

Device Mismatch Effect in Diff. Amplifiers

\Rightarrow up to this point, we've assumed that $Q_1 \neq Q_2$ are perfectly matched

⇒ in actual ctr., got device mismatch due to processing variations

The Results:

→ Output not zero when Input is zero → $N_{sd} \neq 0$ when $N_{sd} > 0$



$$N_o = A_{op} (N_+ - N_-)$$

Ideal Case: $N_0 = 0$

Reality: $N_0 \neq 0$, even w/ $(N_+ - N_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if $Q_1 \neq Q_2$ not matched. (fn BJT & JFET only.)

To model those effects, introduce:

① Input Offset Voltage, V_{IO}

② Input Offset Current, I_{IO}

Type: $I_{CQ} = 10 \text{ mA}$ for BJT

V_{BE} , 1-5 mV for BJT

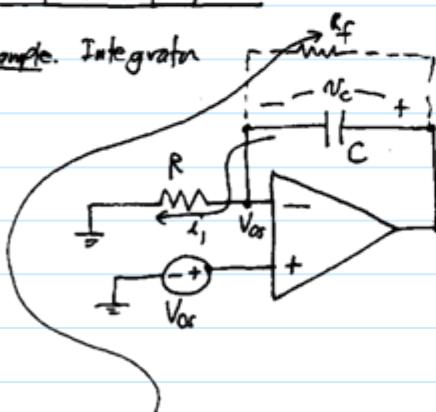
The diagram shows a differential operational amplifier circuit. The non-inverting input terminal (+) is connected to ground through a resistor. The inverting input terminal (-) is connected to the output terminal through a resistor and to a voltage source V_{os} through another resistor. The output terminal is connected to ground through a resistor. A feedback voltage source $\frac{I_{os}}{2}$ is connected between the inverting input terminal (-) and the output terminal.

(difference between
input currents)

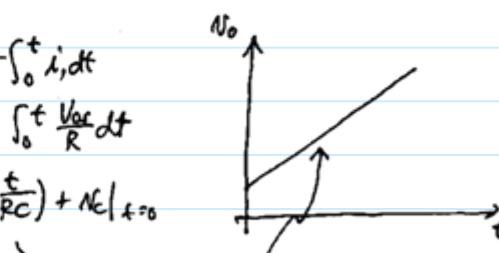
- $\Delta \mu \rightarrow 0$, it could have finite $A_{\nu}, P_1, \& P_0$!
- (negating the effect of mismatched)

Effect of V_{os} on Op Amp Ckt. -

Example. Integration



$$\begin{aligned} &= V_{0R} + \frac{1}{C} \int_0^t i_s dt \\ &= V_{0R} + \frac{1}{C} \int_0^t \frac{V_{0R}}{R} dt \\ &= V_{0R} \left(1 + \frac{t}{RC} \right) + V_{0R} \Big|_{t=0} \end{aligned}$$



Fix: Place an Rf in shunt w/ the C

→ then $V_{S_0} = V_{AS}(1 + \frac{R_f}{R})$, and roiling doesn't happen

\Rightarrow but, usually $R_F = \text{long}$ to allow the C to dominate

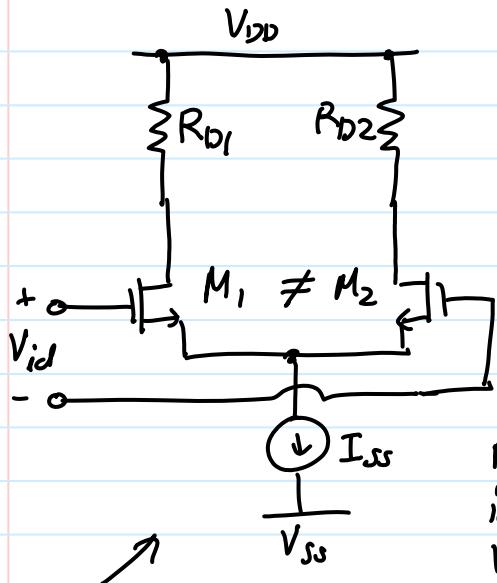
the integrand X for function $\therefore N_0 = V_{\text{eff}} \left(1 + \frac{P_E}{R} \right)$ can be quite large \Rightarrow still want $V_{\text{eff}} = \text{small}$

will continue to increase until
op amp hits the voltage rails

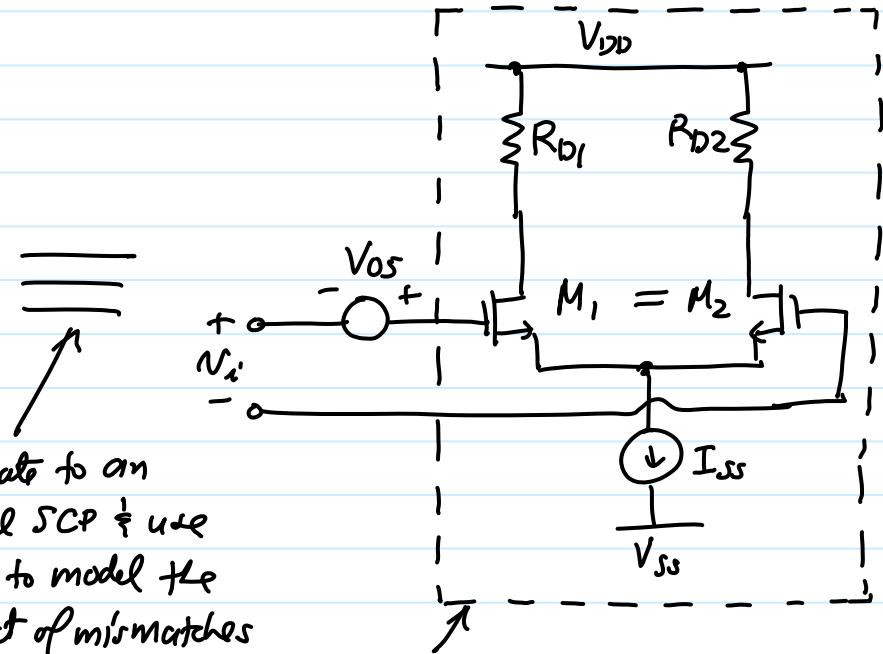
Vos is even more important in setting the resolution of ADC converters and other precision cbts.

V_{OS} of a Mismatched SCP

Objective: Derive an expression for V_{OS} .



Equate to an ideal SCP to use V_{OS} to model the effect of mismatches



Actual SCP w/ Mismatched Xsistors & R's

Ideal SCP w/ Matched $M_1 \neq M_2$ and $R_{D1} = R_{D2}$

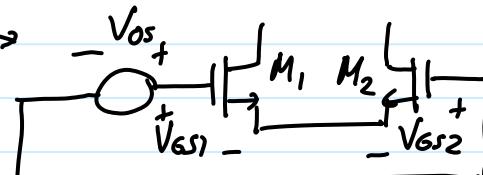
Input offset voltage V_{OS} arises due to variations in:

① Transistors, $M_1 \neq M_2 \rightarrow \frac{W}{L}$ and V_t vary

② $R_{D1} \neq R_{D2} \rightarrow$ causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.

KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$



$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$$\Delta I_D = I_{D1} - I_{D2}$$

$$I_D = \frac{I_{D1} + I_{D2}}{2}$$

$$\Delta \left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$$

$$\left(\frac{W}{L}\right) = \frac{1}{2} \left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2 \right]$$

$$\Delta V_t = V_{t1} - V_{t2}$$

$$V_t = \frac{V_{t1} + V_{t2}}{2}$$

$$\Delta R_D = R_{D1} - R_{D2}$$

$$R_D = \frac{R_{D1} + R_{D2}}{2}$$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2}$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2}$$

$$V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2}$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2}$$

$$V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$2I_D \left(1 + \frac{\Delta I_D}{2I_D}\right)$$

$$2I_D \left(1 - \frac{\Delta I_D}{2I_D}\right)$$

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox} \left[\left(\frac{W}{L}\right) + \frac{1}{2} \Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox} \left[\left(\frac{W}{L}\right) - \frac{1}{2} \Delta\left(\frac{W}{L}\right)\right]}}$$

$$\left[V_{GS}-V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox} (W/L)}}\right]$$

$$= \Delta V_t + (V_{GS}-V_t) \left\{ \sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} \right\}$$

→ Binomial Theorem:

$$(1+nx)^m \xrightarrow{n=\text{small}} 1 + mnx$$

$$V_{OS} = \Delta V_t + (V_{GS}-V_t) \left\{ \left(1 + \frac{1}{4} \frac{\Delta I_D}{I_D}\right) \left(1 - \frac{1}{4} \frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4} \frac{\Delta I_D}{I_D}\right) \left(1 + \frac{1}{4} \frac{\Delta(W/L)}{(W/L)}\right) \right\}$$

$$\cancel{1 + \frac{1}{4} \frac{\Delta I_D}{I_D}} - \cancel{\frac{1}{4} \frac{\Delta(W/L)}{(W/L)}} - \cancel{\frac{1}{16} \frac{\Delta I_D}{I_D} \frac{\Delta(W/L)}{(W/L)}} \cancel{1 + \frac{1}{4} \frac{\Delta I_D}{I_D}} - \cancel{\frac{1}{4} \frac{\Delta(W/L)}{(W/L)}} + \cancel{\frac{1}{16} \frac{\Delta I_D}{I_D} \frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS}-V_t) \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)} \right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS}-V_t) \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0 \therefore I_{D1}R_{D1} = I_{D2}R_{D2} \rightarrow$ mismatch in I_D must be opposite

$$V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS}-V_t) \left\{ -\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

that of R_D

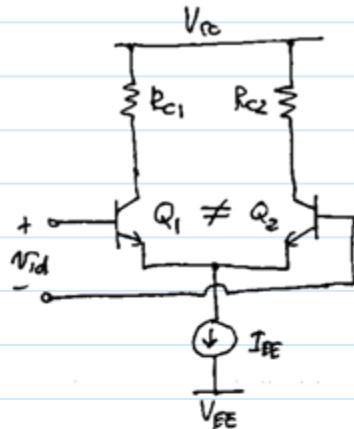
$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

Threshold
Mismatch
bias independent

Geometric (i.e., Layout)
Variation
→ scale w/l overdrive

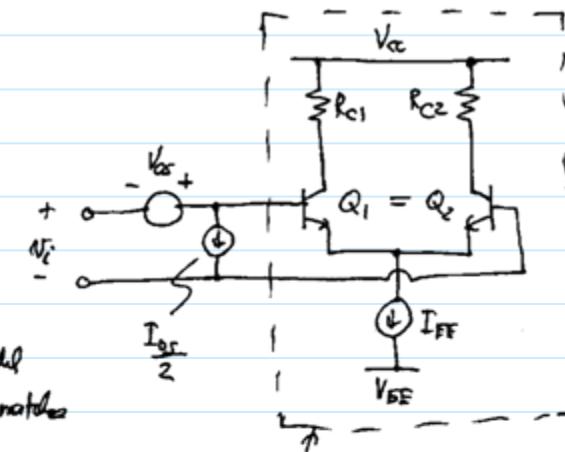
Var in a Mismatched ECP

Objective: Derive an expression for V_{OS}.



Actual ECP w/ Mismatched Xistors & R's

Equate to an ideal ECP + use Var + I_{os} to model the effect of mismatch



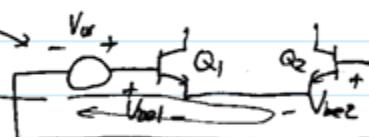
Ideal ECP w/ Mismatched Q₁ & Q₂ and R_{C1} = R_{C2}

Input Offset Voltage Var arises due to variations in:

- ① Xistors, Q₁ & Q₂ → I_s & β vary: $I_s = \frac{qN_A^2 D_n A}{N_A W_B(V_{CB})}$ ↗ "function of" $I_{s1} \neq I_{s2}$ can be caused by:
 (i) $A_1 \neq A_2$ (etching tolerance limits)
 (ii) $N_{A1} \neq N_{A2}$ (doping variations of base)
 (iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)
- ② R_{C1} ≠ R_{C2} → cause gain variation

Definition: $V_{OS} = V_{id}$ to get $V_{od} = 0$, which occurs when:

KVL: $V_{os} - V_{be1} + V_{be2} = 0$



$$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{c1}}{I_{s1}} - V_T \ln \frac{I_{c2}}{I_{s2}} = V_T \ln \left(\frac{I_{c1}}{I_{c2}} \frac{I_{s2}}{I_{s1}} \right)$$

Find $\frac{I_{c1}}{I_{c2}}$ in terms of design elements:

$$[\text{When } V_{id} > V_{OS} \rightarrow V_{od} > 0V] \rightarrow V_{od} = (V_{cc} - I_{c1}R_{c1}) - (V_{cc} - I_{c2}R_{c2}) = 0$$

$$I_{c1}R_{c1} = I_{c2}R_{c2} \rightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_{c2}}{R_{c1}}$$

$$V_{OS} = V_T \ln \left(\frac{R_{c2}}{R_{c1}} \frac{I_{s2}}{I_{s1}} \right)$$

This is an exact equation for Var. It's often more useful & intuitive to express this in terms of percent variations (and eventually standard deviations).

Convert to percent Variation Form -

$$\text{Define. } R_c = \frac{R_{c1} + R_{c2}}{2}, \Delta R_c = R_{c1} - R_{c2} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Objective: Express Var in terms of percent}$$

$$I_s = \frac{I_{s1} + I_{s2}}{2}, \Delta I_s = I_{s1} - I_{s2} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{variations } \frac{\Delta R_c}{R_c} \neq \frac{\Delta I_s}{I_s}.$$



$$\text{In general: } \left. \begin{array}{l} \Delta X = X_1 - X_2 \\ X = \frac{X_1 + X_2}{2} \end{array} \right\} \left. \begin{array}{l} X_1 = X + \frac{\Delta X}{2} \\ X_2 = X - \frac{\Delta X}{2} \end{array} \right\} \Rightarrow \text{Thus: } R_{c1} = R_c + \frac{\Delta R_c}{2}, R_{c2} = R_c - \frac{\Delta R_c}{2}$$

$$I_{s1} = I_s + \frac{\Delta I_s}{2}, I_{s2} = I_s - \frac{\Delta I_s}{2}$$

With these formulations:

$$V_{OS} = V_T \ln \left[\frac{R_{c2}}{R_{c1}} \frac{I_{s2}}{I_{s1}} \right] = V_T \ln \left\{ \frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}} \right\} = V_T \ln \left\{ \frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}} \frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}} \right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right] \Rightarrow V_{OS} \approx V_T \left\{ -\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s} \right\}$$

taking the first term assuming $\Delta R_c \ll R_c$ & $\Delta I_s \ll I_s$

$$V_{OS} = V_T \left\{ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right\}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically ^{varying} parameters for a

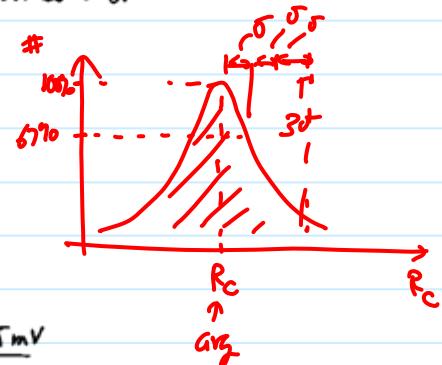
given process run & layout, one usually expresses terms in the form of variances when specifying V_{OS} :

→ since $\frac{\Delta R_c}{R_c} \neq \frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{OS}} = V_T \sqrt{\sigma_{\Delta R_c/R_c}^2 + \sigma_{\Delta I_s/I_s}^2}$$

Ex: Typ. $\sigma_{\Delta R_c/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$$\therefore \sigma_{V_{OS}} = (26m) \sqrt{(0.01)^2 + (0.05)^2} = 1.3mV \quad \text{Typ. Var fn BJT} \sim 1-5mV$$



V_{OS} Drift w/ Temperature

$$\frac{dV_{OS}}{dT} = \frac{kT}{q} \left\{ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right\} \frac{1}{T} = \frac{V_{os}}{T} \quad \underbrace{\text{indep. of } T}_{\text{in Kelvin}}$$

$$\text{Ex. } \frac{dV_{os}}{dT} = \frac{1.3m}{300k} = 4.3 \mu V/\text{°C} \text{ around } T=300\text{K.}$$

I_{OS} in a Mismatched ECP

$$\text{By Definition: } I_{OS} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2} = I_{OS}$$

To express in percent variations:

$$\begin{cases} I_{C1} = I_c + \frac{\Delta I_c}{2} \\ I_{C2} = I_c - \frac{\Delta I_c}{2} \end{cases}$$

$$\begin{cases} \beta_1 = \beta + \frac{\Delta \beta}{2} \\ \beta_2 = \beta - \frac{\Delta \beta}{2} \end{cases}$$

$$\therefore I_{OS} = \frac{I_c + \frac{\Delta I_c}{2}}{\beta + \frac{\Delta \beta}{2}} - \frac{I_c - \frac{\Delta I_c}{2}}{\beta - \frac{\Delta \beta}{2}} = \frac{I_c}{\beta} \left\{ \frac{1 + \frac{\Delta I_c}{2I_c}}{1 + \frac{\Delta \beta}{2\beta}} - \frac{1 - \frac{\Delta I_c}{2I_c}}{1 - \frac{\Delta \beta}{2\beta}} \right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \dots \right] \Rightarrow = \frac{I_c}{\beta} \left\{ \left(1 + \frac{\Delta I_c}{2I_c} \right) \left(1 - \frac{\Delta \beta}{2\beta} \right) - \left(1 - \frac{\Delta I_c}{2I_c} \right) \left(1 + \frac{\Delta \beta}{2\beta} \right) \right\}$$

$$= \frac{I_c}{\beta} \left\{ 1 + \frac{\Delta I_c}{2I_c} - \frac{\Delta \beta}{2\beta} - \frac{\Delta I_c \cancel{\Delta \beta}}{2I_c \cancel{2\beta}} - 1 + \frac{\Delta I_c}{2I_c} - \frac{\Delta \beta}{2\beta} + \frac{\Delta I_c \Delta \beta}{2I_c 2\beta} \right\}$$

$$I_{OS} = \frac{I_c}{\beta} \left\{ \frac{\Delta I_c}{I_c} - \frac{\Delta \beta}{\beta} \right\}$$

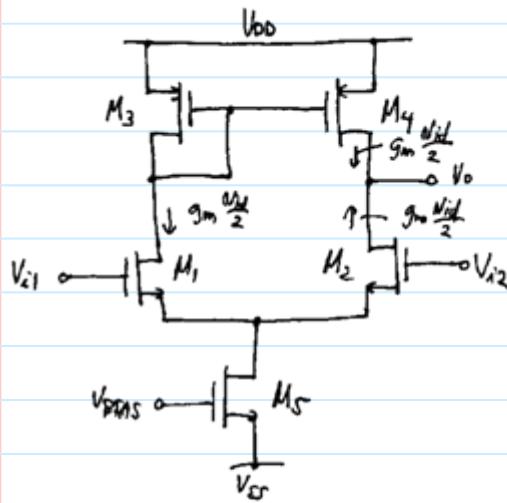
$$\text{But from } V_{od} = 0V \Rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{c2}}{R_{c1}} \rightarrow \frac{\Delta I_c}{I_c} = - \frac{\Delta R_c}{R_c}$$

$$\therefore I_{OS} = - \frac{I_c}{\beta} \left(\frac{\Delta R_c}{R_c} + \frac{\Delta \beta}{\beta} \right)$$

Ex: Typ: $\sigma_{\Delta R_c / R_c} = 0.1$, $\sigma_{\Delta \beta / \beta} = 0.01$

$$\Rightarrow I_{OS} = - \frac{I_c}{\beta} \left[\sigma_{\Delta R_c / R_c}^2 + \sigma_{\Delta \beta / \beta}^2 \right]^{\frac{1}{2}} \approx - 0.1 \frac{I_c}{\beta} \approx \boxed{-0.1 I_B > I_{OS}}$$

MOS Differential Stage w/ Current Mirror Load



→ all the same FET effects, etc...

Small-Signal Gain: (similar to BJT)

$$\frac{V_o}{V_{id}} = \frac{g_{m2}(r_{o2} || r_{o4})}{g_{dr2} + g_{dr4}} = \frac{g_{m2}}{g_{dr2} + g_{dr4}} = \frac{\sqrt{2\mu_n C_{ox} (\frac{W}{L})_2 I_{D2}}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{4\mu_n C_{ox} (\frac{W}{L})_2 I_{D2}}}{\frac{I_{SS}}{2} (\lambda_2 + \lambda_4)} \Rightarrow \frac{V_o}{V_{id}} = \frac{2}{\lambda_2 + \lambda_4} \sqrt{\frac{4\mu_n C_{ox} (\frac{W}{L})_2}{I_{SS}}}$$

$$\left[\frac{\Delta(W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}} \right]$$

Offset Voltage - $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{dd} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{t3,4} \left(\frac{g_{m3,4}}{g_{m1,2}} \right) + \frac{(V_{CS} - V_t)_{1,2}}{Z} \underbrace{\left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}} \right]}_{}$$

For small V_{OS} : ① small ($V_{CS} - V_t$)

$$\textcircled{2} \quad g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2} \quad \frac{1}{2} (\frac{W}{L})_{3,4} < (\frac{W}{L})_{1,2}$$