Lecture 6: Frequency Response Inspection Analysis

- **Announcements**:
- This is our make-up lecture
- We will have lecture tomorrow (Tuesday), as well, at our regular time and place
- **Lecture Topics**:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
  - Frequency Response Examples
- ------------------------------------
- **Last Time**:

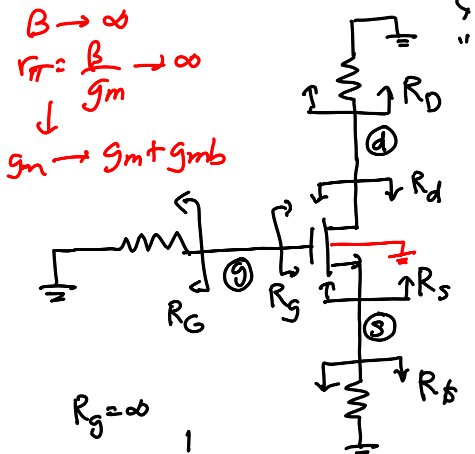
as well as tied to V_{DD}

$\boxed{\text{MOS Inspection Formulas w/ Substrate Grounded}}$

$\beta \to \infty$

$r_\pi = \frac{\beta}{g_m} \to \infty$

$g_m \to g_m + g_{mb}$



only difference from "substrate tied to source" case is that g_m is replaced by $g_m + g_{mb}$ in some of the formulas

particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m + g_{mb}}$

$R_d = r_o\left[1 + (g_m + g_{mb})R_S\right]$

over

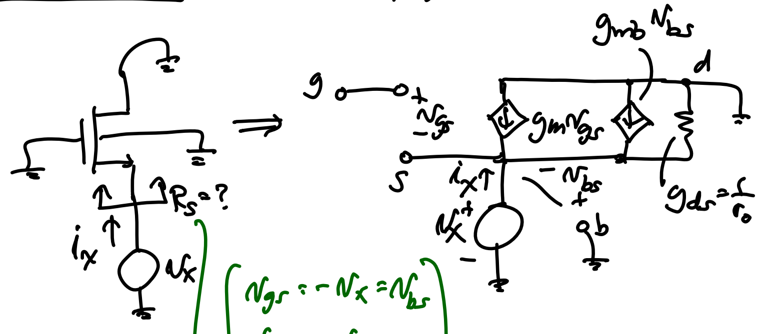
$\frac{v_d}{v_g} = -G_m R_{(d)}$, $G_m = \frac{g_m}{1 + (g_m + g_{mb})R_S}$

$\frac{v_d}{v_s} = -G_m R_{(d)}$, $G_m = -(g_m + g_{mb})$

$\frac{v_s}{v_g} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S}$

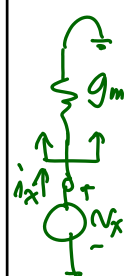
Remark. When the substrate is tied to the source, $g_{mb} = 0$.

$\boxed{\text{Effect of } g_{mb}}$ (one more example)



$v_{gs} = -v_x = v_{bs}$

$v_{ds} = -v_x$

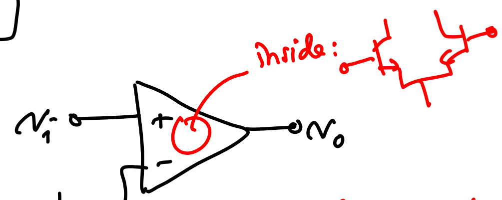


$g_m + g_{mb} + g_{ds} \to R_s = \frac{1}{g_m + g_{mb} + g_{ds}} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o$

$\boxed{R_s \approx \frac{1}{g_m} \| \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}}}$

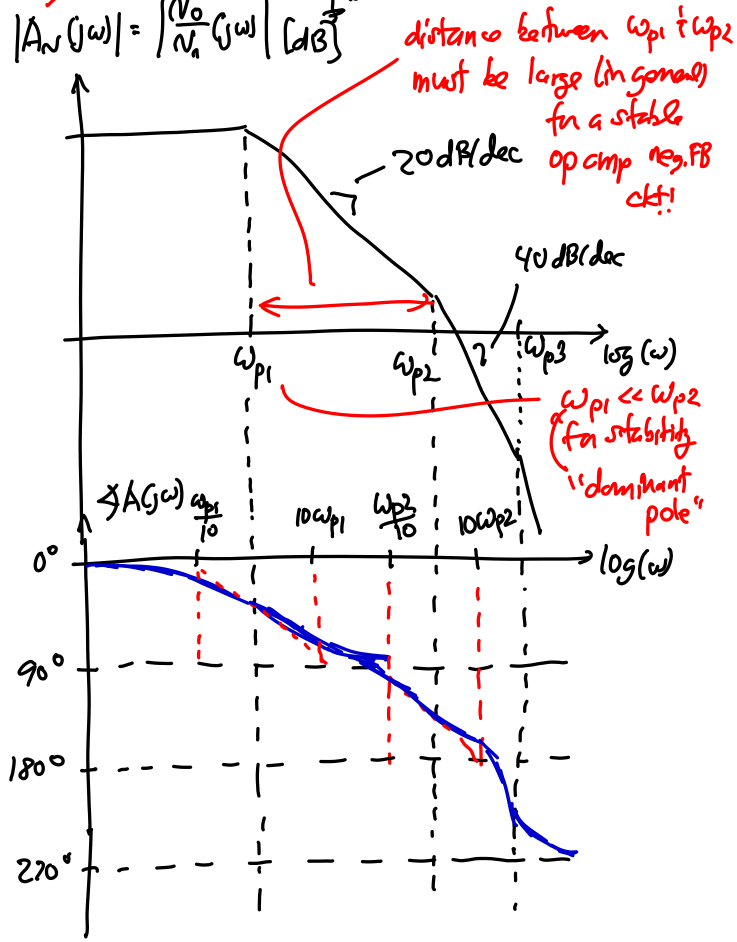
Frequency Response

Dominant Pole



$$|A_v(j\omega)| = \left|\frac{v_o}{v_i}(j\omega)\right| \ [dB]$$

$A(s)$

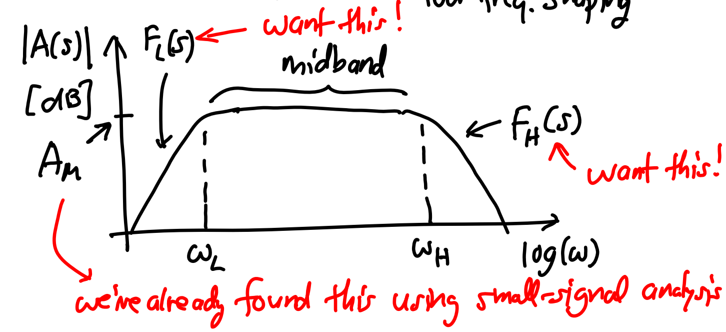


Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s)$$

midband gain — high frequency shaping — low freq. shaping



High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

#poles (n_p), # zeroes (n_z)

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$

from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

time constant

b_1: coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$ ← # poles

(ii) $F_H(s) \cong \dfrac{1}{1+\frac{s}{\omega_H}}$

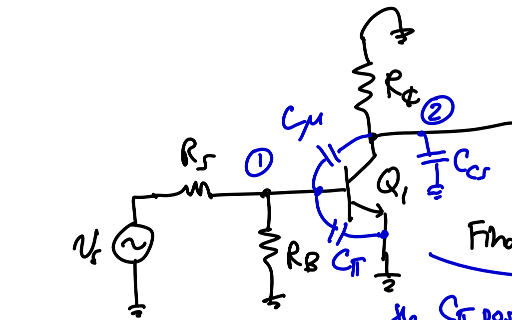
(iii) $b_1 \cong \dfrac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \dfrac{1}{b_1} = \dfrac{1}{\sum_j \tau_{jo}} = \dfrac{1}{\sum_j C_j R_{jo}}}$

When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}+\cdots-\frac{1}{\omega_{z1}^2}-\frac{1}{\omega_{z2}^2}-\cdots}}$$

(just FYI)

Example. H.F. Analysis of a C.E. Ckt.

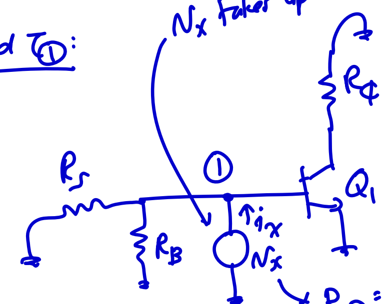


Find the cut-off freq., ω_H. ("3dB freq")

<1nF caps: C_π, C_μ, & C_{cs} all open-ckted

Find τ_1:

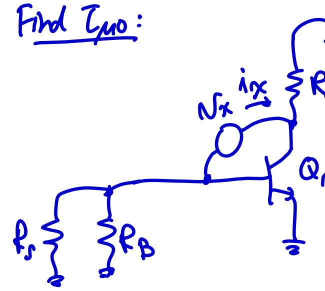
v_x takes up the C_π position.



$R_1 = \dfrac{v_x}{i_x} = r_\pi \| R_s \| R_B$

$C_1 = C_\pi \rightarrow \tau_1 = C_\pi(r_\pi \| R_s \| R_B)$

Find $\tau_{\mu o}$:



"O" means "in short"

$R_{\mu o} = \dfrac{v_x}{i_x} = R_1 + R_2 + g_m R_1 R_2$

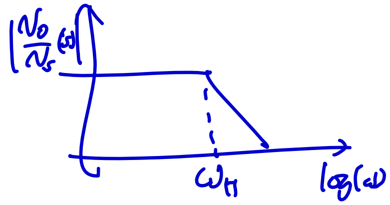
$R_{\mu o} = R_s \| R_B \| r_\pi + r_o \| R_C + g_m(R_s \| R_B \| r_\pi)(r_o \| R_C)$

$\tau_{\mu o} = R_{\mu o} C_\mu$

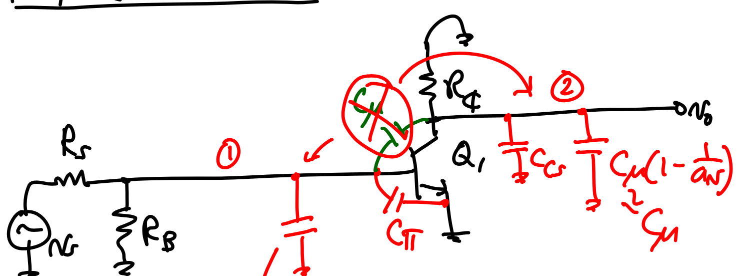
$G_m = \dfrac{g_m}{1+g_m R_E} = g_m$ (R_E = 0)

Find $\tau_{②}$: $\tau_{②} = (r_o \| R_C) C_{cs}$

$$\omega_H = \frac{1}{\tau_{①} + \tau_{\mu o} + \tau_{②}}$$



Now, use Miller's Theorem:



$C_\mu(1-a_v) = C_\mu(1+g_m R_C)$

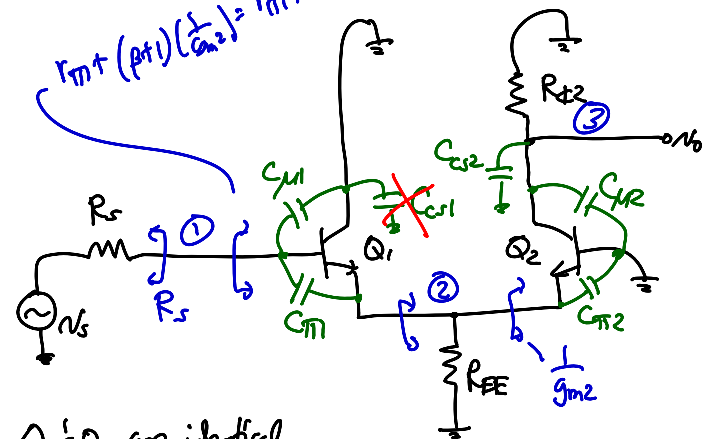
gain across Q (from base to collector)

$\tau_{①} = (r_\pi \| R_s \| R_B)(C_\pi + C_\mu(1+g_m R_C))$

$\tau_{②} = (R_C \| r_o)(C_\mu + C_s)$

$$\omega_H = \frac{1}{\tau_{①} + \tau_{②}}$$

Multi-Stage Ex.



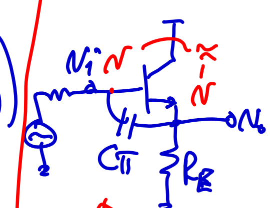
Q_1 & Q_2 are identical.

Find ω_H.

$\tau_{①} = (R_s \| (2r_\pi)) C_{\mu 1}$

$\tau_{\pi 1} = C_\pi \left(r_\pi \left\| \frac{R_s + 1/g_{m2}}{1 + g_{m1}\left(\frac{1}{g_{m2}}\right)} \right. \right)$

Aside,



if gain $\frac{v_o}{v_i} \simeq 1$, then can often neglect the $\tau_{\pi o}$

R_{EE} = large!

neg.

$\tau_{②} = C_{\pi 2}\left[\frac{1}{g_{m2}} \| R_{EE} \| \left(\frac{1}{g_{m1}} + \frac{R_s}{(\beta+1)} \right) \right]$

$\tau_{③} = (C_{\mu 2} + C_{cs2}) R_{C2}$

$$\omega_H = \frac{1}{\tau_{①} + \tau_{\pi 1} + \tau_{②} + \tau_{③}}$$

MOS Two-Stage Amplifier