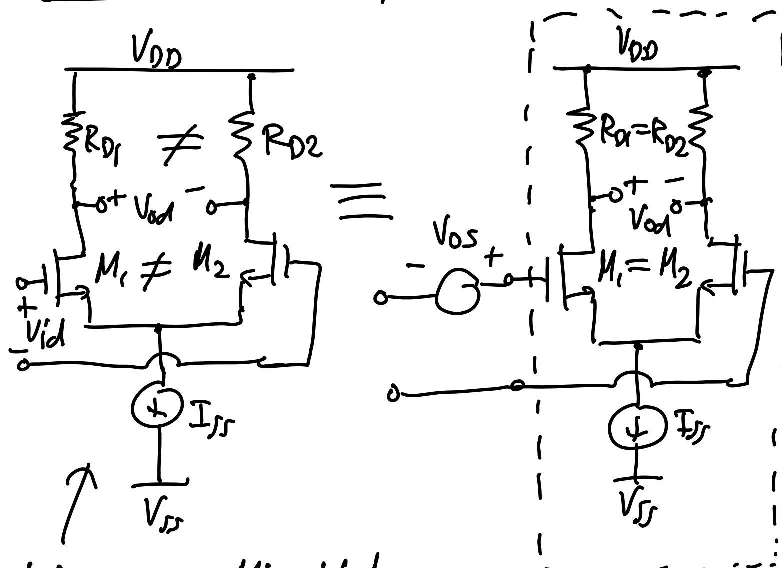


Lecture 15: Vos & Finite Gain-BW

- Announcements:
  - HW#7 will be online soon
  - Pre-Lecture materials for this lecture (on Vos) are already online
  - Lab#2 will be due the week AFTER Spring Break; this should give ample time to write up a good lab report
  - I will miss Thursday's lecture
  - We will make it up on Friday, at 3 p.m., in the Hogan Room ⇨ it will be recorded via software on my computer
  - Midterm Exam next week, Thursday, March 17, in 213 Wheeler (this room), during the regular class period
- ------------------------------------
- Lecture Topics:
  - Midterm Exam Info
  - Offset Voltage (Vos)
  - Finite Gain-BW Product
- ------------------------------------
- Last Time:
- Going through prepared notes on op amps … almost finished … finish the notes
- Then go to the Vos notes

Vos of a Mismatched SCP

Objective: Derive an expression for V_{OS}.



Actual SCP w/ Mismatched Xsistors & R's

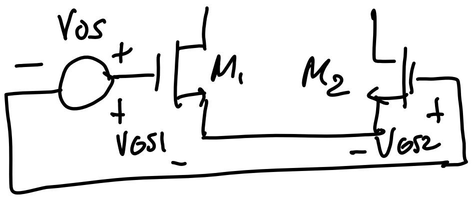
Ideal SCP w/ matched $M_1 = M_2$ & $R_{D1} = R_{D2}$

Vos enters due to variations in:

① Xsistors M_1 & M_2 → $\frac{W}{L}$ & V_t vary

② $R_{D1} \neq R_{D2}$ → cause variations in gain

Definition. $V_{OS} = V_{id}$ needed to get $V_{od} = 0\,V$ in this def:



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2}$$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \qquad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$ | $\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$

$I_D = \frac{I_{D1} + I_{D2}}{2}$ | $\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$

$\Delta V_t = V_{t1} - V_{t2}$ | $\Delta R_D = R_{D1} - R_{D2}$

$V_t = \frac{1}{2}(V_{t1} + V_{t2})$ | $R_D = \frac{1}{2}(R_{D1} + R_{D2})$

Rearranging:

$I_{D1} = I_D + \frac{\Delta I_D}{2}$ | $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2}$ | $V_{t1} = V_t + \frac{\Delta V_t}{2}$

$I_{D2} = I_D - \frac{\Delta I_D}{2}$ | $\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2}$ | $V_{t2} = V_t - \frac{\Delta V_t}{2}$

Substituting into (1): $2I_D\left(1 + \frac{\Delta I_D}{2I_D}\right)$

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$\frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{OS} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0 \quad \therefore I_{D1}R_{D1} = I_{D2}R_{D2}$

mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

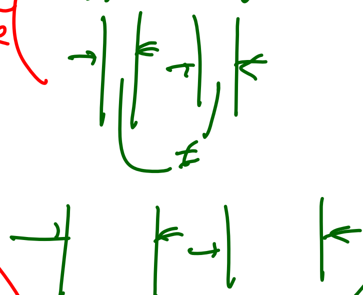
Pay no attention to these signs!

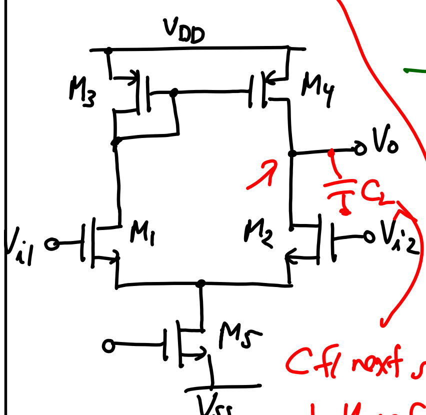
$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS}-V_t)\left\{-\frac{\Delta R_D}{R_D} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch

Geometric Variations (e.g., lithography limitations)

scale w/ overdrive





C f/ next stage + those f/ M_2 & M_4

$\frac{\Delta R_D}{R_D}$: (-1 , $\frac{\Delta(W/L)}{(W/L)}$: (-1

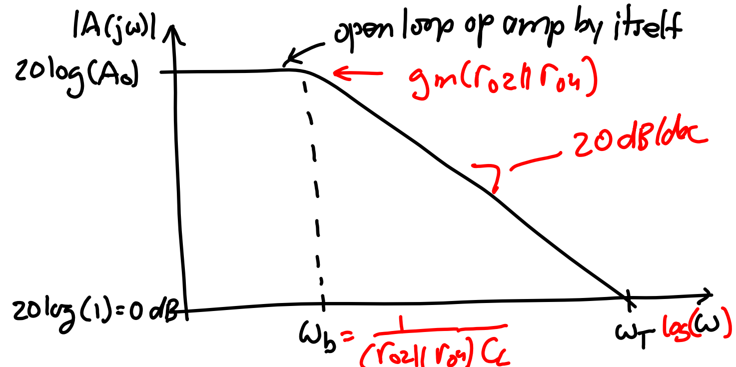
can easily be the case!

Freq. Response: $\omega_H = \frac{1}{(r_{o2} \| r_{o4})C_L}$ ← dominant pole

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$.

In reality, the gain is given by: $A(s) = \frac{A_o}{1 + s/\omega_b}$



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ (= 0dB)

At ω_T:

$$|A(j\omega_T)| = 1 = \frac{A_o}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}}$$

$$[\omega_T \gg \omega_b] \Rightarrow \frac{A_o}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \underbrace{\boxed{\omega_T = A_o\omega_b}}_{\text{Gain-Bandwidth Product}}$$

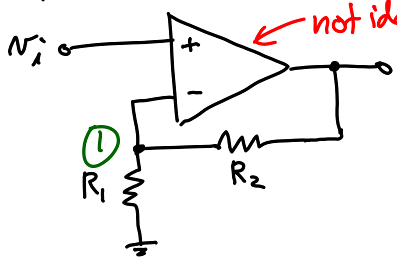
For $\omega \gg \omega_b$:

$$A(s) \approx \frac{A_o}{\frac{s}{\omega_b}} = \frac{A_o\omega_b}{s} = \frac{\omega_T}{s} = \frac{f_T}{f} \quad \left[\text{Integrator w/ time constant } \tau = \frac{1}{\omega_T}\right]$$

The unity gain bandwidth f_T is usually specified on op amp data sheets. Knowing f_T, one can easily determine the op amp gain at a given frequency f.

Frequency Response of Closed Loop Amplifiers

Example. Non-Inverting Amplifier



not ideal → finite gain, finite BW

$$v_o = A(s)(v_+ - v_-)$$

$$A(s) = \frac{A_o}{1+\frac{s}{\omega_b}}$$

Find an expression for the gain as a function of frequency.

① Brute force derivation:

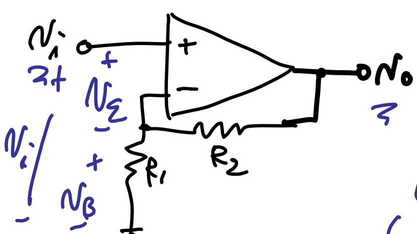
KCL @ ①: $\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} \rightarrow \frac{v_o}{R_2} = v_-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

$$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \longrightarrow \frac{v_o}{v_i}(s) = \frac{1+\frac{R_2}{R_1}}{1+\frac{1}{A(s)}\left(1+\frac{R_2}{R_1}\right)}$$

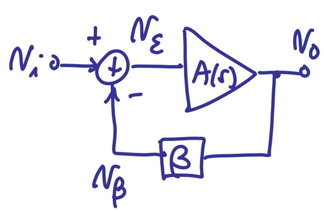
$$\left[A(s) = \frac{A_o}{1+\frac{s}{\omega_b}}\right] \Rightarrow \boxed{\frac{v_o}{v_i}(s) = \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{s}{A_o\omega_b\left(\frac{R_1}{R_1+R_2}\right)}}}$$

(after algebra)

② More insightful way to do this:

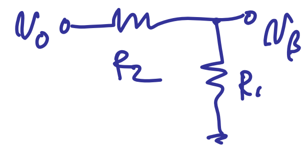


Neg. FB Block Diagram



$$v_\epsilon = v_i - v_\beta$$

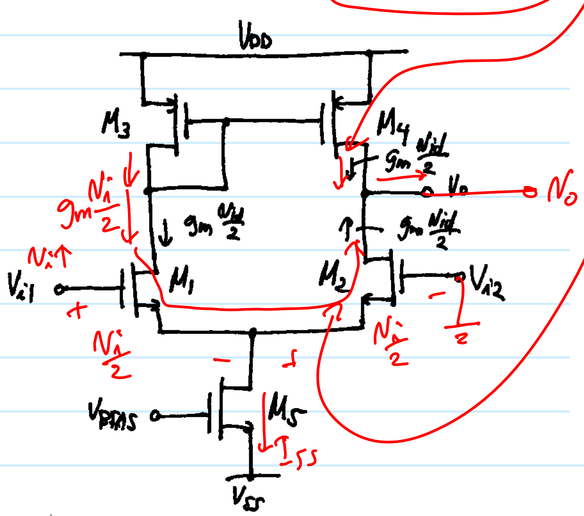
$$\text{ç } v_i = v_\epsilon + v_\beta$$



$$v_\beta = \frac{R_1}{R_1+R_2}v_o$$

$$\boxed{\beta = \frac{v_\beta}{v_o} = \frac{R_1}{R_1+R_2}}$$

MOS Differential Stage w/ Current Mirror Load

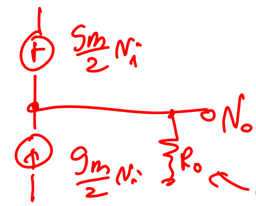


$$v_o = \left(\frac{g_m}{2} v_i + \frac{g_m}{2} v_i\right) R_o \Rightarrow \frac{v_o}{v_i} = g_m R_o$$

from bottom → $\frac{g_m}{2} v_i$; from top → $\frac{g_m}{2} v_i$; output R_o of the ckt. → R_o

$$= \boxed{g_m (r_{o2} \| r_{o4}) = \frac{v_o}{v_i}}$$

A_{dm}

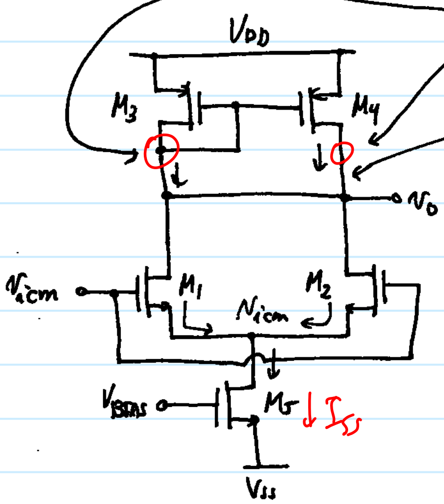


What is R_o?

↳ go to the main lecture page

↳ $R_o = (r_{o2} \| r_{o4})$ ... but not by inspection!

CMRR –

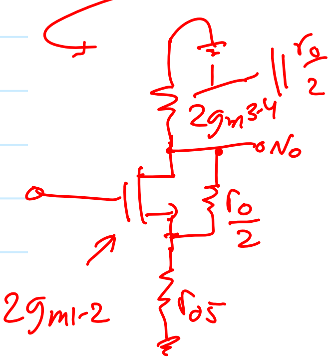


$V_{D3} = V_{D4}$ (they track each other) → for $M_1 = M_2$ & $M_3 = M_4$ → provided the ckt. is completely symmetrical.

⇒ Thus, can short the drains of M_3 & M_4

↳ truly symmetric → we can draw a half ckt.

{ or just combine the two ckts. together!



$$\boxed{A_{cm} = -\frac{2g_{m1\text{-}2}\left(\frac{1}{2g_{m3\text{-}4}}\right)}{1 + 2g_{m1\text{-}2} r_{o5}}}$$

(Get the same answer if use the CM Half Ckt.)

Thus:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \cancel{g_{m1\text{-}2}} (r_{o1\text{-}2} \| r_{o3\text{-}4})(1 + 2g_{m1\text{-}2} r_{o5})\left(\frac{g_{m3\text{-}4}}{\cancel{g_{m1\text{-}2}}}\right)$$

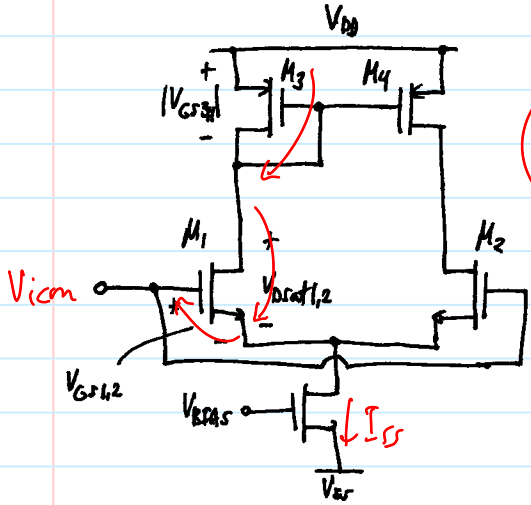
$$\boxed{CMRR = (1 + 2g_{m1\text{-}2} r_{o5})\, g_{m3\text{-}4} (r_{o1\text{-}2} \| r_{o3\text{-}4})}$$

Common-Mode Input Range – Range of input voltages for which all devices remain in saturation.

Low End – must keep M_5 saturated

$$V_{icm(min)} = CMR^- = V_{SS} + V_{ov5} + V_{GS1,2} = \boxed{V_{SS} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} (W/L)_5}} + V_{t1\text{-}2} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox} (W/L)_{1\text{-}2}}}}$$

High End - keep M_1, M_2 saturated



$$V_{icm(max)} = CMR+ = V_{DD} - |V_{GS3,4}| - V_{ov1,2} + V_{GS1,2}$$

$$V_{icm(max)} = V_{DD} - \sqrt{\frac{I_{SS}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4}}} - |V_{t3,4}| + V_{t1,2}$$

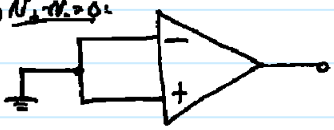
Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched

⇒ in actual ckt., get device mismatches due to processing variations

The Result:

① $v_+ - v_- = 0$ → Output not zero when input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!



$v_o = A_{dm}(v_+ - v_-)$

Ideal Case: $v_o = 0$

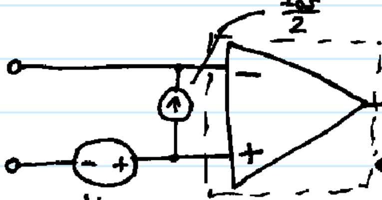
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model these effects, introduce:

① Input Offset Voltage, V_{OS} → Typ. 1-5 mV for BJT

② Input Offset Current, I_{OS} → Typ. I_{OS} = 10 nA for BJT



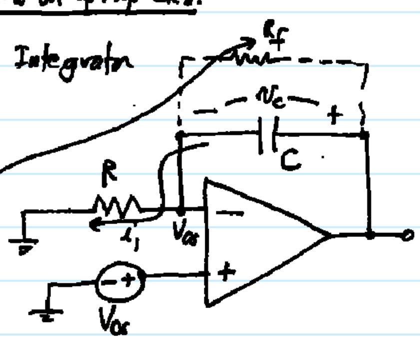
I_{OS} → $I_{OS} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{OS} → voltage needed to set $v_o = 0$ in this ckt. (negating the effect of mismatches)

Effect of V_{OS} on Op Amp Ckts. –

Example. Integrator



$$v_o = V_{OS} + \frac{1}{C}\int_0^t i_1 dt$$

$$= V_{OS} + \frac{1}{C}\int_0^t \frac{V_{OS}}{R} dt$$

$$= V_{OS}\left(1 + \frac{t}{RC}\right) + v_C\big|_{t=0}$$

→ will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C

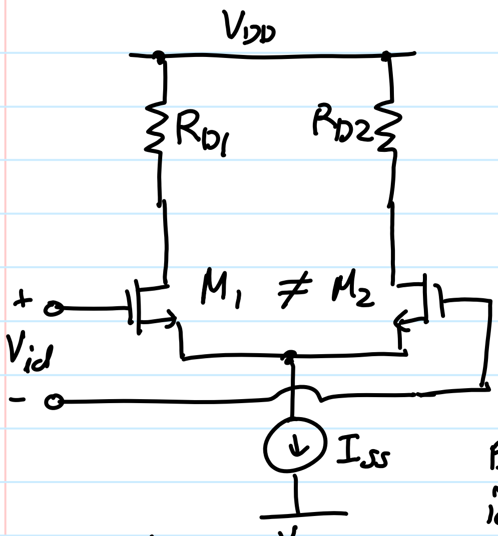
→ then $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen

→ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{OS} = small

V_{OS} is even more important in setting the resolution of A/D converters and other precision ckts.

V_{OS} of a Mismatched SCP

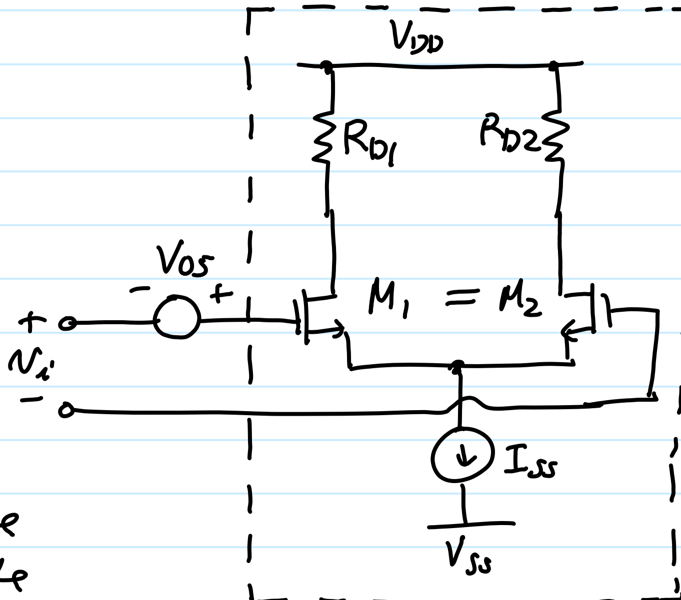
<u>Objective</u>: Derive an expression for V_{OS}.



Actual SCP w/ Mismatched Xsistors & R's

$\equiv$

Equate to an ideal SCP & use V_{OS} to model the effect of mismatches

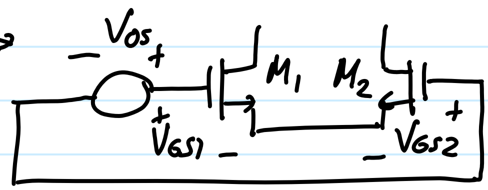


Ideal SCP w/ Matched M_1 & M_2 and $R_{D1} = R_{D2}$

<u>Input offset voltage V_{OS} arises due to variations in:</u>

1. Xsistors, M_1 & M_2 $\longrightarrow$ $\frac{W}{L}$ and V_t vary
2. $R_{D1} \neq R_{D2}$ $\longrightarrow$ causes gain variation

<u>Definition</u>. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.



<u>KVL</u>: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore\ V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \qquad (1)$$

<u>Define difference and average quantities:</u>

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$	$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$	$V_t = \frac{V_{t1} + V_{t2}}{2}$	$R_D = \frac{R_{D1} + R_{D2}}{2}$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{os} = \Delta V_t + \sqrt{\frac{\overbrace{2(I_D + \Delta I_D/2)}^{2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1+\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}} - \sqrt{\frac{\overbrace{2(I_D - \Delta I_D/2)}^{2I_D\left(1-\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1-\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0 \quad \therefore I_{D1}R_{D1} = I_{D2}R_{D2} \rightarrow$ mismatch in I_D must be opposite that of R_D

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch

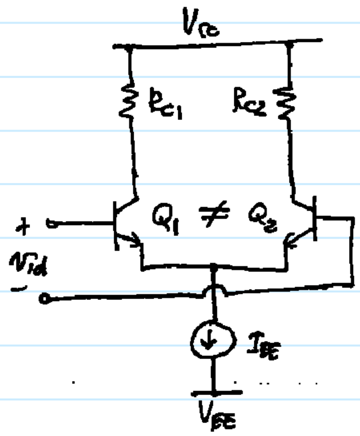
bias independent

Geometric (i.e., Layout) Variation

scale w/ overdrive

V_{OS} in a Mismatched ECP

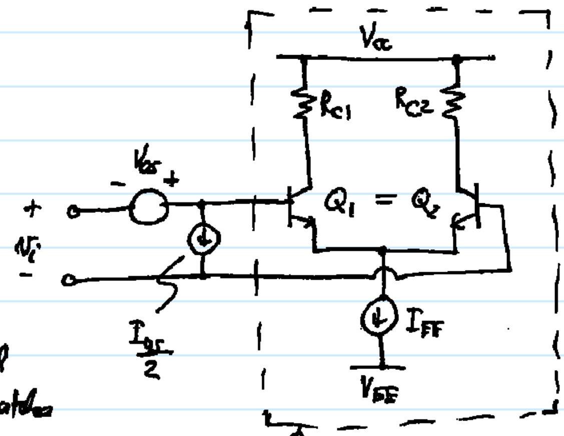
Objective: Derive an expression for V_{OS}.



Actual ECP w/ Mismatched Xsistors + R's

$\equiv$

Equate to an ideal ECP + use V_{OS} + I_{OS} to model the effect of mismatches



Ideal ECP w/ Matched Q_1 + Q_2 and $R_{C1} = R_{C2}$

Input Offset Voltage V_{OS} arises due to variations in:

① Xsistors, Q_1 + Q_2 $\longrightarrow$ I_s + β vary: $I_S = \dfrac{q n_i^2 \overline{D}_n A}{N_A W_B(V_{CB})}$ "function of" $\therefore$ $I_{S1} \neq I_{S2}$ can be caused by:

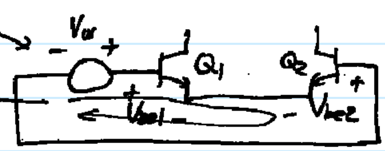
(i) $A_1 \neq A_2$ (etching tolerance limits)

(ii) $N_{A1} \neq N_{A2}$ (doping variations of base)

(iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)

② $R_{C1} \neq R_{C2}$ $\rightarrow$ causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$, ~~which occurs when~~ in this ckt.:



KVL: $V_{OS} - V_{be1} + V_{be2} = 0$

$$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = V_T \ln\left(\frac{I_{C1}}{I_{C2}} \frac{I_{S2}}{I_{S1}}\right)$$

Find $\frac{I_{C1}}{I_{C2}}$ in terms of design elements:

[When $V_{id} = V_{OS} \rightarrow V_{od} = 0V$] $\longrightarrow$ $V_{od} = (\cancel{V_{CC}} - I_{C1}R_{C1}) - (\cancel{V_{CC}} - I_{C2}R_{C2}) = 0$

$$I_{C1}R_{C1} = I_{C2}R_{C2} \longrightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}}$$

$$\boxed{V_{OS} = V_T \ln\left(\frac{R_{C2}}{R_{C1}} \frac{I_{S2}}{I_{S1}}\right)}$$

This is an exact equation for V_{OS}. It's often more useful + intuitive to express this in terms of percent variations (and eventually standard deviations).

Convert to Percent Variation Form –

Define. $R_c = \frac{R_{c1}+R_{c2}}{2}$, $\Delta R_c = R_{c1}-R_{c2}$; $I_s = \frac{I_{s1}+I_{s2}}{2}$, $\Delta I_s = I_{s1}-I_{s2}$

Objective: Express V_{os} in terms of percent variations $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$.

In general: $\Delta X = X_1 - X_2$, $X = \frac{X_1+X_2}{2}$ $\Rightarrow$ $X_1 = X + \frac{\Delta X}{2}$, $X_2 = X - \frac{\Delta X}{2}$ $\Rightarrow$ Thus: $R_{c1} = R_c + \frac{\Delta R_c}{2}$, $R_{c2} = R_c - \frac{\Delta R_c}{2}$; $I_{s1} = I_s + \frac{\Delta I_s}{2}$, $I_{s2} = I_s - \frac{\Delta I_s}{2}$

With these formulations:

$$V_{os} = V_T \ln\left[\frac{R_{c2}}{R_{c1}}\frac{I_{s2}}{I_{s1}}\right] = V_T \ln\left\{\frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \cdot \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}}\right\} = V_T \ln\left\{\frac{1-\frac{\Delta R_c}{2R_c}}{1+\frac{\Delta R_c}{2R_c}} \cdot \frac{1-\frac{\Delta I_s}{2I_s}}{1+\frac{\Delta I_s}{2I_s}}\right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right] \Rightarrow V_{os} \cong V_T\left\{-\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s}\right\}$$

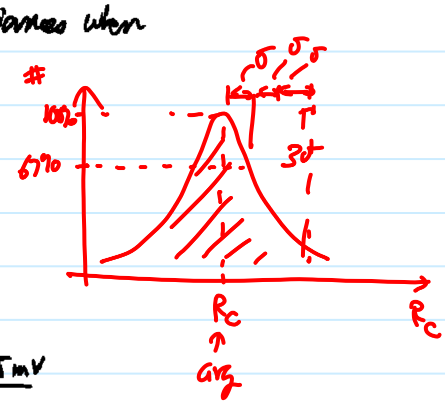
taking the first term assuming $\Delta R \ll R_c$ & $\Delta I_s \ll I_s$

$$\boxed{V_{os} = V_T\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically varying parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V_{os}:

→ since $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{os}} = V_T\sqrt{\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta I_s/I_s}}$$



Ex: Typ. $\sigma_{\Delta R/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$\therefore \sigma_{V_{os}} = (26m)\sqrt{(0.01)^2 + (0.05)^2} = \underline{1.3mV}$ Typ. V_{os} for BJT ~ $\underline{1\text{-}5mV}$

$\boxed{V_{os} \text{ Drift w/ Temperature}}$

$$\frac{dV_{os}}{dT} = \underbrace{\frac{kT}{q}\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}_{\text{indep. of } T}\frac{1}{T} = \frac{V_{os}}{T}$$

[In kelvin]

Ex. $\frac{dV_{os}}{dT} = \frac{1.3m}{300k} = 4.3\mu V/°C$ drift around $T = 300K$.

I_{os} in a Mismatched ECP

By Definition: $I_{os} = I_{B1} - I_{B2} = \boxed{\frac{I_{c1}}{\beta_1} - \frac{I_{c2}}{\beta_2} = I_{os}}$

To express in percent variations:

$$\begin{cases} I_{c1} = I_c + \frac{\Delta I_c}{2} \\ I_{c2} = I_c - \frac{\Delta I_c}{2} \end{cases} \qquad \begin{cases} \beta_1 = \beta + \frac{\Delta\beta}{2} \\ \beta_2 = \beta - \frac{\Delta\beta}{2} \end{cases}$$

$$\therefore I_{os} = \frac{I_c + \frac{\Delta I_c}{2}}{\beta + \frac{\Delta\beta}{2}} - \frac{I_c - \frac{\Delta I_c}{2}}{\beta - \frac{\Delta\beta}{2}} = \frac{I_c}{\beta}\left\{\frac{1 + \frac{\Delta I_c}{2I_c}}{1 + \frac{\Delta\beta}{2\beta}} - \frac{1 - \frac{\Delta I_c}{2I_c}}{1 - \frac{\Delta\beta}{2\beta}}\right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \ldots\right] \rightarrow \; = \frac{I_c}{\beta}\left\{\left(1 + \frac{\Delta I_c}{2I_c}\right)\left(1 - \frac{\Delta\beta}{2\beta}\right) - \left(1 - \frac{\Delta I_c}{2I_c}\right)\left(1 + \frac{\Delta\beta}{2\beta}\right)\right\}$$

$$= \frac{I_c}{\beta}\left\{\cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} - \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}} - \cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} + \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}}\right\}$$

$$\boxed{I_{os} = \frac{I_c}{\beta}\left\{\frac{\Delta I_c}{I_c} - \frac{\Delta\beta}{\beta}\right\}}$$

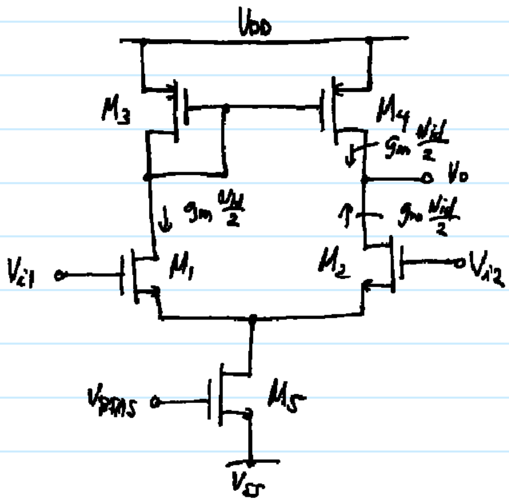
But for $V_{id} = 0V \Rightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_{c2}}{R_{c1}} \longrightarrow \frac{\Delta I_c}{I_c} = -\frac{\Delta R_c}{R_c}$

$$\therefore \boxed{I_{os} = -\frac{I_c}{\beta}\left(\frac{\Delta R_c}{R_c} + \frac{\Delta\beta}{\beta}\right)}$$

Ex. Typ: $\sigma_{\Delta\beta/\beta} = 0.1$, $\sigma_{\Delta R_c/R_c} = 0.01$

$$\longrightarrow I_{os} = -\frac{I_c}{\beta}\left[\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta\beta/\beta}\right]^{1/2} \cong -0.1\frac{I_c}{\beta} \approx -0.1 I_B = I_{os}$$

MOS Differential Stage w/ Current Mirror Load



all the same FB effects, etc...

Small Signal Gain: (similar to BJT)

$$\frac{v_o}{v_{id}} = g_{m2}(r_{o2} \| r_{o4}) = \frac{g_{m2}}{g_{ds2}+g_{ds4}} = \frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{SS}}}{\frac{I_{SS}}{2}(\lambda_2+\lambda_4)} \Rightarrow \boxed{\frac{v_o}{v_{id}} = \frac{2}{\lambda_2+\lambda_4}\sqrt{\frac{\mu_n C_{ox}(W/L)_2}{I_{SS}}}}$$

$$\left[\frac{\Delta(W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}}\right]$$

Offset Voltage – $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{od} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{t3,4}\left(\frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{(V_{GS}-V_t)_{1,2}}{2}\left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}}\right]$$

Via similar derivation to what we just did

For small V_{OS}: ① small $(V_{GS}-V_t)$

② $g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2}$ & $\left(\frac{W}{L}\right)_{3,4} < \left(\frac{W}{L}\right)_{1,2}$