approx -\frac{R_C}{2R_{EE}}}$$

Want small for large CMRR ∴ want R_{EE} = large!

Common-Mode Rejection Ratio = $\text{CMRR} = \dfrac{A_{dm}}{A_{cm}} = \dfrac{-g_m R_C}{-\dfrac{g_m R_C}{1+g_m(2R_{EE})}} \Rightarrow \boxed{\text{CMRR} = 1 + 2g_m R_{EE}}$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

Large Signal ECP Performance



Find I_{c1} & I_{c2}:

KVL: $V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$

$I_{c1} = I_{s1}\exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right)$, $V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right)$

$V_{i1} - V_T \ln\left(\frac{I_{c1}}{I_{c2}} \frac{\cancel{I_{s2}}}{\cancel{I_{s1}}}\right) - V_{i2} = 0 \rightarrow \ln\frac{I_{c1}}{I_{c2}} = \frac{V_{i1}-V_{i2}}{V_T} = \frac{V_{id}}{V_T}$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{id}}{V_T}\right) \qquad (1)$$

$$I_{EE} = I_{e1} + I_{e2} = \frac{1}{\alpha}(I_{c1}+I_{c2}) \qquad (2)$$

Combine (1) & (2) to get:

$$I_{C1} = \frac{\alpha I_{EE}}{1+\exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{C2} = \frac{\alpha I_{EE}}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \qquad (3)$$

Find V_{od}:

$$\left.\begin{aligned} V_{o1} &= V_{cc} - I_{c1}R_c \\ V_{o2} &= V_{cc} - I_{c2}R_c \end{aligned}\right\} \quad V_{od} = V_{o1} - V_{o2} = (I_{c2} - I_{c1})R_c$$

using (3)

$$= \alpha_F I_{EE} R_c \left\{ \frac{1}{1+\exp\left(\frac{V_{id}}{V_T}\right)} - \frac{1}{1+\exp\left(-\frac{V_{id}}{V_T}\right)} \right\}$$

$$\times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)} \qquad \times \frac{\exp\left(+\frac{V_{id}}{2V_T}\right)}{\exp\left(+\frac{V_{id}}{2V_T}\right)}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

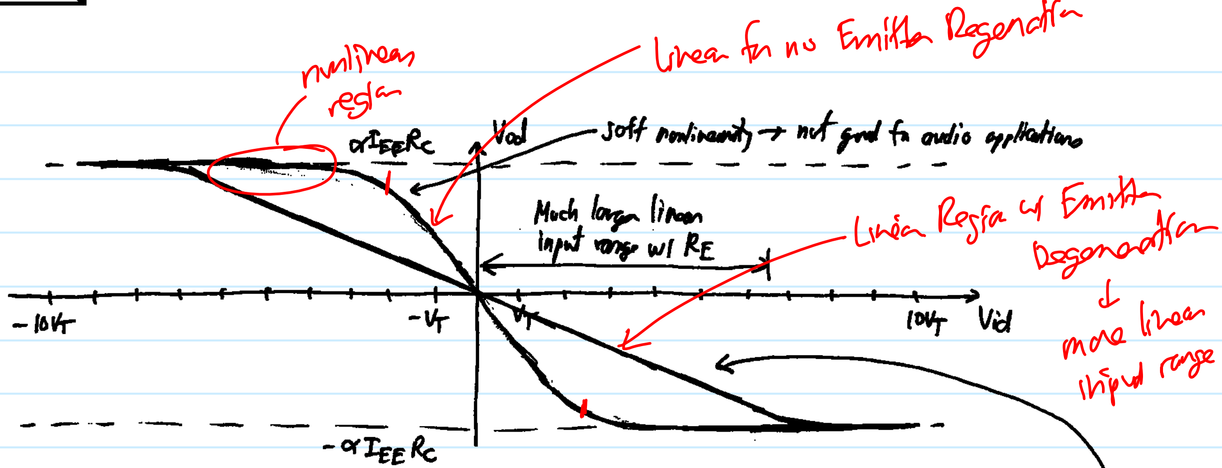
$$= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_c \frac{\sinh\left(-\frac{V_{id}}{2V_T}\right)}{\cosh\left(-\frac{V_{id}}{2V_T}\right)}$$

$$\left[\left.\begin{aligned} \sinh u &= \tfrac{1}{2}(e^{u} - e^{-u}) \\ \cosh u &= \tfrac{1}{2}(e^{u} + e^{-u}) \end{aligned}\right\} u = -\frac{V_{id}}{2V_T} \right]$$

$$\therefore \boxed{V_{od} = \alpha_F I_{EE} R_c \tanh\left(-\frac{V_{id}}{2V_T}\right)}$$

From our knowledge of the Taylor series for
$\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \dots$

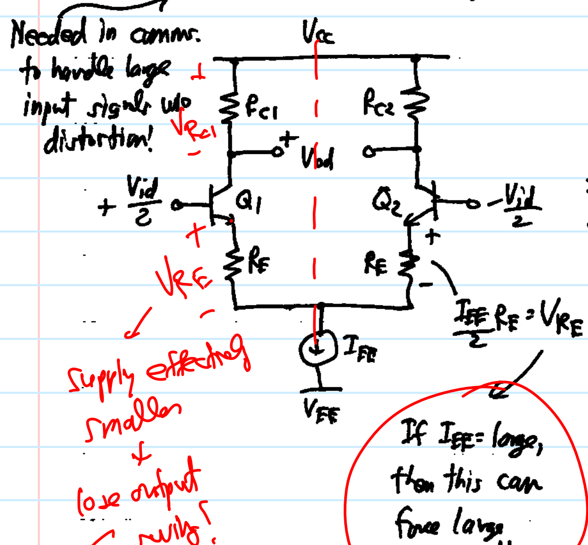
this is fairly linear for small Vid, but gets nonlinear abruptly when Vid approaches a threshold value!

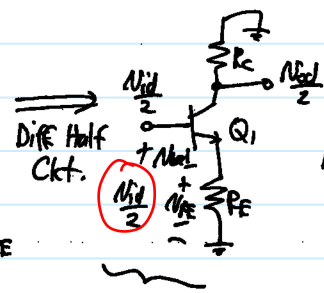


In the above curve, the $\frac{V_{od}}{V_{id}}$ Xfer function is really only linear for $V_{id} \ll V_T$ → beyond V_T, start to enter the nonlinear realm of curve → causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xistor amplifiers)

Needed in comm. to handle large input signals w/o distortion!





$$A_{dm} = -\frac{g_m R_C}{1 + g_m R_E}$$

⇒ s.s. gain reduced, but the linear range is increased

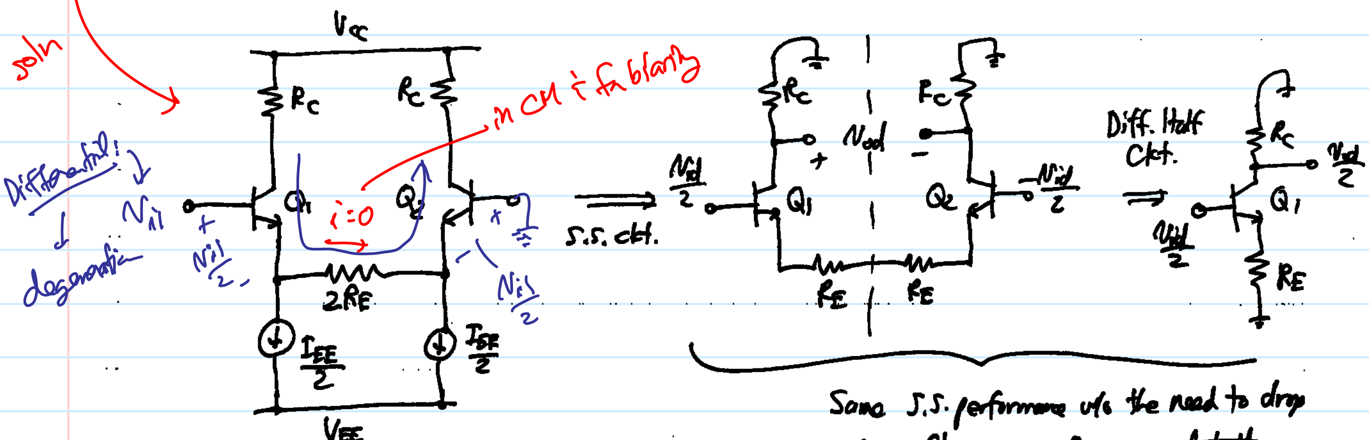
If I_{EE} = large, then this can force large supply voltages

$\frac{V_{id}}{2} = V_{be1} + V_{RE}$

This can still be $V_{be1} \ll V_T$ if this absorbs some of the input voltage!

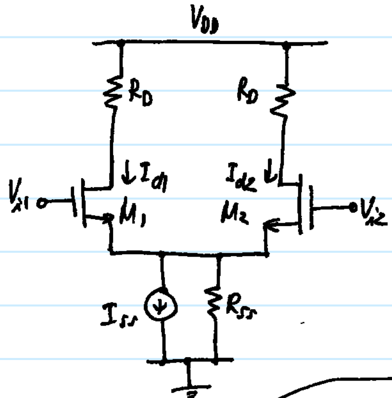
Alternative Biasing Technique If Need Larger DC Current:–

soln



Same s.s. performance w/o the need to drop a DC voltage across R_E → ~~get both~~ Can use lower V_{CC} & V_{EE}.

MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

$\Rightarrow$ approach: get $V_{id} = f(\Delta I_d) \longrightarrow$ then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \underbrace{\tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_1}_{k}(V_{gs1} - V_t)^2 \Rightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore\ V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define. $\left.\begin{aligned} \Delta I_d &= I_{d1} - I_{d2} \\ I_d &= \frac{I_{d1} + I_{d2}}{2} \end{aligned}\right\}\quad \begin{aligned} I_{d1} &= I_d + \frac{\Delta I_d}{2} \\ I_{d2} &= I_d - \frac{\Delta I_d}{2} \end{aligned}$

$$V_{id} = \sqrt{\frac{2\left(I_d + \frac{\Delta I_d}{2}\right)}{k}} - \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{k}} \Rightarrow \frac{k}{2}V_{id}^2 = I_d + \cancel{\frac{\Delta I_d}{2}} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \cancel{\frac{\Delta I_d}{2}}$$

$$\frac{k}{2}V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

$\Rightarrow$ now rearrange to get ΔI_d (algebra)

Solve for ΔI_d:

$$\Delta I_d = \frac{k}{2}V_{id}\left(\frac{2I_{SS}}{k/2} - V_{id}^2\right)^{1/2} = \boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{id}\sqrt{\left(\frac{2I_{SS}}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right) - V_{id}^2} = \Delta I_d}$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

V_{GS} for $I_D = \frac{I_{SS}}{2}$

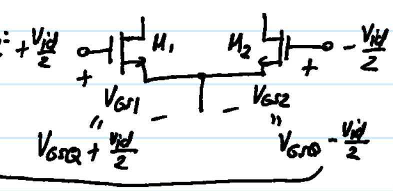
$$|V_{id}| \le \sqrt{\frac{2I_{SS}}{k}} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{2}\,(V_{GS} - V_t)$$

if true then input devices are both saturated

Thus, to extend the linear input range:

1. $I_{SS}\uparrow \longrightarrow (V_{GS} - V_t)\uparrow$
2. $W\downarrow$
3. $L\uparrow$

To derive this:



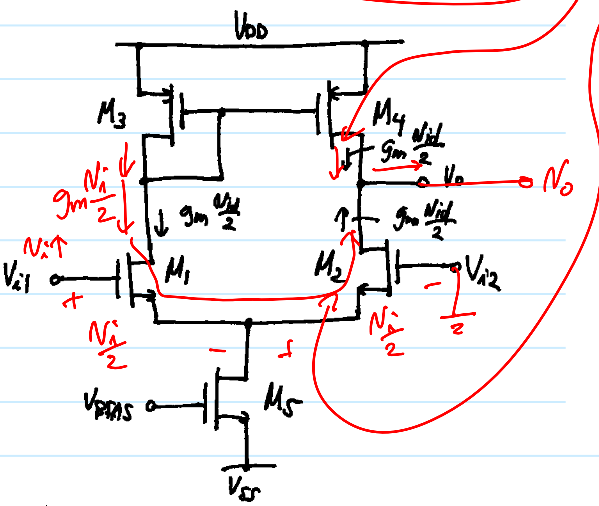
When $\frac{V_{id}}{2} \ge V_{GSQ} - V_t = \Delta V$ then M_2 will cut-off

$\therefore\ V_{id} \le 2(V_{GSQ} - V_t) \longrightarrow$ to maintain saturation

$$V_{GSQ} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \frac{V_{id}}{2}$$

then plug in ΔI_d & solve for V_{id}

MOS Differential Stage w/ Current Mirror Load



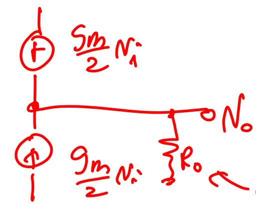
from bottom / from top

$$v_o = \left(\frac{g_m}{2}v_i + \frac{g_m}{2}v_i\right)R_o \Rightarrow \frac{v_o}{v_i} = g_m R_o$$

output R_o of the ckt.

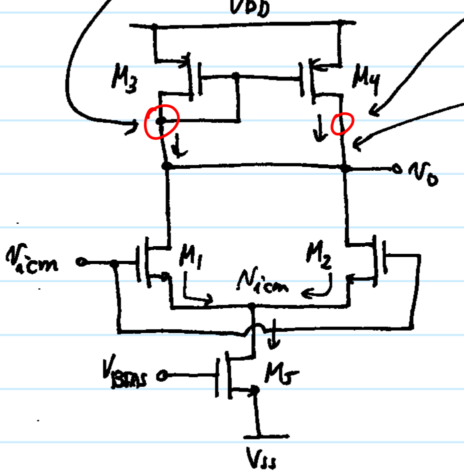
$$\boxed{= g_m(r_{o2} \| r_{o4}) = \frac{v_o}{v_i}}$$

A_{dm}



What is R_o?
go to the main lecture page
$R_o = (r_{o2} \| r_{o4})$ ... but not by inspection!

CMRR –



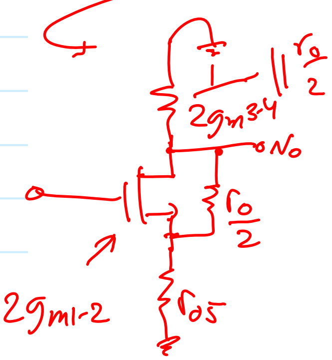
for $M_1 = M_2$ & $M_3 = M_4$

$V_{D3} = V_{D4}$ (they track each other) provided the ckt. is completely symmetrical.

$\Rightarrow$ Thus, can short the drains of M_3 & M_4

truly symmetric $\rightarrow$ we can draw a half ckt.

or just combine the two ckts. together!



$$\boxed{A_{cm} = -\frac{2g_{m1\text{-}2}\left(\frac{1}{2g_{m3\text{-}4}}\right)}{1 + 2g_{m1\text{-}2}r_{o5}}}$$

(Get the same answer if use the CM Half ckt.)

Thus:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \cancel{g_{m1\text{-}2}}(r_{o1\text{-}2} \| r_{o3\text{-}4})(1 + 2g_{m1\text{-}2}r_{o5})\left(\frac{g_{m3\text{-}4}}{\cancel{g_{m1\text{-}2}}}\right)$$

$$\boxed{CMRR = (1 + 2g_{m1\text{-}2}r_{o5})\, g_{m3\text{-}4}(r_{o1\text{-}2} \| r_{o3\text{-}4})}$$

Common-Mode Input Range – Range of input voltages for which all devices remain in saturation.

Low End – must keep M_5 saturated

$$V_{icm}(min) = CMR^- = V_{SS} + V_{ov5} + V_{GS1,2} = \boxed{V_{SS} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}(W/L)_5}} + V_{t1\text{-}2} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox}(W/L)_{1\text{-}2}}}}$$