

1. Solving Systems of Equations

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following system of equations state whether or not a solution exists. If a solution exist list all of them.

i. $49x + 7y = 49$
 $42x + 6y = 42$

ii. $5x + 3y = -21$
 $2x + y = -9$

iii. $49x + 7y = 60$
 $42x + 6y = 30$

iv. $2x + 2y + 4z = -1$
 $y + z = -2$
 $x + 2y + 3z = 2$

v. $2x + 2y + 4z = 6$
 $y + z = 1$
 $x + 2y + 3z = 4$

vi. $x + y + z = 4$
 $3z = 6$
 $y + z = 3$

(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow the instructions in dis1.ipynb.

Answer all the questions in the IPython notebook.

2. Gaussian Elimination

Convert the following equations into an augmented matrix. Solve using Gaussian elimination.

(a)

$$\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases} \quad (1)$$

(b)

$$\begin{cases} x + y + z = 2 \\ x - y = 1 \\ y - z = 1 \end{cases} \quad (2)$$

Use Gaussian Elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} \quad (3)$$

(b)

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ 3 \end{bmatrix} \quad (4)$$

(c)

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \quad (5)$$

(d)

$$\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \quad (6)$$

3. Energy Disaggregation

Recently, energy companies like PG&E have put a lot of thought into a problem called *energy disaggregation*. The energy disaggregation problem is to take measurements of the total amount of electricity that a house uses and then try to determine which appliances are being used in the house. Energy companies want to do this because it allows them to better predict how much electricity they will need to produce on a given day and also so they can offer suggestions to their customers for how they can save energy.

To get an idea for how this works, suppose you live in a very simple house with just an air conditioning unit, a refrigerator, and a television that all use power measured in Watts. Suppose you want to figure out how much energy your TV and refrigerator use but the only measuring device you have is the meter on the outside of the house that measures the total power the house is using. You can turn off the TV at any time, but you don't want to unplug the refrigerator because you don't want the food to go bad. The air conditioner stays off in the morning but then turns on in the afternoon.

- Design a method for determining how much power each appliance uses. How many measurements will you need to make?
- Write the system of equations you would need to solve this problem in terms of the unknowns (the power of the air conditioner x_{AC} , the power of the refrigerator x_R , and the power of the TV, x_{TV}) as well as the measurements you make of total power (labeled T_1, T_2 , etc).