1. **Mechanical Problems**

   (a) Compute the determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

   (b) Compute the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

   (c) Compute the determinant of $\begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 \\ 0 & 0 & -31 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

2. **Row Operations and Determinants**

   In this question we explore the effect of row operations on the determinant of a matrix. Note that scaling a row by a will increase the determinant by a, and adding a multiple of one row to another will not change the determinant. Swapping two rows of a matrix and computing the determinant is equivalent to multiplying the determinant of the original matrix by -1. The determinant of an identity matrix is 1. Feel free to prove these properties to convince yourself that they hold for general square matrices.

   (a) An upper triangular matrix is a matrix with zero below its diagonal. For example a 3×3 upper triangular matrix is :

   $$\begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{bmatrix}$$

   By considering row-operations and what they do to a determinant, argue that the determinant of a general $n \times n$ upper-triangular matrix is the product of its diagonal entries, if they are non-zero. For example, the determinant of the 3×3 matrix above is $a_1 \times b_2 \times c_3$ if $a_1, b_2, c_3 \neq 0$.

   (b) If the diagonal of an upper-triangular matrix has a zero entry, argue that its determinant is still the product of its diagonal entries.

3. **Eigenvalues and Special Matrices - For Visualization** The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

   (a) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalue $\lambda \in \mathbb{R}$?

   (b) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (c) Does a projection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (d) If a matrix M has an eigenvalue 0, what does this say about its nullspace? What does this say about the solution(s) of the system of linear equations $M\vec{x} = \vec{b}$?

4. Gram-Schmidt Procedure and QR Factorization

(a) Compute the QR Factorization of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$