1. Exploring Nullspaces

(a) The **column space** of a matrix is the **range** or possible outputs of a transformation/linear operation/-function. It is also the **span** of the vectors that form the columns of the matrix.

(b) The **nullspace** is the set of input vectors that output a zero vector

For the following five matrices, answer the following questions:

(a) What is the column span of A? What is its dimension?

(b) What is the nullspace of A? What is its dimension?

(c) (optional) Do the columns of A form a basis of $\mathbb{R}^2$? Why or why not?

(a) \[
\begin{bmatrix}
1 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
0 & 1 \\
0 & 1 \\
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 2 \\
-1 & 1 \\
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
-2 & 4 \\
3 & -6 \\
\end{bmatrix}
\]

(e) \[
\begin{bmatrix}
1 & 2 & 1 \\
-1 & 0 & 3 \\
0 & -1 & -2 \\
\end{bmatrix}
\]