For Reference: Circuits Cookbook, Abridged

**Voltage Divider**

\[ V_{R2} = V_S \left( \frac{R_2}{R_1 + R_2} \right) \]

**Voltage Summer**

\[ V_{out} = V_1 \left( \frac{R_2}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right) \]

**Unity Gain Buffer**

\[ \frac{v_{out}}{v_{in}} = 1 \]

**Inverting Amplifier**

\[ v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right) + V_{REF} \left( \frac{R_f}{R_s} + 1 \right) \]

**Non-inverting Amplifier**

\[ v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left( \frac{R_{top}}{R_{bottom}} \right) \]

**Transresistance Amplifier**

\[ v_{out} = i_{in}(-R) + V_{REF} \]
1. Noise Cancelling Headphones Part 2

The basic goal of noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

(a) In the previous discussion, we had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound more real to us. For similar reasons, having multiple microphones to pick up ambient sounds from multiple different locations can help us do a better job of cancellation, if we can use that information in the right way.

Let’s now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo – i.e., we have two different audio streams $s_{left}$ and $s_{right}$ (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. We have three microphone audio signals $s_{mic1}$, $s_{mic2}$, and $s_{mic3}$, and let’s assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example, $a_{1left}$ would represent the fraction of the signal picked up by microphone 1 that will be heard in the user’s left ear, $a_{2right}$ would represent the fraction of the signal picked up by microphone 2 that will be in the user’s right ear, etc.

Let the vector $\vec{s}_{noise}$ represent the noise heard in each ear and $\vec{s}_{mic}$ represent the sound in each mic. Find a matrix $A$ such that $\vec{s}_{noise} = A\vec{s}_{mic}$.

(b) Assume no noise canceling, find an equation for $\vec{s}_{ear}$, the sound heard in each ear in terms of the two audio streams and $\vec{s}_{noise}$.

(c) In order to cancel the noise, we want to create a signal that is the inverse of $\vec{s}_{noise}$. Let $\vec{s}_{cancel}$ be the vector representing the cancel signal in each headphone. Find a matrix $B$ in terms of the matrix $A$ such that $\vec{s}_{cancel} = B\vec{s}_{mic}$. 
(d) Assume that the microphones can be modeled as voltage sources, whose value $v_{micn}$ is proportional to $s_{micn}$, design and sketch a circuit that would implement the cancellation matrix $B$. You should assume that this circuit has three voltage inputs $v_{mic1}$, $v_{mic2}$, and $v_{mic3}$ and two voltage outputs $v_{cancel\_left}$ and $v_{cancel\_right}$ (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). In order to simplify the problem, you can assume that all of the $v_{mic}$ voltages are already centered at 0V (relative to the DAC ground). Furthermore, assume all entries of the $A$ matrix are positive. You may use op-amps and resistors to implement your circuit. You do not have to pick specific resistor values, but write expressions for each resistor value.

(e) **PRACTICE:** Building upon your solutions to all previous parts, and otherwise making the same assumptions about the relative voltage ranges of $v_{mic1}$, $v_{mic2}$, and $v_{mic3}$ and available supply voltages, sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.