EECS 16A Designing Information Devices and Systems I
Fall 2016 Babak Ayazifar, Vladimir Stojanovic **Discussion 15A**

1. Diagonalization

One of the most powerful ways to think about matrices is to think of them in diagonal form [1].

(a) Consider a matrix A, a matrix V whose columns are the eigenvectors of A, and a diagonal matrix Λ with the eigenvalues of A on the diagonal (in the same order as the eigenvectors in the columns of V). From these definitions, show that

$$AV = V\Lambda \tag{1}$$

(b) We now multiply both sides on the right by V^{-1} and get $A = V\Lambda V^{-1}$, the diagonal form of A. Consider the action of A on a coordinate vector $\vec{x}_u$ in the standard basis. Interpret each step of the following calculation in terms of coordinate transformations and stretching by eigenvalues.

$$A\vec{x}_u = V\Lambda V^{-1}\vec{x}_u \tag{2}$$

[1] Not all matrices can be put in this form but most can. The ones that can't be diagonalized can be put in similar form called Jordan form.

2. **Spectral Mapping Theorem**

One of the most powerful things about matrix diagonalization is that it gives us insight into polynomial functions of matrices.

(a) Write A^N using the diagonalization of A and simplify as much as possible. What do you get?

(b) How could you raise A to any power while only doing three matrix multiplications.

(c) Can you suggest an easy way to compute any polynomial function of A?