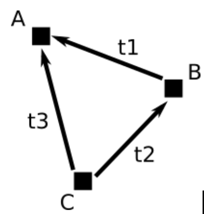
1. **Lecture Review: 9/20/16**

2. **Traffic Flows** Suppose your goal is to measure flow rates of vehicles along roads in Berkeley. However, the city's limited budget prohibits you from placing a traffic sensor along every road. Fortunately, you realize that there is a specific constraint that traffic must obey: the number of cars entering an intersection must equal the number of cars exiting the intersection. We'll see how this constraint helps us determine how many sensors you need for a given set of roads, and where we should place them.

   (a) Begin with a loop of road with three intersections, A, B, and C. t_1 cars flow from B to A per hour. t_2 cars flow from C to B per hour. And t_3 cars flow from C to A per hour.



[H!]

   Because we have determined that the number of cars in the network is conserved, the total number of cars per hour flowing into each node is zero. For example, at node B, $t_2 - t_1 = 0$. Let's write this constraint as a system of linear equations. We can represent the flows on each road as a vector $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$.

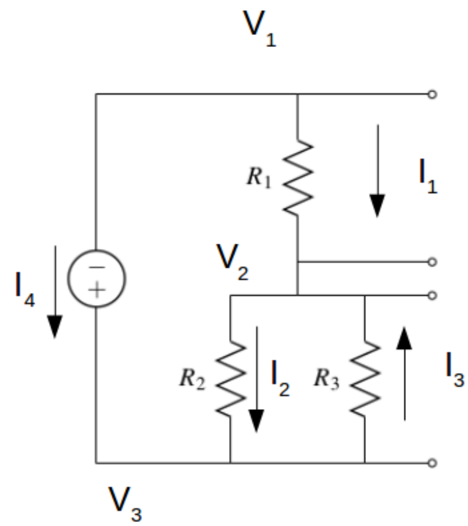
   Find the matrix G such that the equation

$$\begin{bmatrix} & G & \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

   represents the constraint that the sum of flows into each node is zero. This matrix is called the *incidence matrix*. What does each row of this matrix represent? What does each column of this matrix represent?

   (b) We can place sensors on a road to measure the flow through it. But, as we mentioned earlier, the budget is limited. Our goal is to figure out the minimum number of sensors needed to measure flow along every road.

   Suppose for the network above we have one sensor reading, $t_1 = 10$. Is it possible to calculate the flow rates t_2 and t_3?



3. "Traffic flows in circuits"

Although we have not yet covered much into circuits, circuits are actually a natural application of linear algebra and an extension of the ideas of traffic flows. In fact a circuit can be considered like a traffic flows, except with charge carriers as the "cars".

(a) Translate the above circuit into a directed graph, ignore the elements for now. Notice we can define the direction of flow for any element in any way we like. How many vertices are in the graph? How many edges?

(b) Write down the incidence matrix A of the graph. What are the do each of the rows this matrix represent? What do all the columns represent?

(c) What is the dimension of the column space of this matrix?

(d) Now let's consider A^T. Let $\vec{i}$ be a vector containing the flows through each element. What is the product $A^T\vec{i}$? What must this value be equal to? HINT: Consider the conservation laws.