Design Process

When we build a system or device, we often want to systematically figure out how to achieve our goal. At a high level, the process involves the following steps:

- Step 1: Concretely state your goal for the system
- Step 2: Describe a strategy for achieving your goal
- Step 3: Implement the components within your strategy
- Step 4: Verify your design meets the original requirements
- Iterate through these more than once

We will apply the design method above to the development of a touchscreen. Now for simplicity, let’s only consider the one-dimensional case. Suppose we have a touchscreen as follows:

The touchscreen has two layers, the vertical blue wire and the black plane. The red arrow indicates the user and the user is only allowed to touch somewhere on the vertical blue wire. Looking closer, the portion of the touch screen we are concerned with is
When we press the screen, the above becomes

Let’s apply the design method to this problem.

• **Step 1: Concretely state your goal for the system**
  We would like to find out the vertical position at which the finger touches the screen.

• **Step 2: Describe a strategy for achieving your goal**
  We can’t directly measure the state of the finger. However, we will see that the finger touch can affect certain electrical quantity associated with the touchscreen. By measuring that electrical quantity and figuring out how it maps to what we originally want to detect, we can figure out what position on the screen the finger is touching.

\[
\text{Position } y \implies ? \implies \text{Measurement of electrical quantity}
\]

As depicted above, we need to figure out a way to relate what we’re looking for position \( y \) with the measurement of the electrical quantity.

• **Step 3: Implement the components within your strategy**
  It turns out what we can do is to connect the touch screen with a battery as follows
It turns out we can figure out the state $y$ if we measure the voltage $V_{ab}$ across point $a$ and $b$! However, before we can understand why, we need some basic knowledge of circuits. We will return to this at the end of the lecture.

Basic Circuits

The first thing we need to know in designing the touch screen is that applying voltage across a metal (or any conductive material) causes currents to flow through the material. Currents are charges moving in a particular direction. We can think of voltage as providing the energy to force the charges to move in a particular direction, just like we need to exert force on a box to push it. Let’s formally explain these basic circuit components below:

**Charge:** There are two types of charges, positive charge (+) and negative charge (−). We measure charges with the unit coulomb (C).

**Current:** Current is the net amount of charge crossing a surface in a unit time. We usually use the symbol $I$ to denote current. The unit for currents is ampere (A), which is equivalent to 1 coulomb per second. If $\Delta Q$ is the net charge crossing surface in $\Delta t$, then the current is

$$I = \frac{\Delta Q}{\Delta t}.$$  

We typically specify the direction of the net positive charge flow using an arrow. For example, the following arrow

\[ \text{5A} \]

represents that net +5 coulombs of charges are crossing the surface per second to the right. We could also say that net −5 coulombs of charges are crossing the surface per second to the left.

**Voltage:** Voltage is the amount of energy needed to move a unit charge between two points. We usually denote voltage with symbol $V$. It has unit volt (V), which is voltage difference between two points that will require 1 joule of energy to move 1 coulomb unit of charge between the points, i.e. $1V = 1J/C$. Voltage is a relative quantity defined between two points. An absolute voltage is meaningless and usually is implicitly referenced to a known point in the circuit (ground) or in some cases a point at infinity.

Kirchoff’s Current Law (KCL)

KCL states that the net charge flowing into any node of a circuit is identically zero. This follows from the conservation of charge – charge cannot vanish or appear at any node. For example, in the following circuit, the currents must satisfy

$$i_1 + i_2 - i_3 = 0.$$  

(1)

In addition, when current is travelling through elements in series, the currents must be equal.

$$i_A = i_B.$$  

(2)
Kirchoff’s Voltage Law (KVL)

Kirchoff’s Voltage Law states that the net potential around any loop in a circuit is zero.

\[ \sum_{\text{Loop}} V_k = 0. \]  \hspace{1cm} (3)

This follows from the conservation of energy – the net energy in going around a loop is zero. This is just like what you may have seen in physics – if you’re climbing a hill, the net potential energy you gain climbing up the hill and going back down the hill is zero because the amount of potential energy gained climbing up the hill is equal to the amount of energy lost when you go down the hill.

Let’s look at the example in figure 1. Adding the voltage "drops" around the loop, we have

\[ -V_A + V_B + V_C = 0. \]  \hspace{1cm} (4)

Note that if we had defined the loop in the opposite direction,

\[ V_A - V_C - V_B = 0. \]  \hspace{1cm} (5)

Resistors

Real conductors have "resistance", which impedes the flow of current by producing a voltage drop across the path of the current. The higher the current, the higher the voltage drop. Remarkably, the voltage drop is exactly proportional to the current. This is a statement of Ohm’s Law:

\[ V = IR. \]  \hspace{1cm} (6)
The proportionality constant is known as the resistance of the conductor. The unit of R is Voltage/Ampere, or more commonly "Ohms" (Ω). Such a conductor is known as a resistor. The value of resistance can be altered by using different materials in the construction of the component, or by modifying the physical dimensions of the conductor.

![Figure 2: Current flow into a resistor results in a voltage drop proportional to the current.](image)

For a wire of uniform cross area, the resistance is calculated as follows

\[ R = \frac{\rho l}{A}. \]

It’s proportional to the length \( l \) of the wire, inversely proportional to the cross sectional area \( A \), and proportional to the material resistivity \( \rho \). Intuitively, this makes sense because the longer the wire, the more impedance the current face flowing through, and the larger the cross sectional area, the easier it is for current to flow through.

Let’s look at the following example involving a voltage source and a resistor:

![Diagram](image)

Using Ohm’s law, we know that the voltage drop across the resistor on the right is equal to \( \Delta V = 5I \) and the voltage gain when going through the voltage source on the left is 10V. Using KVL, we have

\[ 10 - 5I = 0. \]

Hence, we have that the current going through the loop (in the specified direction) is \( I = 2A \) and the voltage drop across the resistor is \( 5\Omega \times 2A = 10V \).

**Passive sign convention**

Notice the + and − signs we draw across resistors. In electrical engineering, we often adopt the passive sign convention – positive current should go into the positive terminal of the resistor.

A circuit often consists of many resistors. The way the resistors are connected can often be categorized into the two types: (1)series, (2)parallel. Let’s get some intuition and perform analysis on each type below:
Resistors in Series

Now, let’s see what happens if we connect resistors in series.

Intuitively, the amount of impedance the current faces by going through the two resistors in series would just be the sum of the impedance faced by the current going through each individual of the resistor. Let’s analyze this rigorously.

We know by Ohm’s law that the voltage drop across $R_1$ is $IR_1$ and the voltage drop across $R_2$ is $IR_2$. Similarly, applying KVL on the loop, we have

$$V_S - IR_1 - IR_2 = 0 \quad (8)$$

which leads us to

$$V_S - I (R_1 + R_2) = V_S - IR_{eq} = 0 \quad (9)$$

where $R_{eq} = R_1 + R_2$. So essentially, we can think of the circuit as

where the two resistors in series are replaced by a resistor with resistance $R_{eq} = R_1 + R_2$. We say that the equivalent resistance for resistors in series is the sum of their resistance.
Resistors in Parallel

How about resistors in parallel?

Let’s think about this intuitively. Having resistors in parallel is like taking taking two boxes and combining them via their sides. This increases the cross-sectional area in the front, as illustrated below. So intuitively, the equivalent resistance of resistors in parallel would be smaller than the resistance of the original resistors. In addition, intuitively, more current would choose to go to the side with less resistance. We can show this rigorously.

We have the following circuits

First, let’s apply KCL on the circuit. We see that $I_1$ and $I_2$ branch off from $I$ and converge back together. We have

$$I = I_1 + I_2. \quad (10)$$

Now, applying KVL on the loop that contains $R_1$ and $V_S$, we have

$$V_S - I_1 R_1 = 0. \quad (11)$$
Applying KVL again on the loop that contains $R_2$ and $V_s$, we have

$$V_s - I_2R_2 = 0.$$  \hfill (12)

Now substituting $I_1 = \frac{V_s}{R_1}$ and $I_2 = \frac{V_s}{R_2}$ in the first equation, we have

$$I = I_1 + I_2 = \frac{V_s}{R_1} + \frac{V_s}{R_2} = V_s \left( \frac{1}{R_1} + \frac{1}{R_2} \right).$$  \hfill (13)

This is equivalent to

$$V_s = I \left( \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) = IR_{eq},$$  \hfill (14)

where we set $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$. Hence, the resistors in parallel circuit shown earlier is equivalent to

\[\text{Voltage Divider}\]

Notice that in the circuits we’ve seen so far, the voltage source always creates a set value of potential difference across itself. What if we need to a different potential difference across a device? We can achieve this using a voltage divider. Recall we have the resistors in series circuit

\[\text{Voltage Divider}\]

We figured out earlier that

$$V_s = I \left( R_1 + R_2 \right).$$  \hfill (15)

Hence, $I = \frac{V_s}{R_1 + R_2}$. So the voltage drop across $R_2$ is

$$V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_s.$$  \hfill (16)

Notice that we can create any output voltage of $V_{out} = \alpha V_s$ for any $\alpha \in [0, 1]$ by varying the resistance $R_1$ and $R_2$. 
Resistive Touchscreen Revisited

We now have the knowledge we need to design resistive touchscreen! Continuing with step (3) of our design process in the first section, we have the following design

Now that you’ve been introduce to some knowledge in circuits, what does the above look like to you? A voltage divider! Concretely, we can think of the thin piece of metal (blue line) as a resistor of resistance $R$ and length $l$. We let $l_{ab}$ denote the length of the metal between point (a) and (b), and $l_{bc}$ denote the length of the metal between point (b) and (c). First, we know that

\[ l_{ab} + l_{bc} = l. \]  

(17)

We can essentially model the above touch screen with the following circuit

We know that resistance is proportional to its length. Hence, we have

\[ \frac{R_{ab}}{R_{bc}} = \frac{l_{ab}}{l_{bc}}. \]  

(18)

From the voltage divider in the last section, we know that

\[ V_{bc} = \frac{R_{bc}}{R_{ab} + R_{bc}} V_S = \frac{l_{bc}}{l_{ab} + l_{bc}} V_S = \frac{l_{bc}}{l} V_S. \]  

(19)
Reorganizing the terms, we have that

\[ l_{bc} = \frac{V_{bc}}{V_S} l. \]  

(20)

Since we know our input voltage \( V_S \) and the length of the entire metal \( l \), we can completely figure out the state \( l_{bc} \) by measuring the output voltage \( v_{bc} \). \( l_{bc} \) would just be the position at which our finger touches the touchscreen, with the reference point being node \( c \) the bottom of the metal.

2D Resistive Touchscreen

In the last section, we illustrated how to figure out the one-dimensional position at which you touched on a resistive touchscreen. What if we would like to figure out the two-dimensional (vertical and horizontal) position at which you touched? It turns out we can use a similar technique that we’ve seen in the one-dimensional case – voltage dividers.

The touchscreen has two layers – the top layer is represented in red and the bottom layer is represented in blue. The thick strips are made out of highly conductive material and they are used to make sure that every point on the same strip has the same potential. Suppose we would like to measure the vertical position, we can connect the two terminals \( E_1 \) and \( E_2 \) with a voltage source as follows.
We represent the point you touched as the bold black dot. Additionally, let the height of the touchscreen be $h$, the length of height above the point you touched be $h_{\text{above}}$, and the length of height below the point you touched be $h_{\text{below}}$. (Hence we know that $h_{\text{above}} + h_{\text{below}} = h$). Let us specifically point out a few things:

- Everything in red belongs to the top plate and everything in blue belongs to the bottom plate.

- The positive terminal of the voltage source is connected to $E_1$ and the negative terminal of the voltage source is connected to $E_2$. Note that we apply the voltage across $E_1$ and $E_2$. The voltage source, $R_{\text{above}}$ and $R_{\text{below}}$ form a close loop. $R_{\text{right}}$ models the resistance in the bottom plate, and there should be no current going through $R_{\text{right}}$. Simplifying the figure above, the circuit in consideration is

Notice that this is just a voltage divider! We measure $V_{\text{out}}$, which is the voltage drop from $E_4$ (connected to the bottom plate) to the negative terminal of the voltage source on the left. So similarly, we can obtain the vertical position (with the reference point being the bottom of the plate) as follows

$$h_{\text{below}} = \frac{V_{\text{out}}}{V_S} h$$

To measure the horizontal position, we can do something similar – instead of connecting $E_1$ and $E_2$, we connect $E_3$ and $E_4$ with a voltage source.
Note for clarity, even though the blue plate is the bottom plate, we illustrated the entire blue plate in the figure above. The point you touched is represented as the bold black dot. Let the width of the touchscreen be \( w \), the length of width on the left of the point you touched be \( w_{\text{left}} \), and the length of width on the right you touched be \( w_{\text{right}} \). (Hence we know that \( w_{\text{left}} + w_{\text{right}} = w \)). We can simplify what we have above to

\[
\text{Notice this is just a voltage divider. If we measure the voltage } V_{\text{out}}, \text{ we can figure out the horizontal position (with the reference point being the very right of the plate) with}
\]

\[
w_{\text{right}} = \frac{V_{\text{out}}}{V_S} \cdot w
\]