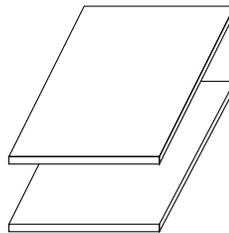


Touchscreen Revisited

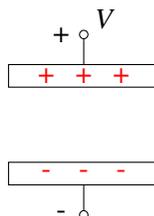
We've seen how a resistive touchscreen works by using the concept of voltage dividers. Essentially, for a resistive touchscreen, we map the value of analog voltage to the position touched. Another way to design a touchscreen is to break the screen down into a bunch of pixels. At each one of the pixels, we can detect whether the finger is touching it or not. This allows the touchscreen to sense multiple touch points at a time. In order to do this efficiently, we need a new element: **capacitors**. The beauty of the capacitor is that you don't have to bend anything like we did for the resistive touchscreen — the presence or absence of the finger can modify the capacitance directly!

Capacitor

A capacitor is two pieces of metal that are separated by some other material that is not in general conductive.



If there is voltage across the two pieces of conductors, charges will build up on the surface of the capacitor. As depicted below, when we apply voltage V across the two plates, positive charges build up on the (bottom) surface of the plate connected to the positive terminal and negative charges build up on the (top) surface of the plate connected to the negative terminal.



Capacitance represents how much charge can be stored for a given amount of voltage. We usually denote capacitance with C . Let's represent voltage across the capacitor as V and the charge stored on the capacitor

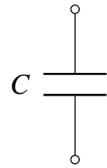
as Q . In mathematical form, we have the following definition

$$C \equiv \frac{Q}{V}, \quad (1)$$

This relation is often written as

$$Q = CV. \quad (2)$$

When we draw circuits, we usually use the two short parallel lines to represent a capacitor:

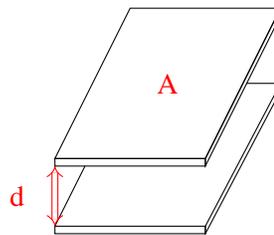


The unit of capacitance is **Farad (F)**. A capacitor with capacitance 1F is charged with 1C of charge when 1V of voltage is applied across it.

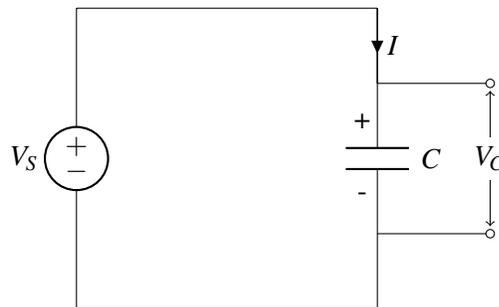
Let's try to get some intuition on capacitors by drawing an analogy between capacitors and water buckets — simply put, we can think of a capacitor as a bucket. The more water (charge) you put in the bucket, the higher the water level (voltage) would be. The water level in the bucket is determined by its dimensions. Similarly, the capacitance is set by the dimensions of the conductors and some properties of the material separating them. In particular, the capacitance of a capacitor is:

$$C = \epsilon \frac{A}{d} \quad (3)$$

where A is the area of the surface of the plates facing each other, d is the separation between the two plates (illustrated below), and ϵ is the "permittivity" of the material between the plates. We can deduce that ϵ has unit F/m . For example, if the material between the capacitors is air (vacuum), then $\epsilon = 8.85 \times 10^{-12} F/m$.



Let's see what happens when we connect a capacitor with a voltage source.



We know that initially, charges are going to build up on the two plates — positive charges on the top plate and negative charges on the bottom plate. Once enough charge has been stored, the voltage across the capacitor becomes $V_C = V_S$; as we will see next, in this next, the current I flowing through the circuit becomes zero.

To understand why $I = 0$ in this case, let's re-examine the defining relationship of a capacitor:

$$Q = CV_C. \quad (4)$$

Differentiating both sides with respect to t , we have

$$\frac{dQ}{dt} = \frac{dCV_C}{dt} = C \frac{dV_C}{dt}. \quad (5)$$

However, we know that current $I = \frac{dQ}{dt}$. Hence,

$$I = C \frac{dV_C}{dt}. \quad (6)$$

Thus, the current through a capacitor is just the product of the capacitance and the rate of change of the voltage across the capacitor. An important implication of this is in order for current to flow through the capacitor, the voltage of that capacitor can change with time.

When a capacitor is charged, there is voltage across it. What does this mean? This means that there is energy stored in the capacitor. Why is there energy stored? Recall that like charges repel each other. So for example if we take a positive charge and move it closer to another positive charge, we need to exert force and thus supply energy to do so. Let's now ask the question: when the capacitor is fully charged, how much energy is stored in it? We know from previous lectures that the energy required to store an additional dq amount of charge when the voltage across the capacitor is V_C is

$$dE = V_C dq. \quad (7)$$

We also know that $dq = CdV_C$. With this in mind, we have

$$dE = V_C (CdV_C) = CV_C dV_C. \quad (8)$$

Integrating both sides, we have

$$\int_0^E dE = \int_0^{V_S} CV_C dV_C = C \int_0^{V_S} V_C dV_C \quad (9)$$

$$E = \frac{1}{2} CV_S^2. \quad (10)$$

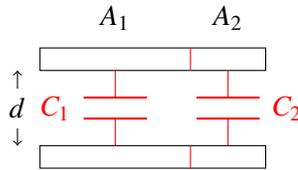
Hence, the energy stored in the capacitor after it's fully charged is $\frac{1}{2} CV_S^2$.

Capacitors in series and capacitors in parallel

Just like resistors, we can connect capacitors in series and in parallel. Let's take a look at what happens when we capacitors are connected in series and in parallel.

Capacitors in parallel

Suppose we combine two capacitors in parallel as follows



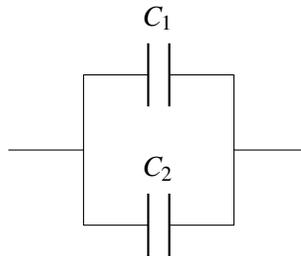
The capacitor on the left has capacitance $C_1 = \epsilon \frac{A_1}{d}$ and the capacitor on the right has capacitance $C_2 = \epsilon \frac{A_2}{d}$. Intuitively, the equivalent capacitance for capacitors in parallel would just be the sum of the capacitance of each of the capacitors since connecting them in parallel is analogous to summing the surface area of the two plates. If we look at the combined capacitor above, its capacitance is equal to

$$C_{eq} = \epsilon \frac{A_1 + A_2}{d}, \quad (11)$$

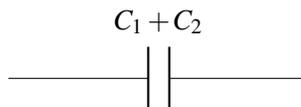
which is equal to the sum of the capacitance of the two capacitors

$$C_{eq} = \epsilon \frac{A_1 + A_2}{d} = \epsilon \frac{A_1}{d} + \epsilon \frac{A_2}{d} = C_1 + C_2, \quad (12)$$

In general, the equivalent capacitance of capacitors in parallel is the sum of their capacitance. Thus, the following circuit

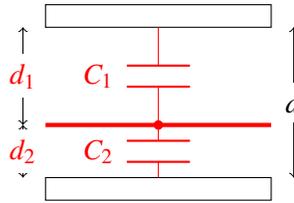


can be reduced to:



Capacitors in series

Now let's look at the case where capacitors are connected in series. If we take the following capacitor and draw a horizontal line in between, then essentially we can view it as two capacitors connected in series



Assume that the surface area is A . Intuitively, we know that the equivalent capacitance should be smaller than both C_1 and C_2 since the plates are farther from each other. We know that $C_1 = \epsilon \frac{A}{d_1}$ and $C_2 = \epsilon \frac{A}{d_2}$. Let the capacitance of the combined capacitor be C_{eq} . It satisfies

$$\frac{1}{C_{eq}} = \frac{1}{\epsilon} \frac{d_1 + d_2}{A} = \frac{1}{\epsilon} \frac{d_1}{A} + \frac{1}{\epsilon} \frac{d_2}{A} = \frac{1}{C_1} + \frac{1}{C_2}. \quad (13)$$

Hence, the equivalent capacitance is equal to

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}. \quad (14)$$

In general, circuits with capacitors in series



can thus be reduced to

