Discussion 1B

Reference Definitions

Inner Product Algebraic definition: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^N, \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \in \mathbb{R}^N : \langle \vec{x}, \vec{y} \rangle = \sum_{i=1}^{N} x_i \cdot y_i$.

Euclidean Norm The *Euclidean Norm* of a vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^N$ is $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_N^2}$

Vector Scaling Let $c \in \mathbb{R}$ and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^N$. Recall that $c \cdot \vec{x} = \begin{bmatrix} c \cdot x_1 \\ c \cdot x_2 \\ \vdots \\ c \cdot x_N \end{bmatrix}$.

1. **Investigating Inner Products** Now follow your TA as we discover some properties of inner products.

2. **Packings**

   (a) Can three vectors in the $\mathbb{R}^2$ plane have all pairwise inner-products be negative? That is, do there exist vectors $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^2$ such that $\langle \vec{u}, \vec{v} \rangle < 0$, $\langle \vec{v}, \vec{w} \rangle < 0$, and $\langle \vec{u}, \vec{w} \rangle < 0$?

   (Hint: Draw a picture!)

   (b) What about four vectors in $\mathbb{R}^2$? That is, do there exist four vectors $\vec{u}, \vec{v}, \vec{w}, \vec{x} \in \mathbb{R}^2$ such that for every pair of vectors $\vec{a}, \vec{b}$: $\left\langle \vec{a}, \vec{b} \right\rangle < 0$?

   (Bonus: What about four vectors in $\mathbb{R}^3$?)