1. **Least Squares: A Toy Example**

Let's start off by solving a little example of least squares.

We're given the following system of equations:

$$\begin{bmatrix} 1 & 4 \\ 3 & 8 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \tag{1}$$

where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

(a) Why can we not solve for $\vec{x}$ exactly?

(b) Find $\vec{\hat{x}}$, the *least-squares estimate* of $\vec{x}$, using the formula we derived in lecture.

(c) Now, let's try to find $\vec{\hat{x}}$ in a different (geometric) way. How might you do it?

2. **Polynomial Fitting**

Least squares may seem rather boring at first glance – we're just using it to "solve" systems of linear equations, after all. But, at further glance, it actually comes in a variety of sizes and flavors! For instance, you can solve problems that have decidedly non-linear elements in them, using least squares. Let's see how.

Last discussion, we had seen how to "fit" data in the form of $(input = x, output = y)$ to a line. This made sense because the input-output relationship was fundamentally linear (Ohm's law).

But what if this relationship was not linear? For instance, the equation of the orbit of a planet around the sun is an ellipse. The equation for the trajectory of a projectile is a parabola. In these sorts of scenarios, how does one fit observation data to the correct curve?

In particular, say we *know* that the output, y, is a *quartic* polynomial in x. This means that we know that y and x are related as follows:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \tag{2}$$

We're also given the following observations:

x	y
0.0	24.0
0.5	6.61
1.0	0.0
1.5	-0.95
2.0	0.07
2.5	0.73
3.0	-0.12
3.5	-0.83
4.0	-0.04
4.5	6.42

(a) What are the unknowns in this question? What are we trying to solve for?

(b) Can you write an equation corresponding to the first observation (x_0, y_0), in terms of a_0, a_1, a_2, a_3 and a_4? What does this equation look like? Is it linear?

(c) Now, write a system of equations in terms of a_0, a_1, a_2, a_3 and a_4 using *all the observations*.

(d) Finally, solve for a_0, a_1, a_2, a_3, and a_4 using IPython. You have now found the quartic polynomial that best fits the data!

(e) We will now do another example in the IPython notebook, and see how to do polynomial fitting quickly using IPython!

3. Demonstration: Triangulation With Noise!