

1. Lecture Recap 8/30/16

2. Gaussian Elimination

(a) Solve the following systems of equations without setting up an augmented matrix.

i.

$$\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases}$$

ii.

$$\begin{cases} x + y + z = 2 \\ x - y = 1 \\ y - z = 1 \end{cases}$$

(b) Use Gaussian Elimination to solve the following systems. Does a solution exist? Is it unique?

i.

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$$

ii.

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

iii.

$$\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

iv.

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

v.

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

3. Finding the bright cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance

emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoulish fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers. But they don't know any linear algebra - and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

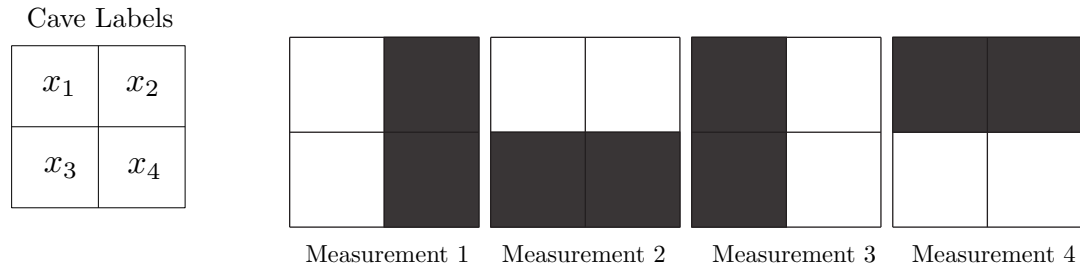


Figure 1: Four image masks.

- (a) Let \vec{x} be the four-element vector that represents the magnitude of light emanating from the four cave entrances. **Write a matrix K that performs the masking process in Figure 1 on the vector \vec{x} , such that $K\vec{x}$ is the result of the four measurements.**
- (b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?
- (c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?