Discussion 5A

1. **Determinants and Characteristic Polynomial**

   (a) Compute the determinant of the 3×3 matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \tag{1}$$

2. **Characteristic Polynomial**

   (a) Find the characteristic polynomial and eigenvalues of A from (1).

   (b) Once you have found the eigenvalues of A, find the corresponding eigenvectors, $\vec{v}_i$, and demonstrate that they are eigenvectors by computing $A\vec{v}_i$.

3. **Geometry of Determinants** Interpret the following properties of determinants geometrically.

   (a) $\det(AB) = \det(A)\det(B)$.

   (b) For invertible matrices A: $\det\left(A^{-1}\right) = \frac{1}{\det(A)}$.

   (c) Let A be the (3×3) matrix:

$$A = \left[\begin{array}{c|cc} \alpha & 0 & 0 \\ \hline 0 & \multicolumn{2}{c}{\multirow{2}{*}{B}} \\ 0 & & \end{array}\right]$$

   For some (2×2) matrix B, and $\alpha \in \mathbb{R}$. Then: $\det(A) = \alpha \det(B)$.

4. **Number of Eigenvalues** Show, without using determinants, that an $(n \times n)$ matrix A can have at most n distinct eigenvalues.

   (Hint: What must be true about the eigenspaces corresponding to different eigenvalues?)