EECS 16A Designing Information Devices and Systems I
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Discussion 4B

1. **Review**

2. **Mechanical Problems**

   In each part, find the eigenspace of M associated with the eigenvalue λ.

   (a) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 1$.

   (b) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 9$.

   (c) $M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $\lambda = 3$.

3. **Eigenvalues and Special Matrices - For Visualization** The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

   (a) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalue $\lambda \in \mathbb{R}$?

   (b) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (c) Does a projection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (d) If a matrix M has an eigenvalue 0, what does this say about its nullspace? What does this say about the solution(s) of the system of linear equations $M\vec{x} = \vec{b}$?

4. **Steady State Reservoir Levels** We have 3 reservoirs, A, B and C. The pumps system between the reservoirs is depicted in Figure 1.

   (a) Write the transition matrix representing the pumps system in the problem.

   (b) Assuming you start the pumps with water levels $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

   Hint: If $\vec{x_{ss}} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs A, B and C with A_{ss}, B_{ss} and C_{ss} kiloliters of water, respectively and apply the pumps once?

   Hint II: Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.

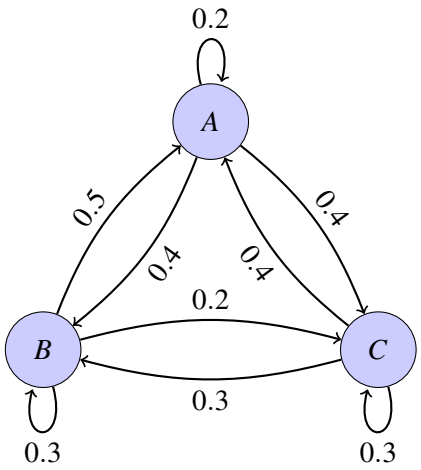

Figure 1: Reservoir pumps system