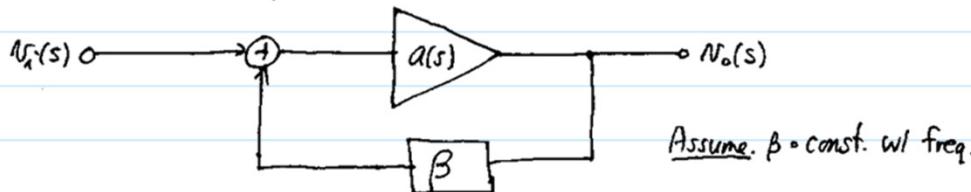


Obtain expressions for overshoot ($V_{\text{overshoot}}$) and settling time (T_s) as functions of phase margin, ϕ_m :



$$A(s) = \frac{N_o(s)}{N_i(s)} = \frac{a(s)}{1+a(s)\beta} \hat{=} \frac{a_0 w_1 w_2}{s^2 + (w_1 + w_2)s + w_1 w_2(1+a_0\beta)} = \frac{A_0 w_n^2}{s^2 + 2s w_n s + w_n^2} = \frac{A_0 w_0^2}{s^2 + \left(\frac{w_0}{\alpha}\right)s + w_0^2}$$

$$\left[A(s) = \frac{a_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = \frac{a_0 w_1 w_2}{(s+w_1)(s+w_2)} \right]$$

General Lourier Biquad Transfer Functions

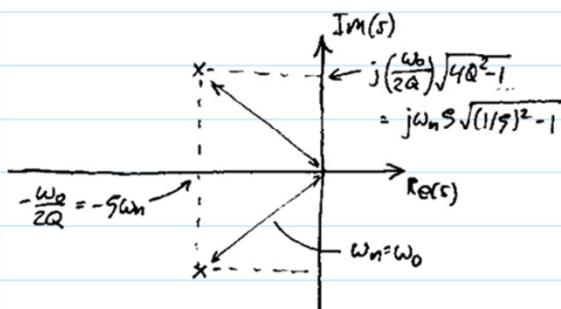
where by direct comparison:

Frequency Response spectra
for various values of s
on next page

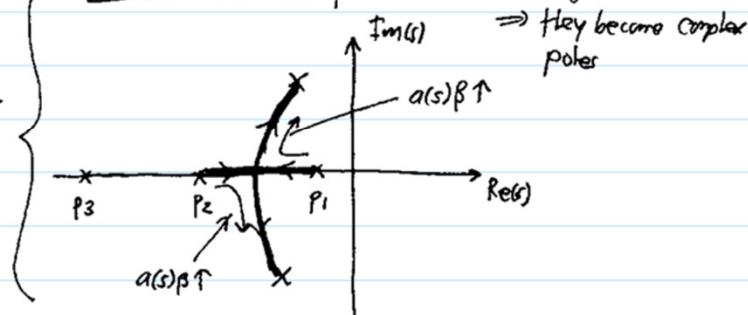
$$\left\{ \begin{array}{l} w_n = \sqrt{w_1 w_2 (1+a_0\beta)} \\ 2s w_n = w_1 + w_2 \rightarrow s = \frac{w_1 + w_2}{2w_n} = \frac{1}{2} \frac{w_1 + w_2}{\sqrt{w_1 w_2 (1+a_0\beta)}} \\ A_0 w_n^2 = a_0 w_1 w_2 \rightarrow A_0 = \frac{a_0}{1+a_0\beta} \end{array} \right.$$

Properties of the General Lourier Biquad Transfer Function (a very well-studied function!)

Pole-Zero Diagram -



Root Locus for FB CLF: poles move as the loop gain increases



Time Domain Behavior -

$$N_o(t) = A_0 V_i \left[1 - \frac{1}{\sqrt{1-s^2}} e^{-s w_n t} \sin(\sqrt{1-s^2} w_n t + \phi) \right], \text{ where } \phi = \tan^{-1} \left[\frac{\sqrt{1-s^2}}{s} \right]$$

For $s < 1$: (for $\text{PM} < 90^\circ$)

$$\% \text{ Overshoot} = \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = \exp \left[\frac{-\pi s}{\sqrt{1-s^2}} \right] \Rightarrow V_{\text{overshoot}} = V_0 \exp \left[\frac{-\pi s}{\sqrt{1-s^2}} \right]$$

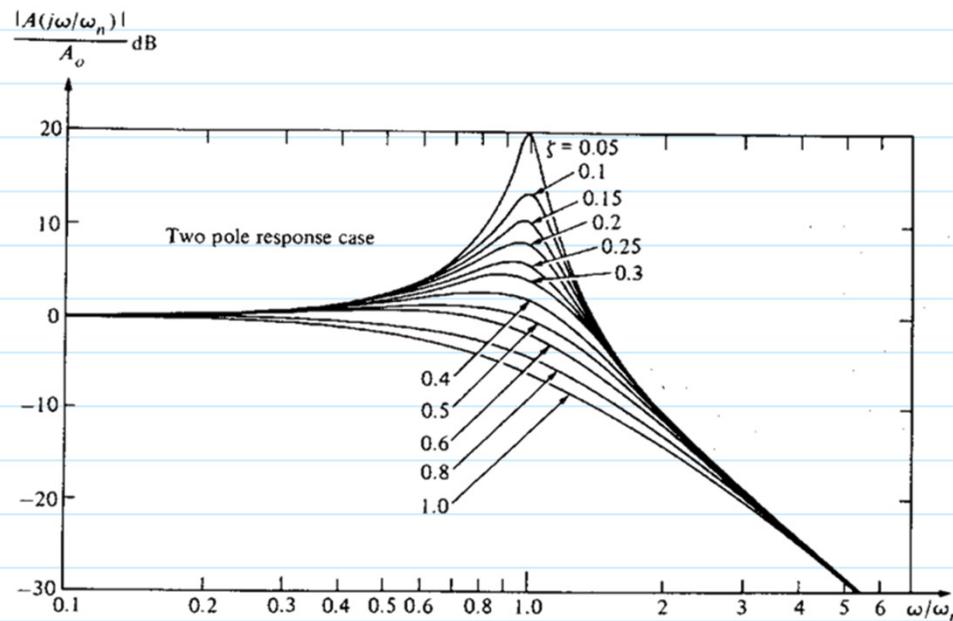
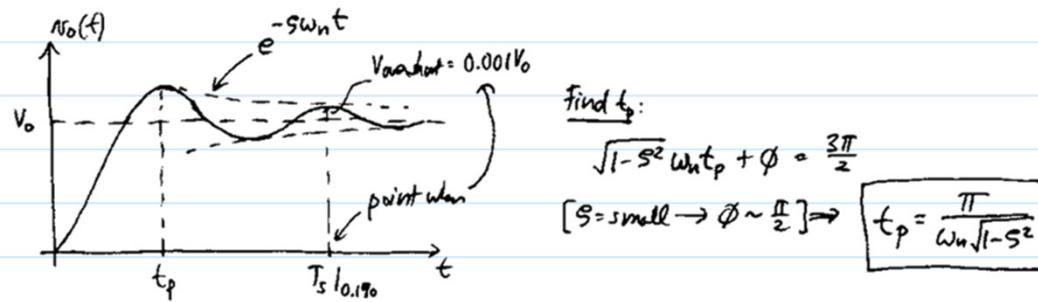


Figure C-2 Gain magnitude response for various values of ξ for a second-order, low-pass system.



Approximate Determination of 0.1% Settling Time -

$$\begin{aligned} V_{overshoot} e^{-s_w n (T_s - t_p)} &= 0.001V_0 \quad \rightarrow \quad T_s = t_p - \frac{1}{s_w n} \ln \left[\frac{0.001V_0}{V_{overshoot}} \right] \\ &\quad \curvearrowright S = \frac{1}{w_n (T_s - t_p)} \ln \left[\frac{0.001V_0}{V_{overshoot}} \right] \end{aligned}$$

Determine $\phi_m = f(S) \approx$

\Rightarrow first get an expression for loop transmission:

$$A(s) = \frac{a(s)}{1+a(s)\beta} \rightarrow a(s) = A(s) + a(s)A(s)\beta \rightarrow a(s) = \frac{A(s)}{1-\beta A(s)}$$

$$\Rightarrow a(s)\beta = \frac{\beta A(s)}{1-\beta A(s)} = \frac{\beta A_0 \omega_n^2}{S^2 + 2s_w n S + \omega_n^2 - \beta A_0 \omega_n^2} \stackrel{S \rightarrow 0}{=} \frac{\omega_n^2}{S^2 + 2s_w n S} = \frac{\omega_n^2}{S(S + 2s_w n)}$$

$\left[\beta \approx \frac{1}{A_0} \right]$ ↑ pole @ origin ↑ LHP pole

\Rightarrow get freq. at which $|a(j\omega)\beta| = 1 \rightarrow \omega_u$: Wurt

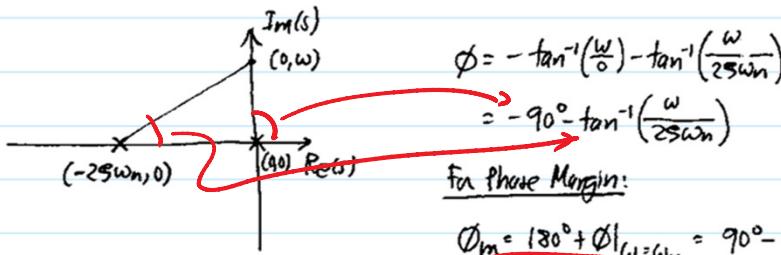
$$a(j\omega)\beta = \frac{\omega_n^2}{-\omega^2 + j25\omega_n\omega} \Rightarrow |a(j\omega)\beta| = \frac{\omega_n^2}{\sqrt{\omega^4 + \omega^2 45^2 \omega_n^2}}$$

$$[|a(j\omega_u)\beta| = 1] \Rightarrow \frac{\omega_n^4}{\omega_u^4 + 45^2 \omega_n^2 \omega_u^2} = 1 \rightarrow \underbrace{\omega_u^4 + 45^2 \omega_n^2 \omega_u^2 - \omega_n^2 = 0}$$

Solve quadratic: $\omega_u = \omega_n \sqrt{[45^2 + 1 - 25^2]^{1/2}}$

C Freq. for phase margin determination.

Find expression for phase:



$$\phi_m = 180^\circ + \phi|_{\omega=\omega_u} = 90^\circ - \tan^{-1}\left(\frac{\omega_u}{25\omega_n}\right)$$

$$\tan(90^\circ - \phi_m) = \frac{\omega_u}{25\omega_n}$$

$$[\tan(A+90^\circ) = -\frac{1}{\tan A}] \Rightarrow -\frac{1}{\tan(-\phi_m)} = \frac{\omega}{25\omega_n}$$

$$\Rightarrow \phi_m = \tan^{-1}\left(\frac{25\omega_n}{\omega_u}\right)$$

a $\phi_m = \cos^{-1}\left[\sqrt{45^2 + 1} - 25^2\right]$

Wurt

Thus:

$$S = \frac{1}{2} \frac{\omega_u}{\omega_n} \tan \phi_m$$

$$\Rightarrow T_s = t_p - \frac{2}{\omega_u \tan \phi_m} \ln \left[\frac{0.001 V_o}{V_{out,initial}} \right]$$

(ignoring effects of slew rate)

\Rightarrow But there's more to it than this. See the following papers:

- ① B.Y. Kamath, R.G. Moyer, and P.R. Gray, "Relationship between frequency response and settling time of operational amplifiers," IEEE J. of Solid-State Circ., vol. SC-9, no. 6, pp. 347-352, Dec. 1974.

- ② R.J. Apfel and P.R. Gray, "A fast-settling monolithic operational amplifier using doublet compression techniques," IEEE J. of Solid-State Circ., Vol. SC-9, no. 6, pp. 332-340, Dec. 1974.